Macroeconomics

Section 1. (Suggested Time: 45 Minutes) *For 3 of the following 6 statements*, state whether the statement is **true**, **false**, or **uncertain**, and give a complete and convincing explanation of your answer. **Note:** Such explanations typically appeal to specific macroeconomic models.

1. U.S. economic data show that the recent increase in the Federal government deficit has been accompanied by an increase in private sector saving. This is evidence in favor of Ricardian equivalence.
2. Firms operating constant returns to scale technologies cannot make positive profits.
3. Increasing the money supply is the most efficient way to pay for government expenditures.
4. Government policy should not be concerned with transitional paths—it is only what happens in steady-state that matters.
5. Increasing the money supply causes inflation.
6. Economies with high real interest rates are more likely to expand.

Section 2. (Suggested Time: 2 Hours, 15 minutes) Answer any 3 of the following 4 questions.

7. **Diamond overlapping generations with specific functional forms**

Time: discrete, infinite horizon

Demography: A mass $N_t \equiv N_0(1+n)^t$ of newborns enter in every period. Everyone lives for 2 periods except for the first generation of old people.

Preferences: For the generations born in and after period 0:

$$U_t(c_{1,t}, c_{2,t+1}) = \ln(c_{1,t}) + \beta \ln(c_{2,t+1})$$

where $c_{i,t}$ is consumption in period t and stage i of life and $\ln(.)$ is the natural logarithm function. For the initial old generation $\tilde{U}(c_{2,0}) = \ln(c_{2,0})$.

Productive technology: The production function available to firms is $F(K,N) = AK^{1/3}N^{2/3}$ where K is the capital stock and N is the number of workers employed. It will be convenient to use the implied per worker production function, $Ak^{1/3}$, where k is the capital stock per worker. Capital is fully used up in production.

Endowments: Everyone has one unit of labor services when young. (Old people cannot work.)

Institutions: There are competitive markets every period for labor and capital. (You can think of a single collectively owned firm that takes wages and interest rates as given.)

(a) Write out and solve the problems faced by generation t households for given prices w_t and R_t (the wage and rental rate of capital).

The household solves

$$\max_{s_{t+1}} \ln(w_t - s_{t+1}) + \beta \ln(R_{t+1}s_{t+1}).$$

FOC:

$$-\frac{1}{w_t - s_{t+1}} + \frac{\beta R_{t+1}}{R_{t+1}s_{t+1}} = 0,$$

which implies

$$s_{t+1} = \frac{\beta w_t}{1+\beta}.$$

(b) Write out and solve the problem faced by the firm in period t.

The firm solves

$$\max_{k_t} Ak_t^{1/3} - R_t k_t - w_t,$$

which implies

$$R_t = \frac{Ak_t^{-2/3}}{3} \qquad \text{and} \qquad w_t = \frac{2Ak_t^{1/3}}{3}.$$

(c) Write down the market clearing condition for capital, define a competitive equilibrium and solve for the implied law of motion for the per-young person stock of capital, k_t, in the economy.

Market clearing requires that $(1+n)k_t = s_t$.

Definition: A competitive equilibrium is an allocation $\{c_{1,t}, c_{2,t}, s_t, k_t\}$ and prices $\{R_t, w_t\}$ such that given prices the allocation solve the household and firm's problems and markets clear.

The law of motion for capital is

$$(1+n)k_{t+1} = \frac{2\beta A k_t^{1/3}}{3(1+\beta)}.$$

(d) Obtain values for each steady-state capital stock, k^*, in terms of the model's parameters. For each determine its dynamic properties (i.e. stability, oscillatory).

To obtain steady-states set $k_t = k^*$ for all t. We get

$$k^{*1/3}\left[(1+n)k^{*2/3} - \frac{2\beta A}{3(1+\beta)}\right] = 0.$$

So steady-states are $k_0^* = 0$ and $k_1^* = \left[\frac{2\beta A}{3(1+n)(1+\beta)}\right]^{3/2}$.

The stability properties of a steady state depend on the slope of the reaction function

$$\left.\frac{dk_{t+1}}{dk_t}\right|_{k_t=k^*} = \frac{2\beta A k^{*-2/3}}{9(1+n)(1+\beta)}.$$

At k_0^* this is clearly very large and positive. The steady-state is unstable monotone. At k_1^*, the slope is $1/3$. Clearly positive but less than 1. The steady-state is stable monotone.

(e) Under what conditions (on parameters) could there be over accumulation of capital?

We need to obtain the interest rate in the non-trivial steady state.

$$R^* = \frac{A k^{*-2/3}}{3} = \frac{(1+n)(1+\beta)}{2\beta}.$$

Over accumulation of capital occurs if $R^* < 1+n$. This will happen if $\beta > 1$.

8. (Inspired by Wang and Wen, 2008.) We are considering a version of the stochastic growth model with intermediate goods. Final output is given by

$$Y = \left(\int_0^1 Y(i)^{1/\mu} di \right)^{\mu}, \quad \mu > 1, \tag{FPRF}$$

while the production function for intermediate goods is given by

$$Y(i) = AL(i)^{\alpha}, \quad 0 < \alpha < 1, \tag{PRF}$$

where A is an aggregate shock.

(a) A final goods producer operates under perfect competition, solving

$$\max_{[Y(i)]_0^1} \left(\int_0^1 Y(i)^{1/\mu} di \right)^{\mu} - \int_0^1 P(i) Y(i) \, di.$$

The first order condition is

$$\mu \left(\int_0^1 Y(i)^{1/\mu} di \right)^{\mu-1} \frac{1}{\mu} Y(i)^{1/\mu - 1} = P(i).$$

Noting that $1/\mu - 1 = (1-\mu)/\mu$, this reduces to

$$P(i) = Y^{(\mu-1)/\mu} Y(i)^{(1-\mu)/\mu}. \tag{iD}$$

With constant returns to scale and perfect competition, there are no profits in the final goods sector. This can be shown more explicitly by inserting equation (iD), and imposing $\int_0^1 Y(i)^{1/\mu} di = Y^{1/\mu}$, in the expression for profits.

(b) Intermediate goods producers are price-setters. Letting W denote the real wage, producer i solves

$$\max_{L(i)} \Pi(i) = P_i [Y(i)] Y(i) - WL(i),$$

subject to equations (PRF) and (iD). Writing this as a Lagrangean, we have

$$\mathcal{L} = Y^{(\mu-1)/\mu} Y(i)^{1/\mu} - WL(i) + \lambda \left(AL(i)^{\alpha} - Y(i) \right).$$

Because the producer is one of infinitely many, it takes aggregate output as given. The first order conditions are

$$\begin{aligned} \frac{1}{\mu} Y^{(\mu-1)/\mu} Y(i)^{1/\mu - 1} &= \lambda, \\ \lambda \alpha AL(i)^{\alpha-1} &= \lambda \alpha Y(i) L(i)^{-1} = W, \end{aligned}$$

which can be combined to yield

$$\frac{\alpha}{\mu} Y^{(\mu-1)/\mu} Y(i)^{1/\mu} L(i)^{-1} = W. \tag{MRPL}$$

(c) In a symmetric equilibrium, where $Y(i) = Y(j)$, $\forall i, j$, it follows from equations (FPRF) and (PRF) that

$$Y = \left(\int_0^1 Y(j)^{1/\mu} di \right)^{\mu} = \left(Y(j)^{1/\mu} \right)^{\mu} = Y(j) = AL(j)^{\alpha}.$$

(Note the different indices!) Likewise

$$L = \int_0^1 L(i) di = \int_0^1 L(j) di = L(j),$$

so that

$$Y = AL^{\alpha}. \tag{APRF}$$

Inserting these results into equal (MRPL) yields

$$\frac{1}{\mu} \alpha \frac{Y}{L} = W. \tag{MRPL$'$}$$

(d) The consumer's problem can be written as a Lagrangean

$$\begin{aligned} \mathcal{L} \;\; = \;\; & \left\{ E_0 \sum\nolimits_{t=0}^{\infty} \beta^t \left[\ln(C_t) - \chi \frac{1}{1+\gamma} L_t^{1+\gamma} \right. \right. \\ & \left. \left. + \lambda_t \left((1 + r_{t-1}) B_t + W_t L_t + \int_0^1 \Pi_t(i) \, di - C_t - B_{t+1} \right) \right] \right\}. \end{aligned}$$

The first-order conditions are

$$\begin{aligned} \frac{1}{C_t} &= \lambda_t, \\ \lambda_t W_t &= \chi L_t^{\gamma}, \\ \lambda_t &= \beta E_t \left((1 + r_t) \lambda_{t+1} \right). \end{aligned}$$

Substituting for λ_t, we get

$$\frac{1}{C_t} = \beta (1 + r_t) E_t \left(\frac{1}{C_{t+1}} \right), \tag{EE}$$

$$W_t \frac{1}{C_t} = \chi L_t^{\gamma}. \tag{LL}$$

(e) The labor-leisure condition can be found by combining equations (MRPL$'$) and (LL):

$$\left(\frac{1}{\mu} \alpha \frac{Y_t}{L_t} \right) \frac{1}{C_t} = \chi L_t^{\gamma}. \tag{LL$'$}$$

The capital accumulation equation can be written as

$$Y_t = C_t, \tag{RC}$$

with Y_t following equation (PRF). This constraint can be derived directly as a resource constraint. Alternatively, one can impose the equilibrium allocation of $B_t = 0$ and note that in a symmetic equilibrium

$$\int_0^1 \Pi_t(i)\, di = Y_t - W_t L_t.$$

Inserting these results into the household's budget constraint yields (RC).

Combining equations (LL′) and (RC) shows that

$$L_t = \left(\frac{\alpha}{\chi}\right)^{1/(1+\gamma)} \mu_t^{-1/(1+\gamma)}.$$

Imposing equations (PRF) and (RC) then yields

$$C_t = \left(\frac{\alpha}{\chi}\right)^{\alpha/(1+\gamma)} A_t \mu_t^{-\alpha/(1+\gamma)}.$$

(f) The coefficient on μ_t is negative. As μ_t increases, the demand for each intermediate good becomes less elastic; equation (iD) shows that the demand curve steepens. This leads intermediate goods producers to impose higher markups, reducing the quantity of intermediate—and ultimately final—goods. The magnitude of the coefficient increases in α. Higher values of α make the supply of intermediate goods more elastic, allowing producers more flexibility to adjust their output when μ_t changes. The magnitude of the coefficent decreases in γ. As γ increases, the supply of labor becomes less elastic, so that changes in the intermediate goods producers' demand for labor generate smaller changes in equilibrium labor and output.

9. Consider the following variant of the Lucas tree model. The preferences of the representative consumer are

$$E_0\left(\sum_{t=0}^{\infty}\beta^t\frac{1}{1-\sigma}\left[C_t^{1-\sigma}-1\right]\right),\quad 0<\beta<1,\quad \sigma>0.$$

where C_t denotes consumption.

Output is produced by an infinite-lived tree: each period, the tree produces d_t units of non-storable output. The **growth** rate of dividends follows a stationary process, with

$$G_{t+1}\equiv\frac{d_{t+1}}{d_t}=\exp(\varepsilon_{t+1}+\theta\varepsilon_t),$$

where $\{\varepsilon_t\}$ is a zero-mean, normally-distributed, i.i.d. process. We will consider two possible values of θ: $\theta=1$, where the effects of growth shocks compound over time; and $\theta=-1$, where the effects of growth shocks are transitory.

(a) We begin by deriving the process for multi-period output growth $g_t(i,j)\equiv\ln(G_t(i,j))\equiv\ln(d_{t+j}/d_{t+i})$. Using the hint, we have

$$\begin{aligned}
g_t(0,1) &= \varepsilon_{t+1}+\theta\varepsilon_t,\\
g_t(0,2) &= g_t(0,1)+g_t(1,2)=\varepsilon_{t+2}+\theta\varepsilon_{t+1}+\varepsilon_{t+1}+\theta\varepsilon_t=\varepsilon_{t+2}+(1+\theta)\varepsilon_{t+1}+\theta\varepsilon_t,\\
g_t(0,3) &= g_t(0,2)+g_t(2,3)=\varepsilon_{t+3}+(1+\theta)\varepsilon_{t+2}+(1+\theta)\varepsilon_{t+1}+\theta\varepsilon_t,\\
g_t(0,j) &= \varepsilon_{t+j}+(1+\theta)\sum_{k=1}^{j-1}\varepsilon_{t+k}+\theta\varepsilon_t.
\end{aligned}$$

Extrapolating from the results for $i=0$, we see that

$$\begin{aligned}
g_t(i,i+1) &= \varepsilon_{t+i+1}+\theta\varepsilon_{t+i},\\
g_t(i,j) &= \varepsilon_{t+j}+(1+\theta)\sum_{k=i+1}^{j-1}\varepsilon_{t+k}+\theta\varepsilon_{t+i},\quad j>i+1.
\end{aligned}$$

Because $\{\varepsilon_t\}$ is i.i.d.,

$$var\left(g_t(0,j)\right)=\left[1+\theta^2+(j-1)(1+\theta)^2\right]\upsilon_\varepsilon^2,$$

where υ_ε^2 is the variance of ε. We see that $var\left(g_t(0,j)\right)$ is higher when $\theta=1$ than when $\theta=-1$.

(b) Writing the consumer's problem as a Lagrangean, we get

$$\begin{aligned}
V(x_t,d_t,\varepsilon_t)=\min_{\lambda_t\geq 0}\ \max_{c_t\geq 0,\ s_{t+1}}\ &\frac{C_t^{1-\sigma}-1}{1-\sigma}+\lambda_t\left(x_t-C_t-P_ts_{t+1}\right)\\
&+\beta E_t\Big(V\big([P_{t+1}(d_{t+1},\varepsilon_{t+1})+d_{t+1}]s_{t+1},\ d_{t+1},\varepsilon_{t+1}\big)\Big).
\end{aligned}$$

The FOC for an interior solution are:

$$\begin{aligned}
C_t^{-\sigma} &= \lambda_t,\\
\lambda_tP_t &= \beta E_t\left(\frac{\partial V\left[t+1\right]}{\partial x_{t+1}}\left[P_{t+1}\left(d_{t+1},\varepsilon_{t+1}\right)+d_{t+1}\right]\right).
\end{aligned}$$

Since (following Benveniste-Scheinkman),

$$\frac{\partial V[t]}{\partial x_t} = \lambda_t,$$

the Euler equation is

$$P_t C_t^{-\sigma} = \beta E_t \left(C_{t+1} \left(d_{t+1}, \varepsilon_{t+1} \right)^{-\sigma} \left[P_{t+1} \left(d_{t+1}, \varepsilon_{t+1} \right) + d_{t+1} \right] \right). \tag{EE}$$

(c) To achieve equilibrium,we impose $c_t = d_t$, $s_{t+1} = 1$, and $b_{t+1} = 0$. Since $0 < \beta < 1$, it makes sense to solve equation (EE) forward:

$$\begin{aligned} E_t \left(\left(1 - \beta L^{-1} \right) \left(P_t d_t^{-\sigma} \right) \right) &= \beta E_t \left(\frac{d_{t+1}}{d_{t+1}^{\sigma}} \right) = \beta E_t \left(d_{t+1}^{1-\sigma} \right) \\ P_t d_t^{-\sigma} &= \frac{1}{1 - \beta L^{-1}} \beta E_t \left(d_{t+1}^{1-\sigma} \right) + f_t \\ &= E_t \left(\sum_{j=1}^{\infty} \beta^j d_{t+j}^{1-\sigma} \right) + f_t, \end{aligned}$$

with the "bubble term" f_t obeying

$$E_t \left(f_{t+1} \right) = \beta^{-1} f_t.$$

We now require that

$$\lim_{J \to \infty} E_t \left(\beta^J P_{t+J} d_{t+J}^{-\sigma} \right) = 0, \quad \forall t. \tag{TVC}$$

Condition (TVC) will be satisfied only if $f_t = 0$ and the price of a stock is

$$P_t = d_t^{\sigma} E_t \left(\sum_{j=1}^{\infty} \beta^j d_{t+j}^{1-\sigma} \right).$$

Recall that

$$\begin{aligned} d_{t+j}^{1-\sigma} &= \left[d_t G_t(0,j) \right]^{1-\sigma} = \left[d_t G_t(0,1) G_t(1,j) \right]^{1-\sigma} = \left[d_t \exp(\varepsilon_{t+1} + \theta \varepsilon_t) G_t(1,j) \right]^{1-\sigma} \\ &= d_t^{1-\sigma} \exp((1-\sigma)\theta\varepsilon_t) \left[\exp(\varepsilon_{t+1}) G_t(1,j) \right]^{1-\sigma} \\ &= d_t^{1-\sigma} \exp((1-\sigma)\theta\varepsilon_t) \exp\left((1-\sigma)[\varepsilon_{t+1} + g_t(1,j)] \right). \end{aligned}$$

It follows that the price of a stock is

$$\begin{aligned} P_t &= d_t \exp((1-\sigma)\theta\varepsilon_t) E_t \left(\sum_{j=1}^{\infty} \beta^j \exp\left((1-\sigma)[\varepsilon_{t+1} + g_t(1,j)] \right) \right) \\ &\equiv d_t \exp((1-\sigma)\theta\varepsilon_t) \Lambda \\ &\equiv d_t p(\varepsilon_t). \end{aligned}$$

The parameter Λ is time invariant because because $\{\varepsilon_t\}$ is i.i.d.

(d) It can be shown that when x is distributed $\mathcal{N}(0,u)$, $E(\exp(x)) = \exp(u^2/2)$. Using this fact, we find that

$$\Lambda = \sum_{j=1}^{\infty} \beta^j \exp\left((1-\sigma)^2 var\left(\varepsilon_{t+1} + g_t(1,j)\right)/2\right).$$

It follows from our answer to part (a) that $var\left(\varepsilon_{t+1} + g_t(1,j)\right)$, and thus Λ, is higher when $\theta = 1$, rather than -1. Holding $d_t \exp((1-\sigma)\theta\varepsilon_t)$ fixed, P_t rises as well. This is akin to precautionary saving. As the variance of future consumption growth—and thus future consumption—rises, the expected marginal utility of consumption rises as well. This raises the value of an asset that delivers consumption in the future, increasing P_t.

10. **On-the-job search with a two-point wage distribution**

Time: Discrete, infinite horizon.

Demography: Single worker who lives for ever.

Preferences: The worker is risk-neutral (i.e. $u(x) = x$). He discounts the future at the rate r.

Endowments: When unemployed the worker receives income b per period. Also with probability α he gets an offer of employment. With probability $\phi > 0$ the offer is a low wage, w_L, and with probability $1-\phi$ the offer is a high wage, w_H, where $w_H \geq w_L \geq b$.

When employed the worker receives the wage, w_L or w_H, every period and continues to get wage offers of the same types and at the same frequency as when unemployed. When employed he also gets laid off (loses his job) with probability λ.

(a) Write down the flow or Bellman type asset equations for each of the states the worker can be in and briefly explain each one.

The full Bellman type equations are

$$\begin{aligned}
V_U &= \frac{1}{1+r}\{b + \alpha\phi \max(V_L, V_U) + \alpha(1-\phi)\max(V_H, V_U) + (1-\alpha)V_U\} \\
V_L &= \frac{1}{1+r}\{w_L + \alpha\phi \max(V_L, V_L) + \alpha(1-\phi)\max(V_H, V_L) + \lambda V_U + (1-\alpha-\lambda)V_L\} \\
V_H &= \frac{1}{1+r}\{w_H + \alpha\phi \max(V_H, V_L) + \alpha(1-\phi)\max(V_H, V_H) + \lambda V_U + (1-\alpha-\lambda)V_H\}
\end{aligned}$$

Note that being laid-off and receiving a new job offer are mutually exclusive events. Assuming the worker prefers high wage employment to low wage employment and prefers low wage employment to unemployment the flow equations are:

$$\begin{aligned}
rV_U &= b + \alpha\phi(V_L - V_U) + \alpha(1-\phi)(V_H - V_U), && (1) \\
rV_L &= w_L + \alpha(1-\phi)(V_H - V_L) + \lambda(V_U - V_L), && (2) \\
rV_H &= w_H + \lambda(V_U - V_H). && (3)
\end{aligned}$$

The first equation reflects the fact that the unemployed worker gets b regardless. With probability $\alpha\phi$ she switches (next period) to employment at a low wage firm and with probability $\alpha(1-\phi)$ she switches (next period) to employment at a high wage firm. With probability $(1-\alpha)$ she remains unemployed. The second equation follows because the low wage worker gets w_L for sure this period but switches to unemployment with probability λ or to a high wage job with probability $\alpha(1-\phi)$. With the remaining probability she stays employed at the low wage. The third equation states that someone employed at a high wage firm gets w_H this period but switches to unemployment with probability λ.

(b) Show that as long as $w_L > b$ the worker will always prefer employment at the low wage to unemployment.

The flow equations are written under the presumption that this is true. To show that under the imposed parameter restrictions this presumption is valid, we need to show that $V_L - V_U \geq 0$. Taking the difference between equations (2) and (1) yields:

$$r(V_L - V_U) = w_L - b + \alpha(1-\phi)(V_H - V_L) - \alpha\phi(V_L - V_U) - \alpha(1-\phi)(V_H - V_U) + \lambda(V_U - V_L)$$

It follows that

$$\begin{aligned}(r + \alpha\phi + \lambda)(V_L - V_U) &= w_L - b + \alpha(1-\phi)\left[(V_H - V_L) - (V_H - V_U)\right] \\ &= w_L - b - \alpha(1-\phi)(V_L - V_U)\end{aligned}$$

and

$$(r + \alpha + \lambda)(V_L - V_U) = w_L - b,$$

which is non-negative.

(c) Now suppose there is a continuum, measure 1, of such workers.

1. Draw a diagram showing the rates of flow between states.

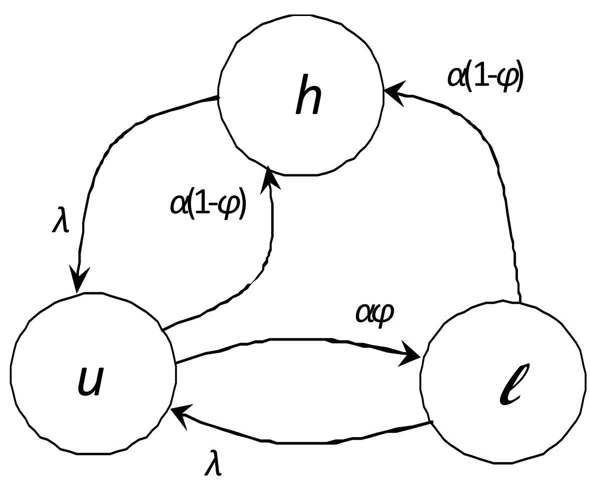

Figure 1:

2. Write out a set of equations that can be solved for the steady-state measures of workers in each of the states.

Any three of the following:

$$
\begin{aligned}
l + h + u &= 1 \\
\alpha(1-\phi)(u+l) - \lambda h &= 0 \\
\lambda(l+h) - \alpha u &= 0 \\
\alpha\phi u - [\alpha(1-\phi)+\lambda] l &= 0
\end{aligned}
$$

3. Solve for the measure of high wage workers and interpret the result.
   Substitute $u + l = 1 - h$, from the first equation in part (ii), into the second equation to get $\alpha(1-\phi)(1-h) = \lambda h$, or

$$h = \frac{\alpha(1-\phi)}{\alpha(1-\phi)+\lambda}.$$

   This is the same as if there were no low wage jobs and high wages job offers were received at probability $\alpha(1-\phi)$. The parameter ϕ has two effects on h. The direct effect is to decrease h, as more people get low wage jobs. But indirect effect increases h, as more people getting low wage jobs increases the set of people able to get offered high wage jobs. The overall effect is negative.