Corporate Finance

Final Exam Solutions

- 1(a)

  The price of the coupon bond is too low relative to the price of the ZCB. Here is the portfolio. Buy the coupon bond for a price

$$C = \frac{6}{1 + 0.093} + \frac{100 + 6}{1.093^2} = 94.2185.$$

  Simultaneously, short-sell enough zero-coupon bonds to cancel out the final cash flow of the coupon bond. This means you want to short sell 1.06 of the ZCBs. You will immediately receive

$$1.06\frac{100}{1.06^2} = 1.06(89.00) = 94.3396.$$

  The net amount of cash in your pocket at year zero.

$$\text{time-zero cash flow} = 94.3396 - 94.2185 = 0.1211.$$

  The year-one cash flow is 6, received from the coupon bond. The year-two cash flow is zero, because the cash flow on the coupon bond exactly offsets the cash flow on the short ZCB position. All net cash flows are either zero or positive. This is an arbitrage opportunity.

- 1(b)

  Price formula using EAR is

$$P = \frac{100}{(1 + y)^2}$$

  First plug in 0.06, then plug in 0.05. Difference between the two is 1.7033, so price rises by $1.7033.

- 1(c)

  Since the price equals the face amount, the bond is a par bond. We know the yield on a par bond equals the coupon rate (as long as the compounding frequency equals the frequency with which the coupon is paid), so the EAR on the bond is the 6% coupon rate.

- 2(a)

  There are a few ways to solve this problem. Here I adopt the method used by most students. The cash flows are the combined cash flows from a long position in an annuity that pays \$1MM/year from years 1 to 23, and a short position in an annuity that pays \$2MM/year from years 24 to 98.

  Use the annuity formula

  $$PV = \frac{C}{r}\left(1 - (1+r)^{-N}\right)$$

  with $r = 0.05$. The first annuity has $C = 1$ and $N = 23$. Its value is, in millions,

  $$PV = 13.4886$$

  The second annuity has $C = 2$ and $N = 75$. Plugged into the same formula produces a PV of 38.9699. But remember the annuity formula determines the value, as of time zero, of an annuity beginning in time one. Therefore 38.9699 is value of the second annuity as of time 23. We have to discount that back to time zero. The answer is

  $$\text{time-zero}PV = 38.9699/1.05^{23} = 12.68749.$$

  Therefore the total value of the cash flows is, in millions,

  $$\text{NPV} = 13.4886 - 12.68749 = 0.8011$$

- 2(b)

  The proposed rule is backwards. This project has positive cash flows in the beginning and negative cash flows at the end, unlike a usual project. Therefore, the higher the discount rate, the *larger* is the NPV – the high discount rates reduce the PV of the distant negative cash flows. The correct rule is for discount rates *greater* than the IRR, the project should be adopted.

  In case anyone cares, the IRR for this project is 4.801%. I calculated this using Solver.

- 3(a)

  The incremental EBIT from sales of the printer is

  $$(\text{new price}-\text{COGS})\times(\text{new forecast of sales})-(\text{old price}-\text{COGS})\times(\text{old forecast of sales})$$

  This incremental EBIT is received only in year one, since that is the only year in which the printer will be sold. Plugging in the correct values produces an incremental EBIT of $-900,000$.

  But we are not done. There is also incremental EBIT from sales of replacement cartridges. This is

  $$(\text{new forecast of cartridge sales} - \text{old forecast of cartridge sales}) \times (\text{price} - COGS)$$

This formula applies to years two and three. For both years, the incremental EBIT is 900,000, based on an additional 30,000 printers sold in year 1, which corresponds to an additional 30,000 cartridges sold in years 2 and 3, with a EBIT per cartridge of \$30.

Therefore the total incremental EBITs are

Year 1: \$ −900K

Year 2: \$ 900K

Year 3: \$ 900K

- 3(b)

  Discount future cash flows. For this problem there are no capital expenditures, no changes in working capital, no depreciation, so we just multiply EBIT by (1 - tax rate) to get cash flows. The NPV is, in thousands,

$$NPV = -\frac{900(1-0.4)}{1.08} + \frac{900(1-0.4)}{1.10^2} + \frac{900(1-0.4)}{1.12^3} = 300.642.$$

  This is greater than zero, so do the project—cut the price.

- 4(a)

  To fill out the table, we need to know the following three ideas. First, the unlevered project is analyzed assuming no debt is used to finance the project, even if the firm plans to use debt to do so. Second, with straight-line depreciation over three years, the cost of the equipment investment is charged to depreciation in three equal installments. Third, net income is revenues minus (expenses ex depreciation) minus depreciation.

Year	0	1	2	3
Equipment investment	1	0	0	0
Revenue	0	1.5	1.8	1.0
Expenses ex depreciation	0	0.9	1.08	0.6
Depreciation	0	0.3333	0.3333	0.3333
Interest expense	0	0	0	0
Taxes	0	0	0	0
Net income	0	0.267	0.387	0.067

- 4(b)

  First figure out how much working capital is needed in years zero, one, and two. Multiply the year-ahead revenues by 0.2. The working capital requirements are year zero: 0.3

year one: 0.36

year two: 0.2

Now use the standard free cash flow formula.

free cash flow = (revenues minus cash expenses) * (1 - tax rate) - capital expenditures - change in working capital + depreciation * tax rate.

In year zero, there are no revenues and no expenses, so the cash flow is the minus $1 for equipment investment and the minus $0.3 for working capital. The total is -1.3.

In year one, the cash flow is (revenues minus expenses ex depreciation) - change in working capital = 0.6 - (0.36 - 0.3) = 0.54.

In year two, the cash flow is (revenues minus expenses ex depreciation) - change in working capital = 0.72 - (0.2 - 0.36) = 0.88.

In year three, the cash flow is (revenues minus expenses ex depreciation) - change in working capital = 0.4 - (0.0 - 0.2) = 0.6.

- 4(c)

  The second project, the one with certain cash flows, has a higher NPV. The discount rates used for certain cash flows are smaller than those used for the firm's typical projects (this is from the second table in the question). Since this project has expenses up-front and positive cash flows in the future, lower discount rates correspond to higher NPVs.

- 5(a)

  There are a couple of equivalent ways to do this. One way is to discount the expected free cash flows in years 1 to 3 to get year zero values. The discount rate is 10 percent. Then separately calculate the value of the remaining free cash flows using the growing perpetuity formula. Alternatively, discount expected free cash flows in years 1 and 2, then value cash flows in years 3 and up using the growing perpetuity formula. I use the former approach here.

Year	1	2	3
FCF	33	45	50
PV of FCF	30	37.1901	37.5657

The sum of these three PVs is 104.756. Now calculate the value of the remaining free cash flows. Use

$$\text{value as of year 3} = \frac{\text{expected cash flow in year 4}}{0.10 - 0.03}$$

where the growth rate of the cash flows is three percent. The numerator is $50(1.03) = 51.5$. Therefore

$$\text{value as of year 3} = \frac{51.5}{0.07} = 735.714.$$

Discount this year-value back to year zero, using the 10 percent discount rate. The answer is 552.753. Add up the two pieces to get the total NPV

$$104.756 + 552.53 = 657.509.$$

- 5(b)

  The market value of equity is the enterprise value of the firm minus the firm's net debt. Net debt is (gross debt) minus (cash). Therefore the market value of equity is the enterprise value from above, 657.509MM, minus 145MM. The answer is 512.509MM, for a price per share (20 million shares) of 512.509/20 = 25.6254.

- 6

  When there is a riskfree asset, an investor who wants to maximize her expected return for a given level of volatility will pick a portfolio with the highest possible Sharpe ratio. The set of portfolios that achieve the maximum possible Sharpe ratio are a combination of a riskfree asset and a single portfolio of risk assets – the tangential portfolio.

- 7

  Debt raises the cost of equity capital (as long as the equity has a beta greater than zero). It does so because it raises the volatility of returns to equity. Default-free debt has the greatest effect on the volatility of equity returns, because default-free debt absorbs none of the risk of the firm's cash flows. All of the risk is born by equity holders (as in the case of an all-equity financed firm). The key difference driving the increase in risk is that with leverage, the market value of equity is lower because part of the firm's cash flows are owed to debt holders, so a constant dollar amount of volatility corresponds to higher return volatility.

- 8(a)

  The baseline case is no taxes. With no taxes, the share price does not change in a perfect market because the Modigliani-Miller theorem holds. More generally, the value of the firm does not change with the capital structure. Prior to the announcement, the total value of equity was $6.25 times 8MM shares, or $50MM. The announcement cannot change the share price. After spending $22MM to repurchase shares at a price of $6.25, (a total of 3.52MM shares), the equityholders own $28MM with 4.48MM shares for a share price of $6.25.

- 8(b)

  With a constant amount of debt of $22MM and a constant tax rate of 35 percent, the PV of the tax shield is (tax rate) x (constant debt amount). This is $7.7MM. The announcement therefore immediately raises the value of the firm by $7.7MM. All of this increase in value goes to equity holders, so total equity jumps to $50+$7.7 = $57.7MM. There are still 8MM shares, so the share price jumps to $7.2125.

- 8(c)

The market value of equity without distress costs is \$57.7MM. Since the price went up to only \$6.50, the market value of equity went up to only \$52MM. The PV of distress costs is the difference, equal to \$5.7MM.

- 9(a)

  If the firm does not develop, it will default on the debt. Equityholders will get nothing, so the value of the firm's equity is zero. The market value of the debt is \$7MM (received for sure in one year) discounted at the riskfree rate (because default is guaranteed, the cash flow is riskless). This value is \$6.54206MM.

- 9(b)

  The incremental year-1 value is \$23MM. This is riskfree, so its PV is 23/1.07 = \$21.4953MM. Subtract the development cost of \$15MM to produce an NPV of \$6.49533MM.

- 9(c)

  If the project is developed, the debtholders are repaid for sure. They receive the face value of their debt of \$14MM. Equityholders receive the remainder of \$16MM. Discount both payoffs at the riskfree rate. The debt market value is \$13.0841MM and the equity market value is \$14.9533MM.

- 9(d)

  No, they will not do it. The market value of equity after the investment of \$15MM is only \$14.9533MM. They lose money.

- 10(a)

  The unlevered cost of capital is easy to calculate if we assume that Costco maintains a constant debt/equity ratio. In this case, the discount rate for the tax shield equals the discount rate for unlevered equity. We can then derive the formula

  $$\text{unlevered discount rate} = \frac{LE}{LE+D} r_E + \frac{D}{LE+D} r_D.$$

  This formula critically relies on the constant debt/equity ratio assumption.

  Plug in LE/(LE+D) = 0.75 and D/(LE+D) = 0.25. Then the unlevered discount rate is 0.10525.

- 10(b)

  This calculation is standard.

  $$wacc = \frac{LE}{LE+D} r_E + \frac{D}{LE+D} r_D (1 - \text{tax rate})$$

  The wacc is 10 percent.

- 10(c)

  The NPV calculation discounts unlevered free cash flows at the wacc. The wacc takes into account the value of the debt shield.

Year	0	1	2
FCF	-120	80	130
PV of FCF	-120	72.7331	107.455

  The NPV is the sum of the PVs, which is \$60.1822.

- 10(d)

  Debt capacity is Costco's chosen ratio of debt/enterprise value times the increase in the firm's assets. In year 0, the increase in assets is the NPV of the project + 120 for investment. In year 1, the increase in assets is the PV of the remaining cash flow, which is the year-2 cash flow. In year 2, the increase in assets is zero.

Year	0	1	2
Increase in asset value	$120 + 60.1822 = 180.188$	$130/1.1 = 118.191$	0
Debt capacity	$180.188 \times 0.25 = 45.047$	$118.191 \times 0.25 = 29.5478$	0

- 10(e)

  To calculated the unlevered NPV, discount expected free cash flows at the unlevered cost of capital from 10(a), which is 10.525%. The value is 58.8016.

- 10(f)

  First calculate the interest paid on the debt capacity determined in 10(d). Use the debt cost of capital of 6.1%. This interest is paid in the next year, resulting in the interest payments in the table below. Multiply the interest paid by the tax rate to get the increase in free cash flow associated with the tax shield. Discount these cash flows at the unlevered cost of capital to get the present values.

Year	0	1	2
Debt capacity	45.047	29.5478	0
Interest paid	0	2.74787	1.80242
Tax shield at 35 %	0	0.96175	0.63085
PV	0	0.87017	0.51642

- 10(g)

  Add up the value of the unlevered project plus the NPV of the tax shield. The total is $58.8016 + 1.38659 = 60.1882$, which matches the NPV calculated with wacc.