

Corporate Finance

Question 1. The cost of capital (8 points)

St. Claire Enterprises is a levered firm. The equity cost of capital for St. Claire is 7%. The debt cost of capital for St. Claire is 2%. Assume the CAPM is correct and that the market risk premium is positive. Also assume the risk-free rate is positive, but do not make any assumption about the magnitudes of the market risk premium or the risk-free rate. Explain why it is highly unlikely that the stock return has a negative market beta.

This effectively duplicates a question from the 2nd exam. If the market beta of the stock return is negative, then the expected return to the market is greater than the expected return to the stock. Since the expected return to the debt is less than the expected return to the stock, the debt's beta must be even more negative than the stock's beta. Therefore the debt's return is **more** volatile than the stock's return, in the sense that the debt's return is more sensitive to the market return than is the stock's return. But for any firm, the value of its debt is less volatile than the value of its equity.

Question 2. Volatility (8 points)

Safety First, Inc. has unlevered free cash flows of \$3MM per year. These cash flows will be received in perpetuity and are risk-free. SuperRisky, Inc. has expected unlevered free cash flows of \$3MM per year. Each annual cash flow has a standard deviation of 30%. The uncertainty in these cash flows is entirely idiosyncratic. The risk-free rate is 2% per year. The aggregate stock market has a standard deviation of 15% per year and has an expected return of 6% per year. What is the ratio of the unlevered value of Safety First to the unlevered value of SuperRisky? **SHOW YOUR WORK!**

This question effectively duplicates a question from the 2nd exam. The ratio is one. All risks for SuperRisky are idiosyncratic. In any investor's well-diversified portfolio, these idiosyncratic risks are diversified away. Therefore investors value SuperRisky as if the expected cashflows will be received for sure, which is the same as Safety First.

Question 3. Going to Vegas

Hopkins, Inc., is an all-equity software firm. There are 10 shares of stock in the firm. The firm has a single software project that will generate a single cash flow. The cash flow is risk-free, is received one year from today, and will be \$100. The risk-free interest rate is zero percent. There are no corporate or personal taxes. The firm has no cash.

- a. (3 points) What is the current enterprise value of Hopkins, Inc.? SHOW YOUR WORK!

A trivial problem. Discount \$100 at the riskfree rate of zero to produce \$100. This is the enterprise value.

- b. (3 points) Hopkins, Inc is considering issuing a one-year bond. The bond is a promise to pay \$50 a year from today. The firm will use the proceeds from the bond issuance to buy back shares of stock. What will be the sale price of the bond? (To answer this question, ignore any information in parts (c) and (d) below.) SHOW YOUR WORK!

Another trivial problem. The firm will have \$100 to pay back \$50, so the bond is risk-free. Discount the guaranteed cash flow of \$50 at the risk-free rate of 0% to produce \$50.

- c. (3 points) Does issuing this bond make equityholders better off, worse off, or are they indifferent to it? You can answer this question either by referring to your calculations in part (b) or making a more general argument. SHOW YOUR WORK!

They are indifferent. Without issuance, equityholders have something worth \$100. With the issuance, equityholders put \$50 in their pocket. They will then receive \$50 in a year, which has a present value of \$50. This is an application of the Modigliani-Miller theorem in a perfect market without taxes.

- d. (4 points) Assume for part (d) that the firm has chosen to not issue the bond. Also assume that Hopkins, Inc. has an opportunity to modify its existing project. It can, without investing any resources, increase substantially the uncertainty in the project's future cash flow. If it chooses to do this, there is a 40% probability that the single cash flow equals \$200. There is a 60% probability that the single cash flow equals zero. This uncertainty is completely idiosyncratic. What is the NPV of increasing the uncertainty of the cash flows? Will the firm choose to increase the uncertainty of the cash flows? SHOW YOUR WORK! REMEMBER TO ANSWER BOTH QUESTIONS!

This modification has a negative NPV, therefore the firm will not modify the project. The incremental cash flow of the project is received in one year. It is

200 – 100, probability = 0.4;
0 - 100, probability = 0.6.

The expected value is $100*0.4 - 100*0.6 = 40 - 60 = -20$. The risk is idiosyncratic, so discount at the risk-free rate, which is zero. The NPV is -20.

- e. (12 points) Now repeat question (b), but assume that outside investors know that Hopkins has the option to modify its existing project to raise the cash flow's uncertainty, as described in part (d). What will be the sale price of the bond? SHOW YOUR WORK!

If the firm issues the debt, then the payoff to equityholders at time one is [cash flow - \$50], unless this value is negative. If it is negative, the firm defaults on the debt and equityholders receive nothing. If the firm issues the debt and modifies the project, equityholders possible cash flows at time one are

200 – 50, probability 0.4;
Max(0, 0 – 50), probability 0.6.

The expected time-one cash flow to equity is $\$150*0.4 = 60$. The PV of this cash flow is 60. If the firm issues the debt and does not modify the project, the time-one cash flow to equity is a guaranteed \$50. This has a PV of \$50. Therefore if the firm issues the debt, it will also modify the project. But bondholders can do this math as well as we can. Given that the firm will modify the project, bondholders expect to receive

50, probability 0.4;
0, probability 0.6.

The expected value is \$20 and the present value is also \$20. Therefore the bondholders will pay \$20 for the debt that promises to pay off \$50.

- f. (6 points) Does issuing this bond make equityholders better off, worse off, or are they indifferent to it? You can answer this question either by referring to your calculations in part (e) or by making a more general argument. **SHOW YOUR WORK!**

Worse off. They put \$20 in their pocket and, after modifying the project, have an expected cash flow with a PV of \$60. Issuing the bond thereby costs them \$20. This is precisely equal to the NPV of the modification. Equityholders get the NPV of the project, and bondholders get a fair market value. The modification drops the NPV of the cash flows by \$20, therefore equityholders are worse off by \$20.

- g. (5 points) What perfect-market assumption of Modigliani and Miller is violated in this problem?

The assumption: A firm's capital structure does not affect the cash flows generated by its projects. In this case, the existence of the bond modifies the cash flows.

- h. (10 points) Now assume that the firm is considering issuing a one-year convertible bond. The bond promises to pay \$50 at maturity. But at maturity, bondholders can choose to trade in the bond for 8 (eight) shares of company stock. If it chooses the exchange, the bond obligation will be cancelled and the firm will issue eight new shares, raising the total number of shares to 18. The original shareholders will retain 10 of the shares, hence bondholders will own 8/18 of the firm. Keep the assumption that outside investors know that Hopkins has the option to modify its existing project to raise the cash flow's uncertainty, as described in part (d). What will be the sale price of the convertible bond? Will bondholders ever choose to exercise their option to convert the bond into stock? **SHOW YOUR WORK! REMEMBER TO ANSWER BOTH QUESTIONS!**

I announced in class that there was an error in the structure of the problem. We don't know how many shares the company will have at maturity, since the firm will be buying back shares. The problem should have specified 8/18 and 10/18 without specifying a particular number of shares.

The structure of the solution is as follows. Figure out whether equityholders will increase uncertainty or not. Equityholders make this decision by asking what bondholders will do if uncertainty is increased, then asking what bondholders will do if uncertainty is not increased. Once we know what bondholders will do, we can figure out how much money is left for equityholders. Equityholders then compare the value of increasing uncertainty to the value of not increasing uncertainty and make the choice between the two.

Case 1. Assume the convertible bond is issued and the firm modifies the project. The cash flow is either \$200 or \$0. If the cash flow is \$200, bondholders can hand over the bond and receive 8/18 of the \$200, which is \$88.89. This is greater than the \$50 the bondholders would otherwise receive. Therefore the bondholders will convert if the cash

flow is \$200. Existing equityholders will get 10/18 of the 200, or \$111.11. This payoff to existing equityholders implies that the expected payoff to equityholders at time 1 is

$$111.11 * 0.4 + 0 * 0.6 = 44.44$$

Case 2. Now assume the convertible bond is issued and the firm does not modify the project. The cash flow is \$100. If bondholders convert, they get 8/18 of this \$100, or \$44.44. If they do not convert, they get \$50. Hence they will not convert. In this case, equityholders get \$50.

Now equityholders ask whether they prefer Case 1 or Case 2. Since $\$50 > \44.44 , equityholders prefer Case 2. Equityholders will not modify the project.

Everybody understands this, therefore the bond will have a risk-free payoff of \$50 and a sale price of \$50. The option to convert is never exercised because the cash flow is always 100.

Question 4. Believe it or not

JHU Home Furnishings, Inc., is considering adding a new line of beds to sell in their stores. Adding this new line will have the same economic risk as JHU's existing business. Adding the line will require an immediate (year-zero) expenditure of \$10MM. The firm's expectation is that the new line will produce incremental EBIT of \$4MM per year forever (in perpetuity). For simplicity, assume that the first expected EBIT of \$4MM is received one year from today, and subsequent EBIT is received at annual increments. Also for simplicity, assume that capital expenditures are expected to exactly offset depreciation and that the firm expects no changes will be required in its level of working capital.

JHU's existing capital structure is composed of \$80MM in equity and \$30MM in net debt. It currently has 1MM shares of equity. The unlevered cost of capital is 8% and JHU's debt is risk-free with an interest rate of 2%. The corporate tax rate is 30%. There are no personal taxes.

- a. (8 points) JHU plans to pay for the initial investment by selling new shares of stock. When the firm announces its plan to sell additional shares, it also announces the new project and the expected cash flows. Assume that outside investors were not expecting the additional product line from the firm. If outside investors believe the firm's projections, what will be the new stock price immediately after the announcement? SHOW YOUR WORK!

There is no tax shield in this case. The unlevered cash flows are $\$4\text{MM}(1 - 0.30) = \2.8MM . The PV of the unlevered perpetuity is $\$2.8\text{MM}/0.08 = \35MM . The NPV is $\$35\text{MM} - \$10\text{MM} = \$25\text{MM}$. When the project is announced, the market capitalization immediately jumps to $\$80\text{MM} + \$25\text{MM} = \$105\text{MM}$. The stock price is therefore \$105.

- b. (5 points) Given the assumptions of part (a), how many shares will JHU sell? Immediately after the firm makes the initial investment, what will be the enterprise value of the firm? **SHOW YOUR WORK! REMEMBER THERE ARE TWO QUESTIONS TO ANSWER!**

They need \$10MM. They sell $\$10\text{MM}/\105 per share = 95,238 shares. The enterprise value is $(\$105\text{MM} + \$10\text{MM})$ equity + \$30MM debt = \$145MM.

- c. (6 points) Suppose that after hearing JHU's announcement, outside investors believe the incremental EBIT from JHU's new product line will be only \$1MM/year. What will be the new stock price immediately after the announcement? SHOW YOUR WORK!

The unlevered cash flows expected by outside investors are $\$1\text{MM}(1 - 0.30) = \0.7MM . The PV of the unlevered perpetuity is $\$0.7\text{MM}/0.08 = \8.75MM . The NPV is $\$8.75\text{MM} - \$10\text{MM} = -\$1.25\text{MM}$. When the project is announced, the market capitalization immediately drops to $\$80\text{MM} - 1.25\text{MM} = \78.75MM . The stock price is therefore \$78.75.

- d. (4 points) Given the assumptions of part (c), how many shares will JHU sell? Immediately after the firm makes the initial investment, what will be the enterprise value of the firm? SHOW YOUR WORK! REMEMBER THERE ARE TWO QUESTIONS TO ANSWER!

They need \$10MM. They sell $\$10\text{MM}/\78.75 per share = 126,984 shares. The enterprise value is $\$(78.75\text{MM} + \$10\text{MM})$ equity + \$30MM debt = \$118.75MM.

- e. (6 points) Assume that part (c) describes outside investors' initial expectations, and that the firm issues shares as in part (d). Within a few days after the initial investment of \$10MM, new information reveals to outside investors that the firm's projections of future EBIT are accurate. What is the stock price when this information is revealed? What is the enterprise value of the firm? SHOW YOUR WORK! REMEMBER THERE ARE TWO QUESTIONS TO ANSWER!

Outside investors now expect the perpetuity to produce $\$4\text{MM}(1 - 0.3)$ forever. The PV of this perpetuity is, as previously calculated, \$35MM. The old belief was that the perpetuity was worth \$8.75MM, so the market value of equity increases by \$26.25MM to \$115MM, just as in part (a). The enterprise value is \$145MM, just as in part (b). But the stock price is $\$115\text{MM}/(1\text{MM} + 126,984 \text{ shares}) = \$102.04/\text{share}$.

- f. (8 points) Suppose JHU instead pays for the initial investment by issuing \$10MM in perpetual risk-free debt. After the new information discussed in part (e) is revealed, what is the firm's stock price? What is the enterprise value of the firm? Comparing your answers with those in part (e), what are the two advantages of debt financing in this case? **SHOW YOUR WORK! REMEMBER THERE ARE THREE QUESTIONS TO ANSWER!**

The enterprise value is the enterprise value of the unlevered firm, which is calculated in question (e), plus the incremental value of the tax shield. Because it is perpetual, the value of the tax shield is the amount of the debt times the corporate tax rate, or \$10MM * 0.3 = \$3MM. Therefore the enterprise value is \$148MM. The debt is worth \$40MM, therefore the equity is worth \$108MM. There are 1MM shares, so the stock price is \$108. This exceeds the stock price in part (e) of \$102.04 for two reasons. First, issuing the debt avoided the issuance of undervalued equity, benefiting existing shareholders. Second, issuing the debt took advantage of the tax shield of debt.

Question 5. Chaska Bricks

Chaska Bricks Inc. is a levered firm. The expected return to its equity is 12% per year and the expected return to its debt is 8% per year. The market capitalization of its equity is \$200MM. The firm has an enterprise value of \$300MM. The firm is profitable and has a corporate tax rate of 35%. There are no personal taxes.

- a. (4 points) What is the firm's weighted average cost of capital? **SHOW YOUR WORK!**

$$Wacc = (200/300)*0.12 + (100/300)*0.08(1 - \text{tax rate}) = 0.0973333$$

- b. (5 points) Chaska Bricks is considering expanding its production of a special type of brick used in the Olympics. The expected cash flows of this project are listed in the following table. The numbers are in millions of dollars.

Year	0	1	2	3
Unlevered free cash flow	-20	2	20	10

Calculate the NPV of this project, assuming that Chaska Bricks maintains its current debt/equity ratio. **SHOW YOUR WORK!**

$$NPV = -20 + 2/1.0973333 + 20/1.0973333^2 + 10/1.0973333^3 = 6.$$

- c. (6 points) What is the expected debt capacity of this project in years 0, 1, and 2, evaluated immediately after the cash flow in the table? In other words, calculate the debt capacity in year zero immediately after the firm spends 20 on the project, then calculate the debt capacity in year one immediately after the firm receives the year-one cash flow, and so on. SHOW YOUR WORK!

Their debt/enterprise value ratio is $100/300 = 1/3$. The continuation values of the projects at years 0, 1, and 2 are the PVs of the future cash flows, discounted at the weighted average cost of capital. These PVs are 26MM, 26.53068MM, and 9.1113MM respectively. Multiply by 1/3 to get debt capacity. The results are 8.666MM, 8.84356MM, and 3.037MM.

- d. (5 points) Assume that Chaska Bricks takes on the project. At year zero, how much equity should the firm issue (or buy back) and how much debt should it issue (or repay) to maintain a constant D/E ratio, taking into account the funds required for the initial investment? SHOW YOUR WORK!

Because of the project, the firm's enterprise value at time zero grows by the sum of the NPV of the project and the time-zero investment, or $6\text{MM} + 20\text{MM} = 26\text{MM}$. As the debt capacity calculation indicates, 8.666MM of this must be in the form of new debt. Therefore the firm will issue 8.666MM in new debt. But it still needs $\$20\text{MM} - 8.666\text{MM}$ to pay for the investment. Therefore the firm issues $\$11.334\text{MM}$ in equity.

Question 6. Minnesota Moccasins

You are a financial consultant working for Minnesota Moccasins. The firm is considering creating a new shoe line. The initial investment is $\$15\text{MM}$. You estimate that the shoe line will produce unlevered free cash flow of $\$1\text{MM}$ the first year. You believe this free cash flow will grow at a rate of 3% per year. Minnesota Moccasins has an equity cost of capital of 12%, a debt cost of capital equal to the risk-free rate of 7%, and a corporate tax rate of 35%. Minnesota Moccasins has a debt/equity ratio of 0.6. Assume that the firm wants to keep its D/E ratio at this value. The firm has no cash. There are no personal taxes.

- a. (6 points) Assuming the firm does not alter its debt/equity ratio, what is the NPV of the new shoe line, including the tax shield? **SHOW YOUR WORK!**

Since $D/E = 0.6$, $(D+E)/E = 1.6$. Hence $E/(E+D) = 1/1.6 = 0.625$. Therefore $D/(E+D) = 0.375$. The wacc is

$Wacc = 0.625 * .12 + 0.375 * .07 * (1 - 0.35) = 0.0920625$. Use this discount rate to calculate the PV of the growing perpetuity. The value is

$PV = 1/(0.0920625 - 0.03) = \$16.11279MM$. The NPV of the project is this PV minus the initial investment of \$15MM. Therefore the NPV is \$1.11279MM.

- b. (4 points) If the firm adopts the project, how much debt will Minnesota Moccasins take on at year zero? **SHOW YOUR WORK!**

The incremental enterprise value is the initial investment of \$15MM plus the NPV, or equivalently it is the PV of the growing perpetuity. Multiply this by $D/(E+D) = 0.375$ to get the additional debt amount of \$6.0423MM.

- c. (5 points) What is the time-zero present value of the project's tax shield? **SHOW YOUR WORK!**

The easiest way to do this is to subtract from the NPV of the project the NPV of the unlevered firm. To get this value, we need the discount rate for the unlevered firm. Since the firm has a constant D/E ratio, the unlevered discount rate is

$$r_u = (E/(E+D))r_E + (D/(E+D))r_D = 0.625 * 0.12 + 0.375 * 0.07 = 0.10125.$$

Use this discount rate to value the growing perpetuity. The result is

$$1/(0.10125 - 0.03) = \$14.035088MM.$$

The difference between the levered PV and the unlevered PV of the growing perpetuity is the value of the tax shield, or \$2.0777MM.

Another way to calculate this, once we have the unlevered discount rate, is to compute the year-one cash flow to the tax shield of $0.07 * 0.35 * D$, where D is the answer to (b). This cash flow is 0.1480364MM. The cash flow is expected to grow at 3%/year and is discounted at the unlevered discount rate. The answer is identical.

- d. (5 points) The CEO of Minnesota Moccasins tells you that she wants to have a somewhat different capital structure for this project, relative to the other projects of the firm. Since the firm has no cash, its net debt (borrowing minus cash) currently equals its gross debt (borrowing). The CEO wants to finance the project using a ratio of (gross debt)/equity equal to 0.6. However, to cover any unexpected situations, she also wants to have \$2MM in cash available. This cash will be invested at the risk-free rate in perpetuity unless it is needed in an emergency. You believe the cash is not needed, but she's the boss. What is the NPV of the project with this altered capital structure? SHOW YOUR WORK!

The cash is like negative perpetual debt. The tax shield associated with this cash is negative and equal to the usual perpetual debt tax shield formula,

Market value = corporate tax rate * D.

In this case, the market value is $0.35 * (-2\text{MM}) = -0.7\text{MM}$. The new NPV of the project is $\$1.11279\text{MM} - 0.7\text{MM} = 0.41279\text{MM}$.

- e. (5 points) When the capital structure of the project is described by part (d), you expect the project's weighted average cost of capital to decline over time. Explain why, keeping in mind that the cash flows are expected to increase over time.

The project's ratio of (net debt)/(value of the project) will increase over time. Both the equity and the borrowing will increase over time, but the cash holding will stay the same. Eventually the cash holding will be trivially small compared to the value of the project, and the wacc is back to the value calculated in part (a).

Question 7. The U.S. Congress (6 points)

In the wake of the financial crisis, many legislators are concerned about excessive leverage. One way to reduce leverage is to set the corporate tax rate to zero to increase the incentive to finance with equity rather than debt. Of course, that lowers tax revenue, which will increase the deficit. Therefore legislators are considering cutting corporate tax rates by the minimum amount necessary to eliminate the tax advantage of debt. Assume the tax rate on interest income is 30 percent and the tax rate on equity income is 5 percent. Adopt all of the perfect-market assumptions of Modigliani and Miller except the no-tax assumption. What is the highest corporate tax rate that does not encourage firms to issue debt? **SHOW YOUR WORK!**

Use the formula for cash flow to the tax shield,

$$\text{cash flow}_t = r_D \left(\tau_c - \frac{\tau_i - \tau_e}{1 - \tau_e} \right) D_{t-1}$$

What corporate tax rate sets the cash flow to zero? It is $(0.30 - 0.05)/(1 - 0.05) = 0.25/0.95 = 0.26316$, or 26.316 percent.

Question 8. An ex-dividend stock

Q-Group, Inc. is about to pay a \$10/share dividend. The price of the stock the day before it went ex-dividend was \$82 and the price of the stock the next day was \$73.50.

- a. (5 points) Assuming the price drop resulted only from the stock going ex-dividend, what does this decline in the price imply about the effective dividend tax rate for Q-Group? **SHOW YOUR WORK!**

The (negative of) the change in the stock price divided by the dividend amount is one minus the effective dividend tax rate. In this case, we have

$$(82 - 73.50)/10 = 8.5/10 = 0.85 = 1 - (\text{effective dividend tax rate}).$$

Therefore the effective dividend tax rate is 15%.

- b. (5 points) Based on this information and what you know about current tax rates on dividend and capital gains income, which one of these groups is more likely to be the predominant investor in Q-Group stock? The groups are (1) buy-and-hold individual investors (very long-term investing), (2) one-year individual investors (investing with a plan to sell after about a year), and (3) pension funds.
EXPLAIN YOUR ANSWER!

The effective dividend tax rate is

$(\text{tax rate on dividends} - \text{effective tax rate on capital gains}) / (1 - \text{effective tax rate on capital gains})$.

Pension funds have a zero tax rate on both dividends and capital gains. Therefore their effective dividend tax rate is zero. Investors who plan to sell after about a year are not deferring their capital gains. Since the statutory tax rates on capital gains and dividends are equal, the effective dividend tax rate for these investors is also zero. But buy-and-hold investors have an effective capital gains rate close to zero because they do not realize their capital gains for many years. Their effective dividend tax rate is therefore the tax rate on dividend income, which is 15%. Therefore the answer is buy-and-hold individual investors.