

Macroeconomics

Section 1. (Suggested Time: 45 Minutes) *For 3 of the following 6 statements, state whether the statement is **true**, **false**, or **uncertain**, and give a complete and convincing explanation of your answer. **Note:** Such explanations typically appeal to specific macroeconomic models.*

1. Suppose a researcher finds that increases in government spending, when funded by deficits, lead to higher output and employment. Such a finding shows that counter-cyclical government spending is a beneficial policy.
2. An increase in the price of oil will only cause relative prices to change - not general inflation.
3. In recent months, GDP and productivity have risen, but employment has fallen. Such changes cannot be explained by a real business cycle model.
4. An increase in domestic private saving cannot be beneficial if it is simply soaked up by an increased government budget deficit.
5. Taxes on capital income are more distortionary than taxes on labor income.
6. President Obama has said that he intends to allow the “Bush tax cuts” that mainly benefit the rich to expire at the end of 2010. Allowing them to expire will worsen the current recession.

Section 2. (Suggested Time: 2 Hours, 15 minutes) Answer any 3 of the following 4 questions.

7. Diamond Coconut Economy with idiosyncratic preferences

Time: Discrete, infinite horizon

Geography: A trading island and a production island.

Demography: A mass of 1 of ex ante identical individuals with infinite lives.

Preferences: The common discount rate is r , consumption of own produce yields 0 utils, consumption of someone whose good you like yields $u > 0$ utils. The share of individuals whose goods I like is ϕ . Whether an individual likes my good or not is independent of whether I like hers. (So she likes my good with probability ϕ too.)

Productive Technology: On the production island individuals come across a tree with a coconut with probability α each period. The cost of obtaining the coconut is $c \sim F$. The distribution function $F(\cdot)$ is continuous over its support, $(0, \bar{c})$ where $\bar{c} > u$ (so some trees will get rejected).

Matching Technology: On the trading island people with coconuts meet each other with probability γ , a constant.

Navigation: Travel between islands is instantaneous.

Endowments: Everyone has a boat and starts off with one of their own coconuts.

- (a) Write down the asset value (Bellman) equations for this economy. Define the terms you introduce.
- (b) Define a search equilibrium.
- (c) Solve for an implicit equation that specifies the reservation tree “height”, c^* in terms of the parameters of the model.
- (d) How does c^* change with respect to ϕ (i.e. obtain the sign of the comparative static)?
- (e) Draw a diagram showing the population flows between the islands. Write down the steady-state equations and solve for the population on the trading island as a function of c^* .

8. Consider the following version of a stochastic growth model. There are a fixed number of price-taking producers that solve

$$\max_{L_t \geq 0} \Pi_t = Y_t - W_t L_t,$$

$$Y_t = Z_t L_t^{1-\alpha}, \quad 0 < \alpha < 1, \quad (\text{PRF})$$

where: Π_t is profit; L_t is labor; W_t is the real wage; Y_t is output; and Z_t measures the firm's productivity. The population and number of firms are normalized to 1, so that upper case letters denote intensive as well as aggregate quantities.

The preferences of the representative household over consumption, C_t , and labor are given by

$$E_0 \left(\sum_{t=0}^{\infty} \beta^t \left[\ln(C_t) - \chi \frac{1}{1+\gamma} L_t^{1+\gamma} \right] \right),$$

$$0 < \beta < 1, \quad \gamma > 0, \quad \chi > 0.$$

Households receive labor income and profits from firms. They pay lump-sum taxes, T_t , to the government. Households earn a gross return of $(1+r)$ on their assets, K_t , with $\beta(1+r) = 1$. As usual, assume that assets held at the beginning of period $t+1$, K_{t+1} , are chosen in period t . Note that capital is used only as a storage device, and not as a factor of production. Households face the usual initial, non-negativity and No-Ponzi-Game conditions.

The log of technology follows an AR(1) process

$$z_t \equiv \ln(Z_t) = \phi z_{t-1} + \varepsilon_t, \quad 0 \leq \phi < 1, \quad (\text{TS})$$

where the exogenous shock ε_t is i.i.d. and zero-mean.

Lump-sum taxes are set to balance the government's budget:

$$T_t = G_t, \quad (\text{BB})$$

where G_t is government spending, which has no effect on production or household utility. Government spending is in turn a function of productivity:

$$G_t = \xi Z_t^{-\nu}, \quad \xi, \nu > 0. \quad (\text{GS})$$

- Find the equilibrium conditions for this economy, namely: the labor allocation condition; the Euler equation; and the capital accumulation equation.
- Let lower-case letters with carats " $\hat{\cdot}$ " denote deviations of logged variables around their steady state values. Show that the log-linearized expressions for labor hours and output are:

$$\hat{\ell}_t = \theta (z_t - \hat{c}_t), \quad \theta > 0,$$

$$\hat{y}_t = (1 + \lambda) z_t - \lambda \hat{c}_t, \quad 0 < \lambda < \theta.$$

In addition, log-linearize equation (GS).

- (c) Suppose that the steady state consumption-to-capital ratio, C_{ss}/K_{ss} , is ψ , and that the steady state government spending-to-capital ratio, G_{ss}/K_{ss} , is ζ , with $(\psi + \zeta) > r$. It is then straightforward to show that the steady state output-to-capital ratio, Y_{ss}/K_{ss} , is $\psi + \zeta - r$. (Take this as given). Using this result, log-linearize the capital accumulation equation to show that

$$\widehat{k}_{t+1} = (1 + r)\widehat{k}_t + \omega_1 z_t - \omega_2 \widehat{c}_t, \quad \omega_1, \omega_2 > 0. \quad (\text{CA}')$$

- (d) One can log-linearize the Euler equation and solve the resulting system (which also includes (TS) and (CA')) to express consumption as a function of capital and productivity

$$\widehat{c}_t = \eta \widehat{k}_t + \mu z_t. \quad (\text{CF})$$

with

$$\eta = \frac{r}{\omega_2} > 0; \quad \mu = \frac{r\omega_1}{\omega_2(1 - \phi + r)} > 0.$$

(Take this as given.). Using this result, express labor and output as functions of capital (k) and productivity(z). Is labor increasing or decreasing in productivity? Is output increasing or decreasing in productivity? Can the two variables respond in opposite directions? Provide intuitive explanations.

- (e) Suppose that a researcher finds that times of high government spending are times of low employment. Should the researcher conclude that government spending increases unemployment? Using your findings from the preceding questions, briefly discuss.

9. Cash-in-advance in an endowment economy.

Consider the following economy:

Time: Discrete; infinite horizon.

Demography: Continuum of mass 1 of (representative) households that live for ever.

Preferences: The instantaneous household utility function over consumption, c , is $u(c)$. The function $u(\cdot)$ is twice differentiable, strictly increasing and strictly concave, with $\lim_{c \rightarrow 0} u'(c) = \infty$. The discount factor is $\beta \in (0, 1)$.

Endowments: In period t , each household receives a quantity e_t of the perishable consumption good (i.e. it is an endowment economy - no production). Each household has an initial stock, H_0 of money.

Institutions: Every period there are markets in which cash is traded for the consumption good.

There is a government that has the power to make transfers of cash, τ_t . (Transfers can be negative.) It increases the money supply at the rate σ so that the per household stock of cash at time t is given by

$$H_t = H_0 (1 + \sigma)^t.$$

There is a “cash-in-advance” (CIA) requirement. Households cannot directly consume their own endowment. Instead they have to sell e_t in period t to acquire cash. To consume in period t they have to use money they held at the end of period $t - 1$ to buy goods.

- (a) Write down the problem faced by a representative household at time 0. (Hint: they will have a budget as well as a CIA constraint.)
- (b) Rewrite the problem for the representative household in recursive form for period t .
- (c) Write down the necessary conditions for a solution to the household’s problem (including an envelope condition). (You do not need to solve for the Euler equation)
- (d) Write down the market clearing conditions and the government budget constraint.
- (e) Define a competitive equilibrium.
- (f) Could a social planner who is not subject to the CIA constraint do any better in this economy? Explain.
- (g) Suppose the cash in advance constraint **binds**. Define $\pi_t = \frac{p_{t+1}}{p_t}$ as the period t gross rate of inflation where p_t is the price of goods in terms of money. Derive an expression for π_t in terms of endowments and the rate of money growth. Interpret your answer.

10. Consider an economy with a single representative consumer. The consumer's preferences over consumption and leisure are:

$$E_t \left(\sum_{j=0}^{\infty} \beta^j \left[\ln(c_{t+j}) - \chi \frac{1}{1+\gamma} \ell_{t+j}^{1+\gamma} \right] \right),$$

$$0 < \beta < 1, \quad \gamma \geq 0, \quad \chi > 0.$$

The sole source of the single non-storable good is a representative farm that produces the good using labor and an everlasting tree. The farm's objective is to maximize each period's profits:

$$\max_{\ell_t \geq 0} \pi_t = y_t - w_t \ell_t,$$

$$y_t = \ell_t^{1-\alpha}, \quad 0 < \alpha < 1,$$

where: π_t is profit; y_t is the tree's "fruit" or output; ℓ_t is the labor input; and w_t is the real wage.

There is a government that levies a tax on labor income. The labor tax rate τ_t is a non-negative random variable governed by a Markov process with the stationary transition density $f(\tau', \tau)$. Tax revenues are refunded to consumers through the lump-sum transfer T_t . The representative household treats T_t as exogenous.

Consumers receive labor income and profits from farms. They pay taxes to and receive transfers from the government. At the beginning of time 0, each consumer owns one farm. Let $p_t = p(\tau_t)$ be the price at time t of a title to all future profits from a farm, and let $R_t^{-1} = R^{-1}(\tau_t)$ be the price of a risk-free discount bond that pays one unit of consumption at time $t + 1$.

- (a) Write down the consumer's problem in recursive form and find the first order conditions.
- (b) Continuing...
 - i Solving the farm's problem, find wages and profits.
 - ii Imposing clearing in the markets for goods and labor, find the equilibrium labor allocation, $\ell_t = \ell(\tau_t)$.
 - iii Find equilibrium output, $y_t = y(\tau_t)$.
- (c) Derive the equilibrium pricing function for stocks, $p_t = p(\tau_t)$. (This function should not include any expected future prices.)
- (d) Find $R^{-1}(\tau_t)$. Does $R^{-1}(\tau_t)$ depend on the conditional variance of τ_{t+1} ? Briefly explain.