

Macroeconomics

Section 1. (Suggested Time: 45 Minutes) *For 3 of the following 6 statements, state whether the statement is **true**, **false**, or **uncertain**, and give a complete and convincing explanation of your answer. **Note:** Such explanations typically appeal to specific macroeconomic models.*

1. U.S. economic data show that the recent increase in the Federal government deficit has been accompanied by an increase in private sector saving. This is evidence in favor of Ricardian equivalence.
2. Firms operating constant returns to scale technologies cannot make positive profits.
3. Increasing the money supply is the most efficient way to pay for government expenditures.
4. Government policy should not be concerned with transitional paths—it is only what happens in steady-state that matters.
5. Increasing the money supply causes inflation.
6. Economies with high real interest rates are more likely to expand

Section 2. (Suggested Time: 2 Hours, 15 minutes) Answer any 3 of the following 4 questions.

7. Diamond overlapping generations with specific functional forms

Time: discrete, infinite horizon

Demography: A mass $N_t \equiv N_0(1+n)^t$ of newborns enter in every period. Everyone lives for 2 periods except for the first generation of old people.

Preferences: For the generations born in and after period 0:

$$U_t(c_{1,t}, c_{2,t+1}) = \ln(c_{1,t}) + \beta \ln(c_{2,t+1})$$

where $c_{i,t}$ is consumption in period t and stage i of life and $\ln(\cdot)$ is the natural logarithm function. For the initial old generation $\tilde{U}(c_{2,0}) = \ln(c_{2,0})$.

Productive technology: The production function available to firms is $F(K, N) = AK^{1/3}N^{2/3}$ where K is the capital stock and N is the number of workers employed. It will be convenient to use the implied per worker production function, $Ak^{1/3}$, where k is the capital stock per worker. Capital is fully used up in production.

Endowments: Everyone has one unit of labor services when young. (Old people cannot work.)

Institutions: There are competitive markets every period for labor and capital. (You can think of a single collectively owned firm that takes wages and interest rates as given.)

- (a) Write out and solve the problems faced by generation t households for given prices w_t and R_t (the wage and rental rate of capital).
- (b) Write out and solve the problem faced by the firm in period t .
- (c) Write down the market clearing condition for capital, define a competitive equilibrium and solve for the implied law of motion for the per-young person stock of capital, k_t , in the economy.
- (d) Obtain values for each steady-state capital stock, k^* , in terms of the model's parameters. For each determine its dynamic properties (i.e. stability, oscillatory).
- (e) Under what conditions (on parameters) could there be over accumulation of capital?

8. (Inspired by Wang and Wen, 2008.) Consider the following simplified version of a stochastic growth model. Letting $i \in [0, 1]$ index intermediate goods, final output is given by

$$Y = \left(\int_0^1 Y(i)^{1/\mu} di \right)^\mu, \quad \mu > 1. \quad (\text{FPRF})$$

(When possible, time subscripts are omitted.)

The production function for intermediate goods is given by

$$Y(i) = AL(i)^\alpha, \quad 0 < \alpha < 1, \quad (\text{PRF})$$

where $L(i)$ is labor used in sector i , and A is an aggregate technology shifter common to all sectors. The population and steady state technology level are constant and normalized to 1, so that upper case letters denote intensive as well as aggregate quantities.

- (a) Final goods are produced under perfect competition, with each producer solving

$$\max_{\{Y(i)\}_0^1} Y - \int_0^1 P(i) Y(i) di,$$

subject to equation (FPRF). Solving this problem, show that the intermediate goods price $P(i)$ can be written as a function of the intermediate quantity $Y(i)$ and the aggregate quantity Y . What are the aggregate profits of the final goods sector? Why?

- (b) Intermediate goods producers, on the other hand, are price setters. Letting W denote the real wage, producer i solves

$$\max_{L(i)} \Pi(i) = P_i[Y(i)] Y(i) - WL(i),$$

subject to equation (PRF). $P_i[Y(i)]$ is the demand function for good i derived in part (a).

Solve the producer's maximization problem.

- (c) Show that in a symmetric equilibrium, aggregate output, Y , is a function of aggregate labor, $L = \int_0^1 L(i) di$. Using this result, write the optimality condition for an intermediate goods producer in terms of aggregate variables.
- (d) The preferences of the representative household over consumption, C_t , and labor hours, L_t , are given by

$$E_0 \left(\sum_{t=0}^{\infty} \beta^t \left[\ln(C_t) - \chi \frac{1}{1+\gamma} L_t^{1+\gamma} \right] \right), \quad 0 < \beta < 1, \quad \gamma \geq 1, \quad \chi > 0.$$

Households receive labor income and profits from firms. They spend their income on consumption and risk-free bonds. Let B_t denote the quantity of bonds bought at time t , measured in terms of output. 1 unit of bonds delivers $(1 + r_t)$ units of output at time $t + 1$. Households also face the usual initial, non-negativity and No-Ponzi-Game conditions.

Find the first order conditions for utility maximization.

- (e) Assume that **there is no capital** in this economy, so that bonds represent trades between consumers. Imposing equilibrium, find the labor-leisure condition and the resource constraint. Suppose further that the parameters μ and A both vary over time. Write consumption as a function of μ_t and A_t .
- (f) Interpret your answer to part (e). Is the coefficient on μ_t positive or negative? How does it depend on α and γ ? Briefly interpret.

9. Consider the following variant of the Lucas tree model. The preferences of the representative consumer are

$$E_0 \left(\sum_{t=0}^{\infty} \beta^t \frac{1}{1-\sigma} [C_t^{1-\sigma} - 1] \right), \quad 0 < \beta < 1, \quad \sigma > 0.$$

where C_t denotes consumption.

Output is produced by an infinite-lived tree: each period, the tree produces d_t units of non-storable output. The **growth** rate of dividends follows a stationary process, with

$$G_{t+1} \equiv \frac{d_{t+1}}{d_t} = \exp(\varepsilon_{t+1} + \theta\varepsilon_t),$$

where $\{\varepsilon_t\}$ is a zero-mean, normally-distributed, i.i.d process. We will consider two possible values of θ : $\theta = 1$, where the effects of growth shocks compound over time; and $\theta = -1$, where the effects of growth shocks are transitory.

- (a) Derive the process for multi-period output growth $g_t(i, j) \equiv \ln(G_t(i, j)) \equiv \ln(d_{t+j}/d_{t+i})$. (Note that $g_t(0, 1) = \varepsilon_{t+1} + \theta\varepsilon_t$, $g_t(0, 2) = g_t(0, 1) + g_t(1, 2)$, etc.) Which value of θ , 1 or -1 , results in a higher variance of $g_t(0, j)$?
- (b) Let s_t denote the number of trees that the consumer owns at the beginning of time t , and let $P_t = P_t(d_t, \varepsilon_t)$ be the price at time t of all future dividends from the tree. Letting x_t denote the consumer's financial resources, write down Bellman's functional equation for the consumer's problem, and derive the Euler equations associated with stocks and bonds.
- (c) Find the equilibrium price for stocks, and use the definition of $g_t(i, j)$ to show that $P_t = d_t p(\varepsilon_t)$.
- (d) Is the price of stocks higher when the effects of growth shocks are transitory, or when they are persistent? (**Hint:** When x is distributed $\mathcal{N}(0, u)$, $E(\exp(x)) = \exp(u^2/2)$.) Provide an intuitive explanation.

10. **On-the-job search with a two-point wage distribution**

Time: Discrete, infinite horizon.

Demography: Single worker who lives for ever.

Preferences: The worker is risk-neutral (i.e. $u(x) = x$). He discounts the future at the rate r .

Endowments: When unemployed the worker receives income b per period. Also with probability α he gets an offer of employment. With probability $\phi > 0$ the offer is a low wage, w_L , and with probability $1 - \phi$ the offer is a high wage, w_H , where $w_H \geq w_L \geq b$.

When employed the worker receives the wage, w_L or w_H , every period and continues to get wage offers of the same types and at the same frequency as when unemployed. When employed he also gets laid off (loses his job) with probability λ .

- (a) Write down the flow or Bellman type asset equations for each of the states the worker can be in and briefly explain each one.
- (b) Show that as long as $w_L > b$ the worker will always prefer employment at the low wage to unemployment.
- (c) Now suppose there is a continuum, measure 1, of such workers.
 - i Draw a diagram showing the rates of flow between states.
 - ii Write out a set of equations that can be solved for the steady-state measures of workers in each of the states.
 - iii Solve for the measure of high wage workers and interpret the result.