Econometrics

Question 1 Golden Rule and Consumption in the Solow Model

Consider an economy that has access to a production technology

$$Y = K^\alpha L^{1-\alpha}, \quad \text{with } \alpha \in (0,1), \tag{1}$$

where the savings rate is exogenously given and equals s, the population grows at a constant rate n, there is no technology progress $(g = 0)$ and the depreciation rate δ is 0. Also, assume that everybody is employed, thus population equals employment.

(a) Write down an equation for $\dot{Y}$ and $\dot{K}$. Define $k \equiv K/L$ and derive an expression for $\dot{k}$ as a function of n, s and $y \equiv Y/L$.

(b) Using the same definition of long run equilibrium covered in lectures, find the long-run equilibrium values for the capital per capita (k), and the product per capita (y). What is the growth rate of product per capita in the long run?

(c) Write down an expression for consumption per capita $(c \equiv C/L)$ in the long run as a function of n and k^*.

(d) Determine the value of k^* that maximizes c, denote it by k^{gold}. How does this value compare to the k^* you obtained in part (b)? What value(s) of the savings rate s make the economy converge to this k^{gold}? [*Hint: use the functional form, equation (1) to obtain closed form solutions for k^*, k^{gold}, and consumption.*]

Note: if you could not find the savings rate in part (d) you can still answer part (e), thus keep going and do not get stuck, you can go back to part (d) later if you want.

(e) Suppose that this economy starts with a savings rate different than the one required to achieve the maximum consumption per capita in the long run (i.e. one that leads to $k^* \neq k^{gold}$. One politician discovers this and claims that the government should try to correct it and move the savings rate towards the value that maximizes consumption. If the government cares only about people who is alive now, should they implement this policy? Be careful with your argument.

Question 2 Endogenous Growth and Scale Effects

Many endogenous growth models feature so called scale effects: per capita growth rises when population growth rises. Some economists have criticized these models for this reason, since countries with faster population growth do not in general appear to also experience faster per capita income growth.

Consider an economy that has access to a production technology

$$Y = AK^\alpha L^{1-\alpha}, \tag{2}$$

where Y is output, A is the level of technology, K is capital and L is the amount of labor in the economy. Capital evolves according to $\dot{K} = sY$ (thus, the depreciation rate $\delta = 0$). The population growth rate is n. (Throughout, $g_x = \frac{\dot{x}}{x}$, where x can be any of the variables in the model.)

(a) Assume that technology is determined by

$$A = BK^\phi. \tag{3}$$

What sort of endogenous growth model is this? Find $\frac{\dot{K}}{K}$ in terms of the K, L, and other parameters of the model.

(b) Write an expression for g_Y in terms of g_K and g_L. What must be true for a balanced growth path to exist in this model? Solve for the balanced growth path value of g_Y and g_y, where $y = Y/L$. What must we assume about $\alpha + \phi$ in this model for there to be a positive (finite) rate of per capita income growth? How does g_y vary with the rate of population growth?

(c) Now assume instead that technology is determined by $A = B\left(\frac{K}{L}\right)^\phi$. Now, what causes technology to increase? Write down an expression for $\frac{\dot{K}}{K}$. What must be true about $\alpha + \phi$ in order for K to grow continuously at a constant rate? What is the constant rate?

(d) Using our assumption about $\alpha + \phi$ from part (c), write an expression for g_Y in terms of g_K and g_L. What is the relation between g_Y and g_K? Now solve for g_y (where $y = Y/L$). Now, how does g_y vary with the rate of population growth?

(e) Compare your answers for the balanced growth path value of g_y from (b) and (d). What is the main determinant of per capita growth in (b)? What are its main determinants in (d)? Can you think of any intuition concerning the different role of n in the two models?

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Question 3 Essays Based on the Assigned Readings

Be concise, go straight to the point unless explicitly required to link different papers. Write less than 10 lines per question.

Long answers makes it harder for the grader to find the right arguments, thus restrain yourself from using too many words.

(a) José, one of your TAs, claims that much of country Xs economic growth over the last two decades can be attributed to strong increases in productivity. Frantisek, your other TA, however doesnt agree with José. How would the economist Alwyn Young suggest your two TAs should resolve this dispute?

(b) During the 1990s, Eastern European countries experienced various degrees of success in promoting economic growth during the transition from Communism to free markets. How does Svegnar explain these differences? Does he believe that government can play a positive role in promoting growth? If so, how?

(c) Dennison conducted research into the different sources of growth in Western European countries and in the United States. What countries experience the fastest income growth rate during this period according to his results? Can we explain these pattern according to a simple version of the Solow model?

(d) Acemoglu, Johnson and Robinson analyzes the case of Botswana, a very successful African country. What is the main hypothesis the authors elaborate to explain Botswana's relative success? How can you connect that hypothesis to the theories we studied in this course? Is Solow enough or we need something more?

(e) In his review essay of Diamond's book, Peter Temin emphasizes the two main assumptions implicit in Diamond's analysis. In particular, he made the point that even if Diamond does not explicitly incorporated economics, those assumptions have close connections to elements we see in growth theory. Mention and explain the economic meaning of those assumptions. Can you explain why they differ from the basic growth theories? What is special about Diamond's analysis?