

Corporate Finance

Question 1. Finance companies

Finance companies are firms that lend money to people and companies. BlueHorse Finance Co. has 1000 zero-coupon, one-year loans outstanding, each for a face amount of \$1 million. Goldfish Co. has 10 zero-coupon, one-year loans outstanding, each for \$100 million. The face amounts of all the loans are due to be paid one year from today. Each loan has the same probability of default. Each loan has the same rate of loss in the event of default. Assume that the chance of default is completely idiosyncratic for each loan. Therefore the uncertainty of a loan's payoff is uncorrelated with the uncertainty of any other loan's payoff.

- a. (4 points) Is the expected payoff on Bluehorse's portfolio greater, equal, or less than the expected payoff on Goldfish's portfolio? EXPLAIN YOUR ANSWER!

The expected payoffs are equal. The expected payoffs are

Bluehorse : $1000 \times 1\text{MM} \times (\text{expected fraction of loan's face value that is repaid})$

Goldfish : $10 \times 100\text{MM} \times (\text{expected fraction of loan's face value that is repaid})$

Since the expected fractions are the same, the expected payoffs are equal.

- b. (4 points) Is the standard deviation of the payoff on Bluehorse's portfolio greater, equal, or less than the standard deviation of the payoff on Goldfish's portfolio? A solution that briefly explains the intuition is sufficient. You may use a mathematical formula, but you do not need to use one.

The standard deviation of the payoff on Bluehorse's payoff is less than that for Goldfish because of diversification. The standard deviation of a portfolio with 1,000 independent risks is smaller than the standard deviation of a portfolio with 10 independent risks. The following math is not required, but if math is used, it must be done correctly.

Each loan has the same variance of payoff per dollar of face amount. Call this variance $\text{Var}(\text{per dollar})$, and its square root $\text{SD}(\text{per dollar})$. The variance of Bluehorse's portfolio is the sum of the variances of the individual loans, or

$$\text{Var}(\text{portfolio}) = 1000 * \text{Var}(\text{payoff on } \$1\text{MM loan}) = 1000 * (\$1\text{MM})^2 * \text{Var}(\text{per dollar})$$

Therefore the standard deviation is $\text{sqrt}(1000) * \$1\text{MM} * \text{SD}(\text{per dollar})$.

The formula for Goldfish is $10 \cdot \text{Var}(\text{payoff on } \$100\text{MM loan}) = 10 \cdot (\$100\text{MM})^2 \cdot \text{Var}(\text{per dollar})$

Therefore the standard deviation is $\sqrt{10} \cdot \$100\text{MM} \cdot \text{SD}(\text{per dollar})$.

Since $\sqrt{1000} \cdot \$1\text{MM} = \sqrt{10} \cdot \$10\text{MM} < \sqrt{10} \cdot \100MM , we confirm the result.

- c. (4 points) Which loan book has a higher market value as of today?
EXPLAIN YOUR ANSWER!

Their values are equal. All the risks are idiosyncratic. In any investor's well-diversified portfolio, these idiosyncratic risks are diversified away. Therefore in both cases the expected year-ahead values are discounted at the risk-free rate.

Question 2. Portfolio composition

Properties of the returns to two stocks are listed in the table below. The risk-free rate is also reported. The correlation between the stock returns is 40%.

	Expected return	Standard deviation of return
Alta, Inc	14%	30%
Snowbird, Inc.	8%	45%
Risk-free asset	2%	0%

- a. (5 points) What is the expected return and return standard deviation of a portfolio that has a weight of 0.8 on Alta, Inc and a weight of 0.2 on Snowbird, Inc? SHOW YOUR WORK!

$$\text{Expected return} = 0.8 \cdot 14\% + 0.2 \cdot 8\% = 12.8\%$$

$$\text{Variance} = 0.8^2 \cdot 0.3^2 + 2 \cdot 0.8 \cdot 0.2 \cdot 0.3 \cdot 0.45 \cdot 0.4 + 0.2^2 \cdot 0.45^2 = 0.0830$$

$$\text{Standard deviation} = \sqrt{0.0830} = 0.2881 = 28.81\%$$

- b. (5) Assume that the only three assets in the economy are these two stocks and a risk-free asset. Using only the numbers reported in the table and your results for part (a), explain why you know the portfolio in part (a) is not on the efficient frontier.

All portfolios on the efficient frontier have the maximum Sharpe ratio. The Sharpe ratio of the portfolio in part (a) is $(12.8\% - 2\%)/28.81\% = 0.3749$. The Sharpe ratio of Alta, Inc. stock is $(14\% - 2\%)/30\% = 0.40$. Therefore the portfolio in (a) does not have the highest Sharpe ratio.

Question 3. Pet food

WhiteDog, Inc. makes pet food. It has a total market value of equity of \$60MM. It has debt of \$25MM, cash on hand of 35MM, and a market beta of equity of 1.5. The debt has a market beta of zero. The risk-free rate is 3% and the market risk premium is 4%. The corporate tax rate is 35%. The firm will have positive after-tax profits forever.

- a. (3 points) What is WhiteDog's enterprise value? SHOW YOUR WORK!

$$\text{Enterprise value} = 60\text{MM equity} + 25\text{MM debt} - 35\text{MM cash} = 50\text{MM}$$

- b. (5 points) What is the beta of WhiteDog's enterprise value? SHOW YOUR WORK!

$$\text{Beta} = (\text{equity value})/\text{enterprise value} * \text{equity beta} + (\text{net debt value})/\text{enterprise value} * \text{debt beta}$$

$$\text{Beta} = (6/5)*1.5 + 0 = 1.8$$

- c. (6 points) Assume the CAPM is correct. What is the after-tax weighted average cost of capital for WhiteDog's projects? SHOW YOUR WORK!

The CAPM tells us that the expected return to equity is

$$r_E = \text{risk-free rate} + \text{beta} * (\text{market risk premium}) = 3 + 1.5*4 = 9\%$$

$$\text{The same logic implies } r_D = \text{risk-free rate} + 0 * \text{market risk premium} = 3\%$$

$$\text{Wacc} = (\text{equity value})/\text{enterprise value} * r_E + (\text{net debt value})/\text{enterprise value} * r_D * (1 - \text{tax rate}) = (6/5)*9\% - (1/5)*3\% * (1 - 0.35) = 10.41\%$$

- d. (5 points) Assume the CAPM is correct. PurpleCat, Inc. is a large, diversified company that is deciding whether to market a line of pet food. The project has economic risks similar to those of WhiteDog's projects. What is the discount rate appropriate for PurpleCat's unlevered project? SHOW YOUR WORK!

If the project is unlevered, it has zero debt associated with it. The unlevered discount rate can either be calculated from the CAPM using the beta of WhiteDog's enterprise value or using the wacc formula above but dropping the tax term ($1 - \text{tax rate}$). Either way, the result is 10.2%

Question 4. Cost of capital

YellowDuck, Inc. has \$10MM in debt, equity of \$15MM. Equityholders have a required expected return of 7% and debtholders have an expected required return of 2%. The corporate tax rate is 30 percent.

- a. (5 points) Bondholders believe the bonds have a 2 percent chance of default per year. If default occurs, the loss rate in the event of default will be 50 percent. Estimate the yield on YellowDuck's debt. SHOW YOUR WORK!

To approximate the yield, use $r_D = \text{yield} - \text{probability of default} * \text{fraction lost in the event of default}$.

In this case, $0.02 = \text{yield} - 0.02 * 0.5$, or $\text{yield} = 0.02 + 0.01 = 0.03$, 3%

- b. (7 points) Assume the CAPM is correct, but make no assumption about either the risk-free rate or the expected return to the market, other than the assumption that both are positive. Explain why, based on the expected returns specified in this problem, that it is highly unlikely that the stock return has a negative market beta. This is a relatively hard problem.

If the market beta of the stock return is negative, then the expected return to the market is greater than the expected return to the stock. Since the expected return to the debt is less than the expected return to the stock, the debt's beta must be even more negative than the stock's beta. Therefore the debt's return is *more* volatile than the stock's return, in the sense that the debt's return is more sensitive to the market return than is the stock's return. But for any firm, the value of its debt is less volatile than the value of its equity.

Question 5. Beyond Modigliani and Miller

(5 points) Redbird, Inc. is an all-equity firm that has stable, large annual EBITDA and stable, large cash flows. However, because it has substantial depreciation, Redbird's EBIT is zero. Redbird uses the cash flows to invest in new projects and purchase smaller companies. Explain why the shareholders of Redbird may prefer that Redbird take on substantial debt, using the borrowed funds to buy back shares. Do not adopt the perfect market assumptions of Modigliani and Miller.

There is an agency benefit of debt. Managers with lots of free cash flow tend to use the cash in ways that do not benefit the shareholders. They may build empires by buying other companies or by investing in negative NPV projects that will give them personal prestige (for example, buying a sports team), and/or they may spend cash on perks such as corporate jets and fancy corporate retreats. Debt forces managers to spend some of their cash on interest, reducing the amount of free cash that can be wasted.

Another reason is that, although the firm currently has zero EBIT, it might have had positive EBIT in the past or might have positive EBIT in the future. Then debt today will generate negative profits that can be used to offset positive profits in the past or future.

Question 6. Risky debt

BrownBear, Inc. is an all-equity firm. If it does not change its capital structure, the market value of its equity one year from today will be either \$150MM or \$80MM. Both possibilities are equally likely. The risk of the firm is entirely diversifiable. The risk-free rate is 5 percent.

- a. (3 points) What is BrownBear's current market value of equity?

Expected value one year from today is $(1/2)150\text{MM} + (1/2)80\text{MM} = 115\text{MM}$. Discount at the risk-free rate to produce 109.5238MM.

- b. (6 points) Assume that BrownBear issues debt today, and immediately uses the funds to buy back shares. The debt matures a year from today, and pays no interest during the next year. At maturity, the promised payment is \$100MM. Using all of the perfect market assumptions of Modigliani and Miller, including no taxes, what is today's market value of the debt?

If the unlevered value of the firm is \$150MM the debt is paid in full. If the unlevered value of the firm is \$80MM the bondholders will receive \$80MM. The expected payoff is

Expected payoff = $0.5 \cdot 100\text{MM} + 0.5 \cdot 80\text{MM} = 90\text{MM}$.

The risk of the unlevered firm is entirely diversifiable, so the market value of the debt is $90\text{MM}/1.05 = 85.7143\text{MM}$.

- c. (8 points) The CEO of BrownBear wants to issue the debt in order to benefit the shareholders. The CEO tells you, "I know that the perfect market assumptions of M&M imply that issuing the debt has no effect on shareholders. But their argument relies on the ability of shareholders to engage in the same financial transactions that the firm engages in. Our debt will be risky; the firm might default on the debt. Unless we issue that risky

debt, neither shareholders nor anybody else can buy or sell it. Therefore the M&M argument is irrelevant.”

Show the CEO that she is wrong by constructing a portfolio of the unlevered firm and risk-free debt that replicates the payoff of the risky debt.

We want a portfolio that reproduces the payoff of 100MM in the good state and 80MM in the bad state. The difference between these two values is 20MM. The unlevered equity has a difference between the good and bad state of 70MM. Therefore our portfolio will consist of $2/7$ of the unlevered firm and some amount of risk-free debt. The payoff on the portfolio must equal

$$100 = (2/7) * 150 + (\text{payoff on debt}),$$

$$80 = (2/7) * 80 + (\text{payoff on debt}).$$

Solving either of these equations reveals that the payoff on the debt must equal 57.1429. Therefore the market value of the debt today is $57.1429/1.05 = 54.4218$. To summarize, the risky debt cash flows can be reproduced with a portfolio that contains $(2/7)$ of the unlevered firm and 54.4218MM lent at the risk-free rate. Note that the market price today of this portfolio is $(2/7)*109.5238\text{MM} + 54.4218\text{MM} = 85.7143\text{MM}$, which is the market value of the debt calculated in part (b).

Question 7. A recapitalization

CandyCane Inc. is financed entirely with equity, with a market value of \$120MM. Equityholders require an expected return of 10 percent. The risk-free rate is 4 percent. CandyCane is considering issuing perpetual debt to buy back some of the firm's stock. It plans to issue enough debt so that the debt/equity ratio (after buying back the stock) is 0.5. With this amount of debt, bondholders treat the debt as risk-free. Reminder: with perpetual debt, the borrower pays only interest. The principal does not change through time. Assume the firm will have sufficient EBIT to pay the interest every year.

- a. (5 points) Using all of the perfect market assumptions of M&M, **including** the assumption of no taxes, what will be the market value of the firm's debt?
SHOW YOUR WORK!

M&M says value of the firm does not change. Therefore $120\text{MM} = E + D$, and then substitute $D/E = 0.5$, or $E/D = 2$, for

$$120\text{MM} = E/D * D + D = 2*D + D = 3 * D, \text{ so } D = 40\text{MM}$$

- b. (4 points) Using all of the perfect market assumptions of Modgiliani and Miller, **including** the assumption of no taxes, what will be the expected return to equity? **SHOW YOUR WORK!**

Use $r_E = r_U + (D/E)*(r_U - r_D) = 10\% + 0.5*(10\% - 4\%) = 13\%$

- c. (7 points) Assuming a corporate tax rate of 25%, but retaining all other perfect market assumptions of M&M, what will be the market value of the firm's debt? **SHOW YOUR WORK!**

First figure out the value of the levered firm by adding in the value of the interest tax shield. With perpetual debt, the value of the interest tax shield is the perpetuity value of the interest payment times the tax rate. This is equivalent to the market value of the debt times the tax rate. Here, the value of the levered firm is

$$V^L = V^U + D(1 - \text{tax rate}) = 120\text{MM} + D * (\text{tax rate})$$

But we aren't done, since we don't know D. We have to use $V^L = E + D$, then set $D/E = 0.5$ to eliminate E from this equation. This was also done in part (a) The result is $V^L = (E/D)*D + D = 3*D$. Substitute this into the equation above to get

$3*D = 120\text{MM} + D*(\text{tax rate})$, or $2.75D = 120\text{MM}$, or $D = 43.63636$. Therefore $V^L = 130.90909\text{MM}$ and $E = 87.272727$. Check that $130.90909 = 120 + 43.63636*0.25$. It works.

- d. (9 points) Assuming a corporate tax rate of 25%, but retaining all other perfect market assumptions of M&M, what will be the expected return to equity, as a function of the expected return to the unlevered firm, the risk-free rate, the debt/equity ratio, and the tax rate? Your answer should be a formula. Hint and recommendation: we did not derive this formula in class. You should derive it by thinking of equity as a portfolio. The solution to this question does not require the solution to part (c), so you should not use the solution to part (c) here – since you might have done that question incorrectly. **SHOW YOUR WORK!**

Remember $V^L = E + D$, so $E = V^L - D = (V^U + D*\text{tax rate}) - D = V^U - D(1 - \text{tax rate})$.
Work with the equation

$$E = V^U - D(1 - \text{tax rate}).$$

Therefore we can think of equity as a portfolio, with a amount V^U/E invested in the unlevered firm and a short position of $D(1 - \text{tax rate})/E$. Note that the return to D is the same as the return to D^*x , where x is any positive number. (The return to a portfolio is the same as the return to a portfolio that is twice as valuable but otherwise identical.)
Therefore

$$R_E = (V^U/E) * R_U - (D(1 - \text{tax rate})/E)*R_D$$

We can rewrite this as

$$R_E = [(V^U/E) - (1 - \text{tax rate})*D/E] * R_U + [D(1 - \text{tax rate})/E]*(R_U - R_D),$$

Or

$$R_E = R_U + D/E*(R_U - R_D)*(1 - \text{tax rate})$$

This is our formula. It is identical to the formula in the no-tax case aside from the $(1 - \text{tax rate})$ term that appears at the end.