Econometrics

Question 1 Golden Rule and Consumption in the Solow Model

Consider an economy that has access to a production technology

$$Y = K^{\alpha} L^{1-\alpha}, \quad \text{with } \alpha \in (0,1), \tag{1}$$

where the savings rate is exogenously given and equals s, the population grows at a constant rate n, there is no technology progress ($g = 0$) and the depreciation rate δ is 0. Also, assume that everybody is employed, thus population equals employment.

(a) Write down an equation for $\dot{Y}$ and $\dot{K}$. Define $k \equiv K/L$ and derive an expression for $\dot{k}$ as a function of n, s and $y \equiv Y/L$.

Answer.

Take equation (1) and differentiate with respect to (wrt) time

$$\frac{dY}{dt} = \alpha \left(\frac{K}{L}\right)^{\alpha-1} \frac{dK}{dt} + (1-\alpha)\left(\frac{K}{L}\right)^{\alpha} \frac{dL}{dt}$$

which can be rewritten as

$$g_Y = \alpha g_K + (1-\alpha) g_L.$$

The expression for $\dot{K}$ is simpler as there is no depreciation

$$\dot{K} = sY = sK^{\alpha}L^{1-\alpha}.$$

Now, take the definition of k and differentiate wrt time

$$\begin{aligned}
\frac{dk}{dt} = \dot{k} &= \frac{\dot{K}}{L} - \frac{K}{L^2}\frac{dL}{dt} \\
&= s\left(\frac{K}{L}\right)^{\alpha} - \frac{\dot{L}}{L}\frac{K}{L} \\
&= sk^{\alpha} - nk \\
&= sy - ny^{1/\alpha}
\end{aligned}$$

.

(b) Using the same definition of long run equilibrium covered in lectures, find the long-run equilibrium values for the capital per capita (k), and the product per capita (y). What is the growth rate of product per capita in the long run?

Answer.

Here you should proceed exactly as in the lectures. Take the following equation

$$\dot{k} = sk^{\alpha} - nk$$

and use the equilibrium definition as in the lectures, $\dot{k} = 0$, to obtain

$$\begin{aligned} sk^{\alpha} &= nk \\ \Rightarrow \quad (k^*)^{\alpha-1} &= \frac{n}{s} \\ k^* &= \left(\frac{s}{n}\right)^{1/(1-\alpha)}. \end{aligned}$$

Now, use equation (1) to obtain

$$\begin{aligned} \frac{Y}{L} &= \left(\frac{K}{L}\right)^{\alpha} \\ y &= k^{\alpha}, \end{aligned}$$

and then plug in the value for k^*,

$$y^* = (k^*)^{\alpha} = \left(\frac{s}{n}\right)^{\alpha/(1-\alpha)}.$$

Now take logs and the differentiate wrt time

$$\frac{\dot{y}}{y} = \alpha\frac{\dot{k}}{k} = 0,$$

and using the definition of k,

$$\begin{aligned} \frac{\dot{y}}{y} &= \frac{\dot{Y}}{Y} - \frac{\dot{L}}{L} = 0 \\ \frac{\dot{Y}}{Y} &= n. \end{aligned}$$

Exactly as expected we obtain that the growth of rate of total output equals the population growth rate, as this is just the usual result in the Solow model (using the assumptions that $\delta = g = 0$) .

(c) Write down an expression for consumption per capita ($c \equiv C/L$) in the long run as a function of n and k^*.
Answer.
There are many ways to write the consumption per capita, but we are interested in one in particular. It is convenient to write consumption as

$$C = Y - sY$$

divide through by L to obtain

$$c = y - sy$$

and replace with the production function in intensive form and the long run values

$$c = (k^*)^\alpha - s(k^*)^\alpha.$$

In the long run, $s(k^*)^\alpha$ equals nk^*,[1] and we use this to obtain the equation we are looking for

$$c = (k^*)^\alpha - nk^*$$

.

(d) Determine the value of k^* that maximizes c, denote it by k^{gold}. How does this value compare to the k^* you obtained in part (b)? What value(s) of the savings rate s make the economy converge to this k^{gold}? *[Hint: use the functional form, equation (1) to obtain closed form solutions for k^*, k^{gold}, and consumption.]*

Note: if you could not find the savings rate in part (d) you can still answer part (e), thus keep going and do not get stuck, you can go back to part (d) later if you want.

Answer.
Take the final equation from part (c)

$$\max_{k^*} \quad (k^*)^\alpha - nk^*,$$

the first order condition is

$$\alpha(k^{gold})^{\alpha-1} - n = 0$$

and k^{gold} is given by

$$k^{gold} = \left(\frac{\alpha}{n}\right)^{1/(1-\alpha)}.$$

It is easy to see that k^*, which you found in part (b), and k^{gold} differ only in the fact that instead of s, the savings rate, k^{gold} has α in the numerator. This tells us that $s = \alpha$ is the only value of the savings rate that makes the economy converge to k^{gold}, maximizing consumption in the long run .

[1] This is true because the equilibrium condition implies that in the long run, actual investment ($s(k^*)^\alpha$) equals break-even investment (nk^* in this case).

(e) Suppose that this economy starts with a savings rate different than the one required to achieve the maximum consumption per capita in the long run (i.e. one that leads to $k^* \neq k^{gold}$. One politician discovers this and claims that the government should try to correct it and move the savings rate towards the value that maximizes consumption. If the government cares only about people who is alive now, should they implement this policy? Be careful with your argument.

Answer.

It is clear that correcting the savings rate s to reach the golden rule leads to higher consumption in the long run, for any savings rate $s \neq \alpha$.

If the savings rate, s, is higher than α, then in the short run the reduction in the savings rate leads to higher consumption all along the trajectory to the new equilibrium. Basically, the economy reduces k along the trajectory and is able to consume more than what was initially using, thus all generations are better off.

This is not the case if $s < \alpha$. In this case, it is still true that consumption in the long run is higher, but to achieve this the economy needs to invest more, through an increase in the savings rate, as the capital stock is slowly built up, consumption decreases. Then, current generation and some of the future ones will consume less.

If the government care only about those who are alive now it will not allow an increase in the savings rate .

Question 2 Endogenous Growth and Scale Effects

Many endogenous growth models feature so called scale effects: per capita growth rises when population growth rises. Some economists have criticized these models for this reason, since countries with faster population growth do not in general appear to also experience faster per capita income growth.

Consider an economy that has access to a production technology

$$Y = AK^{\alpha}L^{1-\alpha}, \tag{2}$$

where Y is output, A is the level of technology, K is capital and L is the amount of labor in the economy. Capital evolves according to $\dot{K} = sY$ (thus, the depreciation rate $\delta = 0$). The population growth rate is n. (Throughout, $g_x = \frac{\dot{x}}{x}$, where x can be any of the variables in the model.)

(a) Assume that technology is determined by

$$A = BK^{\phi}. \tag{3}$$

What sort of endogenous growth model is this? Find $\frac{\dot{K}}{K}$ in terms of the K, L, and other parameters of the model.

Answer.

This is a model of learning-by-doing.

$$\begin{aligned} \frac{\dot{K}}{K} &= s\frac{Y}{K} \\ &= sBK^{\alpha+\phi-1}L^{1-\alpha} \end{aligned}$$

.

(b) Write an expression for g_Y in terms of g_K and g_L. What must be true for a balanced growth path to exist in this model? Solve for the balanced growth path value of g_Y and g_y, where $y = Y/L$. What must we assume about $\alpha + \phi$ in this model for there to be a positive (finite) rate of per capita income growth? How does g_y vary with the rate of population growth?

Answer.

$$Y = BK^{\alpha+\phi}L^{1-\alpha}$$

Hence,

$$\begin{aligned} g_Y &= (\alpha+\phi)\, g_K + (1-\alpha)\, g_L \\ &= (\alpha+\phi)\, g_K + (1-\alpha)\, n \end{aligned}$$

For a balanced growth path to exist, we need output and capital to grow at the same rate ($g_Y = g_K$). It follows that the balanced growth path value of g_Y is

$$g_Y^* = g_K^* = \frac{(1-\alpha)\, n}{1-\alpha-\phi}$$

Now, since $y = Y/L$

$$\begin{aligned} g_y &= g_Y - g_L, \\ \text{and} \\ g_y^* &= \frac{(1-\alpha)\,n}{1-\alpha-\phi} - n \\ &= \frac{n\phi}{1-\alpha-\phi}. \end{aligned}$$

Now, clearly for a positive finite rate of population growth we need $g_y^* > 0$, which implies that we need $1 - \alpha - \phi > 0$. In turn, this implies $1 > \alpha + \phi$. So long as this is true, per capita income growth is greater when population growth is higher, since

$$\frac{\partial g_y^*}{\partial n} = \frac{\phi}{1-\alpha-\phi} > 0$$

.

(c) Now assume instead that technology is determined by $A = B\left(\frac{K}{L}\right)^{\phi}$. Now, what causes technology to increase? Write down an expression for $\frac{\dot{K}}{K}$. What must be true about $\alpha + \phi$ in order for K to grow continuously at a constant rate? What is the constant rate?

Answer.

In this model, instead if the level of capital causing technological progress, it is the level of capital per worker that causes technological progress. As for $\dot{K}/K$, we proceed as before

$$\begin{aligned} \frac{\dot{K}}{K} &= s\frac{Y}{K} \\ &= sBK^{\alpha+\phi-1}L^{1-\alpha-\phi} \\ &= sB\left(\frac{K}{L}\right)^{\alpha+\phi-1} \end{aligned}$$

Note that in order for K to grow continuously at a constant rate, we need to eliminate the influence of K/L. This is possible only if $\alpha + \phi - 1 = 0$, or equivalently $\alpha + \phi = 1$. With this assumption, the constant rate of growth of aggregate capital is just $g_K = sB$

.

(d) Using our assumption about $\alpha + \phi$ from part (c), write an expression for g_Y in terms of g_K and g_L. What is the relation between g_Y and g_K? Now solve for g_y (where $y = Y/L$). Now, how does g_y vary with the rate of population growth?

Answer.

Now,

$$Y = BK^{\alpha+\phi}L^{1-\alpha-\phi}$$

Therefore

$$g_Y = (\alpha+\phi)\,g_K + (1-\alpha-\phi)\,g_L,$$

but $\alpha + \phi = 1$, so

$$g_Y = g_K.$$

That is, aggregate income and aggregate capital grow at the same rate, sB. Considering growth in per capita income,

$$\begin{aligned} g_y &= g_Y - g_L \\ &= sB - n. \end{aligned}$$

Therefore, in this model, per capita income growth falls when population growth increases. (In fact, it falls one-for-one, since $\partial g_y / \partial n = -1$) .

(e) Compare your answers for the balanced growth path value of g_y from (b) and (d). What is the main determinant of per capita growth in (b)? What are its main determinants in (d)? Can you think of any intuition concerning the different role of n in the two models?

Answer.

The two models provide quite different insights about the determination of per capita growth. In the first model, the main source of per capita growth is population growth (n). In the second model, population growth also exerts an important influence on per capita growth but that influence is negative. The positive sources of economic growth are the savings rate (s) and the productivity of learning-by-doing (B), neither of which even played a role in the first model.

Intuitively, in the first model, ideas were generated by the use of capital. When there are more workers, the economy uses more capital in aggregate (although not necessarily per capita), and hence more ideas are generated. Therefore, population growth increases per capita growth because it raises the total number of ideas. By contrast, in the second model, the number of ideas generated depends on how much capital each worker works with; more workers just reduce this level. Population growth, therefore, has a similar same effect as in the Solow model.

Both models seem to capture a different aspect of reality. It seems likely that more people will have more ideas, which is what the first model suggest. But it also seems plausible to suggest that each person will have more ideas if they have more capital with which to work .

Question 3 Essays Based on the Assigned Readings

Be concise, go straight to the point unless explicitly required to link different papers. Write less than 10 lines per question.

Long answers makes it harder for the grader to find the right arguments, thus restrain yourself from using too many words.

(a) José, one of your TAs, claims that much of country X's economic growth over the last two decades can be attributed to strong increases in productivity. Frantisek, your other TA, however doesn't agree with José. How would the economist Alwyn Young suggest your two TAs should resolve this dispute?

Answer.

To resolve this dispute, it is likely that Alwyn Young would suggest your TAs use data available for country X and do some growth accounting to determine the sources of X's growth. Since Alwyn Young finds that increases in labor force participation rates and increases in capital accumulation helped spur East Asias recent growth, we might also expect to find they played some role in X's growth. However, we wont know for sure until we actually look at the data and do some growth accounting .

(b) During the 1990s, Eastern European countries experienced various degrees of success in promoting economic growth during the transition from Communism to free markets. How does Svegnar explain these differences? Does he believe that government can play a positive role in promoting growth? If so, how?

Answer.

While nearly all of the Eastern European countries implemented "big bang" reforms aimed at freeing up prices and trade, Svegnar argues that only some of the Eastern European countries created laws, regulations, and other institutions designed to promote economic growth. These "Type II" reforms included privatizing firms, establishing a market-oriented legal system, promoting commercial banking via appropriate regulation, establishing labor market regulations, etc.

So yes, Svegnar does believe the government can play a positive role in promoting growth in that it can help promote the above reforms and provide an institutional basis conducive to economic growth. Notice that this role is radically different from the one it originally played in all these countries during the previous decades under the centrally planned system .

(c) Dennison conducted research into the different sources of growth in Western European countries and in the United States. What countries experience the fastest income growth rate during this period according to his results? Can we explain these pattern according to a simple version of the Solow model?

Answer.

Dennison found that the average growth rate in Western Europe was higher than the one in the United States; within Europe, Germany, France, Netherlands and Italy had the highest growth rates. Among European countries, those that showed higher growth rates are exactly those that suffered more destruction during World War II.

Then we can think that an important part of this effect is just countries catching up to their steady state (basically the effect of an exogenous destruction of part of your capital stock in the Solow model), most of the effect being just cumulation of capital. There are two observations here.

First, Belgium had a relatively low growth rate, and it should have had a relatively high one (although it is still high if compared to United Kingdom). Second, the contribution of the cumulation of capital to total growth in these countries is higher than in the United States, thus supporting our point that at least part of the effect can be attributed to a catching up or reconstruction process .

(d) Acemoglu, Johnson and Robinson analyzes the case of Botswana, a very successful African country. What is the main hypothesis the authors elaborate to explain Botswana's relative success? How can you connect that hypothesis to the theories we studied in this course? Is Solow enough or we need something more?

Answer.

The authors linked institutions to Botswana's successful story. In particular, they emphasize the role institutions play in fostering growth through the enforcement of property rights, and the regulation of a market economy. They claim that early institutions in Botswana plus the colonization pattern in the country helped to maintain early institutions. Tribes in Botswana had systems that kept control of the individuals in power and included many other mechanisms that helped the development of "good institutions".

A simple Solow model is not enough. We need to think of an endogenous growth model as say that institutional differences lead to higher growth. Solow tells us we converge to that rate in the long run, but we can not explain why this is higher in Botswana and how institutions help on this process .

(e) In his review essay of Diamond's book, Peter Temin emphasizes the two main assumptions implicit in Diamond's analysis. In particular, he made the point that even if Diamond does not explicitly incorporated economics, those assumptions have close connections to elements we see in growth theory. Mention and explain the economic meaning of those assumptions. Can you explain why they differ from the basic growth theories? What is special about Diamond's analysis?

Answer.

The main assumptions Temin identifies are that bigger population provides opportunities for mutation and innovation, when we think in economic terms, and that geography matters in the development process.

The first assumption is an implicit assumption about the innovation process, in a broad sense that includes biological adaptation, in particular it is similar to think that there are no decreasing returns to population in the innovation process. The main difference here is that innovation and knowledge are introduced as a broader concept.

The second assumption is about geography. Formal models we studied do not contain geography but other authors have added it into their analysis [2]; Diamond's argument about geography is more general and points toward a different dimension. Diamond links geography to development through the ability to connect East-West, emphasizing the similarities in climate, and the transmission of germs, which he connects to domes-

[2] Acemoglu, Johnson and Robinson do it and also Hall and Jones, summarized by Romer.

tication of animals, thus Eurasia is basically a long East-West continent and benefits from it unlike America. This allows the early habitants to achieve a larger size and thus benefit from the economies of scale implicit in the first assumption.
Two things are different. First, broad concept of innovation. Second, the very very long period under analysis .