More tutorial at www.dumblittledoctor.com

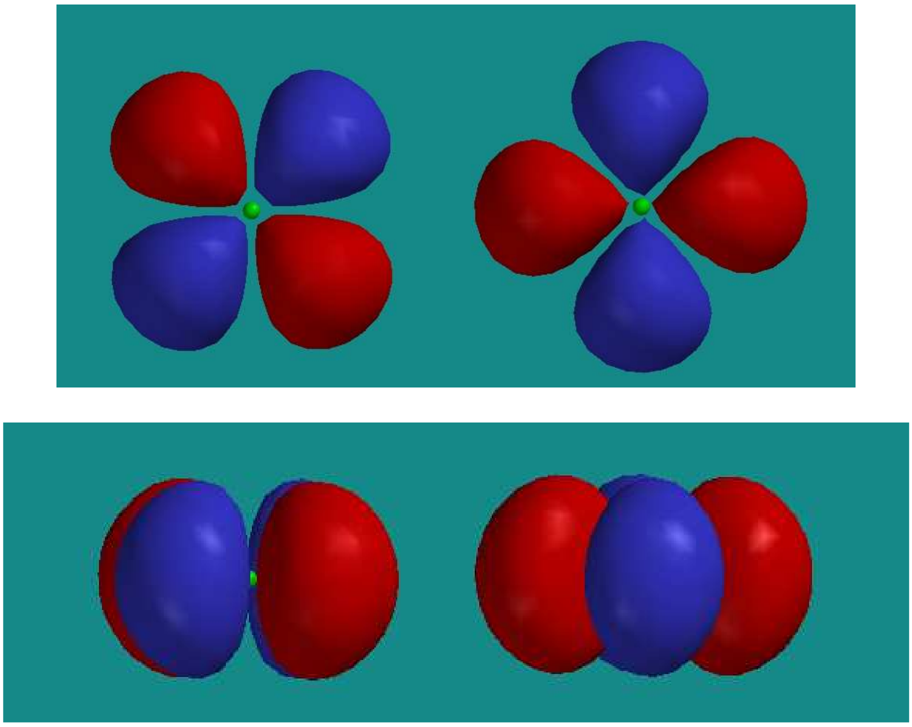
FUNDAMENTAL CONSTANTS AND DATA USED IN THE PROBLEMS

N_A = 6.022 x 10^{23} molecules/mole   h = 6.626 x 10^{-34} J-
c = 2.9979 x 10^{8} m/s   *F* = 96485 Coulomb/mole

m(^{4}He) = 4.0026 u (amu)
m(^{235}U) = 235.0439 u (amu)
m(^{207}Pb) = 206.9759 u (amu)

Addendum: The two orbitals in Problem 3 shown in 2 perspectives

Left **column**: the first orbital from 2 perspectives
Right **column**: the second orbital from 2 perspectives
Top row: the two orbitals shown
with the z axis perpendicular to the page
(presentation shown on the examination)
Bottom row: the **same** two orbitals
shown with the z axis vertical and
in the plane of the page



first orbital   second orbital

The insert contains a periodic table and useful information. You may use the back for scratch work but enter **all** work to be graded in the space provided with each question. Show your work in **all** problems involving computation in order to receive credit.

1) (55 points) Today the earth with a total mass of 6.0×10^{30} g contains approximately 8.0×10^{16} g of uranium-235.

a) The age of the earth is 4.6×10^{9} y. Calculate the mass of uranium-235 present in the earth at its creation. The half life of uranium-235 is 7.0×10^{8} y.

$N = N_0\exp(-kt)$ to $N_0 = N\exp(kt)$. The mass of uranium-235 is directly proportional to the number of atoms so $m_0 = m \times \exp(kt)$.
$k = \ln(2)/\tau = \ln(2)/(4.6 \times 10^{9}\ y) = 9.9 \times 10\text{-}10\ y^{-1}$
$m_0 = (8.0 \times 10^{16}\ g) \times \exp[(9.9 \times 10\text{-}10\ y^{-1})(4.6 \times 10^{9}\ y)] = 7.6 \times 10^{18}$ g
Note that most of the uranium-235 has decayed. This amount is 3.2×10^{16} mole and 2.0×10^{40} atom.

b) The decay of uranium-235 to stable lead-207, which goes through several intermediates, has the **net** stoichiometry $^{235}U \rightarrow {}^{207}Pb + 7\,{}^{4}He$. Using the data on the insert, calculate the total energy per atom that is released from the decay of uranium-235 and its daughters.

$\Delta m = m(^{235}U) - m(^{207}Pb) - 7m(^{4}He) = 235.0439 - 206.9759 - 7(4.0026) = 0.0498$ u $= 0.0498$ g/mole $= 4.98 \times 10^{-5}$ Kg/mole $= 8.27 \times 10\text{-}^{29}$ Kg/atom$\Delta E = (\Delta m)c^2$
$\Delta E = (8.27 \times 10\text{-}^{29}\ \text{Kg/atom})(2.9979 \times 10^{8}\ m/s) = 7.43 \times 10^{-12}$ J/atom
[Alternatively, one could multiply the mass defect in amu by the conversion factor 931.494 Mev/u to obtain the answer in Mev.]
Multiply this result by the total number of uranium-235 atoms present at the origin of the earth and one obtains a large number, 1.5×10^{29} J!! The decay of uranium-235 is the major engine that drives tectonic activity on the earth.

c) Suggest why ^{4}He is such a prominent product in the decay of uranium-235?

The result is a consequence of the instability of the reactant and the stability of the product. Nuclei beyond bismuth, e.g. uranium-235, are unstable. Alpha emission yields a decrease in the mass and atomic numbers. A cascade of alpha emissions is required to reach the region of stability with a desirable binding energy per nucleon. The helium nucleus with a closed nuclear shell configuration is very stable.

d) Lead-211, one of the daughters in the decay scheme of uranium-235, decays via beta emission. Provide the other product(s) produced in lead-211's beta decay.

^{211}Bi and an antineutrino. The reaction is $^{211}Pb \rightarrow {}^{211}Bi + \nu + e^{-1}$

2) (75 points) Rubidium (Z = 37) is a rare member of the family of alkali metals. Its first ionization energy is 4.18 eV (403 kJ/mol).

a) Show via a calculation that the effective nuclear charge Z_e of rubidium's valence electron is 2.77. (Hint: use the value of the ionization energy.)

The configuration of ground-state rubidium is [Kr] (5s) so n of the sole valence electron is 5. Recall that $E(n) = -RZ_e^2/n^2$.
$IE = E(n = \infty) - E(n = 5) = -E(n = 5) = RZ_e^2/5^2$
$Ze = [(25)IE/R]^{0.5} = [(25 \times 4.18 \text{ eV})/(13.6 \text{ eV})]^{0.5} = 2.77$

b) Interpret the value of Z_e. That is, is a result with this order of magnitude expected or is it anomalous?

If the 5s electron never penetrated the Krypton core, the core electrons would screen out all of the nuclear charge except for one. Hence, Z_e would be exactly one. However, electrons do not move and fixed orbits and the 5s electron which has no centrifugal force will spend part of its time inside the core and see more of the full nuclear charge. Z_e is expected to be greater than one but still close to 1.

c) Estimate via a calculation the atomic radius of rubidium.

$\langle r \rangle = a_0 n^2/Z_e = (0.529 \text{ Å})(5^2)/(2.77) = 4.7 \text{ Å}$

d) When gaseous rubidium is excited in an electrical discharge, the excited atoms emit light. The wavelength of a prominent line in rubidium's emission spectrum is 420.2 nm. Calculate the energy of one mole of photons with this wavelength.

$E = N_A E_\gamma = N_A h\nu = N_A hc/\lambda$
$E = (6.0221 \times 10^{23})(6.6261 \times 10^{-34} \text{ J-s})(2.9979 \times 10^8 \text{ m/s})/(420.2 \times 10^{-9} \text{ m})$
$E = 284.7 \text{ kJ}$

e) As one increases the energy of the valence electron in rubidium, does the uncertainty in its position increase or decrease? What is the corresponding change in the uncertainty of the electron's momentum?

An increase the energy of the valence electron in accompanied by an increase in n. Therefore the radius of the atom increases. The electron is found in a larger space and the uncertainty of its exact position increases. By the Heisenberg Uncertainty Principle, the uncertainty of the electron's momentum will decrease.

f) Will a magnetic field deflect ground-state rubidium atoms? Briefly explain.

Rubidium has one unpaired electron and therefore is paramagnetic. As a result of its magnetism, it will be deflected by a magnetic field. This is the basis of famous Stern-Gerlach experiment where silver was used.

3) (30 points) A pair of orbitals from the same atomic species is shown below. In the figure, the z axis is perpendicular to the page and the x,y plane lies the plane of the sheet.

a) What is the value of the *l* (azimuthal) quantum number of an electron in these orbitals? Briefly provide the basis for your answer.

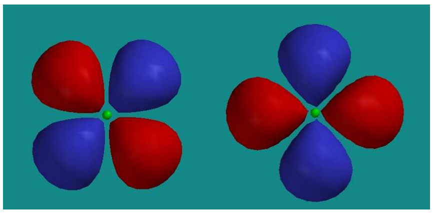
If there are no hidden nodes, the total number of angular nodes is 2 so *l* equals 2.

b) What is the relationship between these orbitals? How is this relationship tied to quantum numbers?

We see two nodal planes that contain the z axis so $|m_l|$ is 2. The two orbitals are related by a 90º/2 = 45º rotation about the z axis.

c) Will an electron in these orbitals ever be found at or close to the nucleus?

The angular nodes have the result that the Wavefunction and therefore the electron density will be zero at the nucleus. The orbital angular momentum of the electron leads to a strong centrifugal force preventing the electron from getting close.



4) (40 points) The division between metals and metals is shown on periodical tables as a roughly diagonal, zig-zag line at the right of the table. [This boundary is a bold-faced line on the insert.] Discuss succinctly but thoroughly how the characteristics of this boundary follow from the fundamental principles of atomic structure.

The location of non-metals to the right of the periodic table follows from the Bohr equation for orbital energies which was used in Problem 2a. In any row of the table, the n quantum number of the valence electrons is fixed. Valence electrons provide incomplete screening for one another so as one moves from left to right, the effective nuclear charge increases with the full nuclear charge. Consequently, the ionization energy (IE) which is directly proportional to the square of Ze shows a striking increase. At the left, the IE is small and the element is metallic; at the right the IE is large and the material is a non-metal. The transition occurs somewhere in between and is not sudden. One observes a band of elements with properties of both metals and non-metals. The key feature is the location of the boundary to the right. The boundary shifts further to the right as one moves down the periodic table. The decrease of IE in a column is due to approximate constancy of Ze and the steady increase in n. That is, elements at the bottom are more metallic than those at the top. As a result, the boundary between metal and non-metal is shifted to the right as one moves down the table.

A correct answer here focuses on the primary sources of the result rather than “symptoms” such as electron configuration. Electron affinities can be misleading as both metals and non-metals have negative EA’s.