11 Mathematics

11.1 Introduction

TEX is at its best while producing mathematical documents. If you want to test the power of TEX, do typeset some mathematics. In the foreword of the TEX book, Knuth writes: "TEX is a new typesetting system intended for the creation of beautiful books—and especially for books that contain a lot of mathematics".

LATEX has a special mode for typesetting mathematics. Mathematical text within a paragraph (in-line) is entered between `\(` and `\)`, between `$` and `$` or between `\begin{math}` and `\end{math}`.

Normally larger mathematical equations and formula are typesetted in separate lines, in display mode. To produce this, we enclose them between `\[` and `\]`, between `$$` and `$$` or between `\begin{displaymath}` and `\end{displaymath}`. This produces formula, which are not numbered. If we want to produce equation number, we have to use equation environment.

The spacing for both in-line and displayed mathematics is completely controlled by TEX.

11.2 Maths in text

input—file

```
Using~(5.64) and the fact that the
$c_n=\langle\psi_n\vert\Psi\rangle$
and $d_n^*=\langle X\psi_n\rangle$,
the scalar product $\langle X\vert
\Psi\rangle$ can be expressed in the
way as $\langle X\vert\Psi\rangle=
\sum_nd_n^*c_n = \mathbf{d}^\dagger
\boldsymbol{\cdot}\mathbf{c}$ where
\(\mathbf{c}\) is a column vector
with elements $c_n$ and row vector
$\mathbf{d}^\dagger$ with elements
$d_n^*$. The inverse $\mathbf{A}^{-1}$
of a matrix $\mathbf{A}$ is such that
$\mathbf{AA}^{-1}=\mathbf{A}^{-1}
\mathbf{A}= \mathbf{I}$.
```

output—dvi

Using (5.64) and the fact that the $c_n = \langle\psi_n|\Psi\rangle$ and $d_n^* = \langle X\psi_n\rangle$, the scalar product $\langle X|\Psi\rangle$ can be expressed in the way as $\langle X|\Psi\rangle = \sum_n d_n^* c_n = \mathbf{d}^\dagger \cdot \mathbf{c}$ where $\mathbf{c}$ is a column vector with elements c_n and row vector $\mathbf{d}^\dagger$ with elements d_n^*. The inverse $\mathbf{A}^{-1}$ of a matrix $\mathbf{A}$ is such that $\mathbf{A}\mathbf{A}^{-1} = \mathbf{A}^{-1}\mathbf{A} = \mathbf{I}$.

Where $\mathbf{I}$ is the unit matrix, elements $I_{mn} = \delta_{mn}$. For a *stationary state* $\Psi_E = \psi_E \exp(-\mathrm{i}Et/\hbar)$ and a *time-independent* operator A it is clear that the expectation value $\langle\Psi_E|A|\Psi_E\rangle = \langle\psi_E|A|\psi_E\rangle$ does not depend on the time.

```
Where $\mathbf{I}$ is the unit matrix, elements $I_{mn}=\delta_{mn}$. For a
\emph{stationary state} $\Psi_E=\psi_E\exp(-{\rm i}Et/\hbar)$ and a
\emph{time-independent} operator $A$ it is clear that the expectation value
\begin{math}\langle\Psi_E\vert A\vert\Psi_E\rangle=\langle\psi_E\vert
A\vert\psi_E\rangle\end{math} does not depend on the time.
```

11.3 Fraction

```
$$
   \frac{{\rm d}\varepsilon}{{\rm d}\varepsilon}\qquad
   \frac{\frac{a}{x-y}+\frac{b}{x+y}}{1+\frac{a-b}{a+b}}
$$
```

$$\frac{\mathrm{d}\varepsilon}{\mathrm{d}\varepsilon} \qquad \frac{\frac{a}{x-y}+\frac{b}{x+y}}{1+\frac{a-b}{a+b}}$$

11.4 Equation

Don't put blank lines between the dollar signs delimiting the mathematical text. TEX assumes that all the mathematical text being typeset is in one paragraph, and a blank line starts a new paragraph; consequently, this will generate an error message.

11.4.1 Equation with numbers

```
\begin{equation}
 \varphi(x,z) = z - \gamma_{10} x - \sum_{m+n\ge2} \gamma{mn} x^m z^n
\end{equation}
```

$$\varphi(x,z) = z - \gamma_{10} x - \sum_{m+n\ge2} \gamma{mn} x^m z^n \tag{1}$$

11.4.2 Equation without numbers

```
\begin{displaymath}
   \left(\int_{-\infty}^{\infty} e^{-x^2}\right)
   =\int_{-\infty}^{\infty}\int_{-\infty}^{\infty}e^{-(x^2+y^2)}dx\,dy
\end{displaymath}
```

OR

```
$$
   \left(\int_{-\infty}^{\infty} e^{-x^2}\right)
   =\int_{-\infty}^{\infty}\int_{-\infty}^{\infty}e^{-(x^2+y^2)}dx\,dy
$$
```

$$\left(\int_{-\infty}^{\infty} e^{-x^2}\right) = \int_{-\infty}^{\infty}\int_{-\infty}^{\infty} e^{-(x^2+y^2)}dx\,dy$$

```
\[
   \left(\int_{-\infty}^{\infty} e^{-x^2}\right)
   =\int_{-\infty}^{\infty}\int_{-\infty}^{\infty}e^{-(x^2+y^2)}dx\,dy
\]
```

$$\left(\int_{-\infty}^{\infty} e^{-x^2}\right) = \int_{-\infty}^{\infty}\int_{-\infty}^{\infty} e^{-(x^2+y^2)}dx\,dy$$

11.4.3 Subequations[1]

```
\begin{subequations}
\begin{equation}
   \langle\Psi_1\vert\Psi_2\rangle\equiv\int\Psi_1^*
   (\mathbf{r})\Psi_2 (\mathbf{r}){\rm d}\mathbf{r}
\end{equation}
and
\begin{equation}
   \langle\Psi_1\vert\Psi_2\rangle\equiv\Psi_1^*(\mathbf{r}_1,\ldots,
   \mathbf{r}_N)\Psi_2(\mathbf{r}_1,\ldots,\mathbf{r}_N){\rm d}
   \mathbf{r}_1\ldots{\rm d}\mathbf{r}_N.
\end{equation}
\end{subequations}
```

$$\langle\Psi_1|\Psi_2\rangle \equiv \int \Psi_1^*(\mathbf{r})\Psi_2(\mathbf{r})\mathrm{d}\mathbf{r} \tag{2a}$$

and

$$\langle\Psi_1|\Psi_2\rangle \equiv \Psi_1^*(\mathbf{r}_1,\ldots,\mathbf{r}_N)\Psi_2(\mathbf{r}_1,\ldots,\mathbf{r}_N)\mathrm{d}\mathbf{r}_1\ldots\mathrm{d}\mathbf{r}_N. \tag{2b}$$

11.4.4 Framed displayed equation

```
\begin{equation}
   \fbox{$\displaystyle\int_0^\infty f(x)\,{\rm d}x
   \approx\sum_{i=1}^nw_i{\rm e}^{x_i}f(x_i)$}
\end{equation}
```

$$\boxed{\int_0^\infty f(x)\,\mathrm{d}x \approx \sum_{i=1}^{n} w_i \mathrm{e}^{x_i} f(x_i)} \tag{3}$$

11.4.5 Multiline equations – Eqnarray

```
\begin{eqnarray}
   \bar\varepsilon &=& \frac{\int_0^\infty\varepsilon
   \exp(-\beta\varepsilon)\,{\rm d}\varepsilon}{\int_0^\infty
   \exp(-\beta\varepsilon)\,{\rm d}\varepsilon}\nonumber\\
&=& -\frac{{\rm d}}{{\rm d}\beta}\log\Biggl[\int_0^\infty\exp
   (-\beta\varepsilon)\,{\rm d}\varepsilon\Biggr]=\frac1\beta=kT.
\end{eqnarray}
```

$$\begin{aligned}\bar{\varepsilon} &= \frac{\int_0^\infty \varepsilon \exp(-\beta\varepsilon)\,\mathrm{d}\varepsilon}{\int_0^\infty \exp(-\beta\varepsilon)\,\mathrm{d}\varepsilon} \\ &= -\frac{\mathrm{d}}{\mathrm{d}\beta}\log\left[\int_0^\infty \exp(-\beta\varepsilon)\,\mathrm{d}\varepsilon\right] = \frac{1}{\beta} = kT.\end{aligned} \tag{4}$$

\nonumber is used for suppressing number.

[1] subeqn.sty package should be loaded.

11.4.6 Matrix

```
$$
   \matrix{1 & 2 & 3\cr 2 & 3 & 4\cr 3 & 4 & 5}\qquad
   \left(\matrix{1 & \cdots & 3\cr 2 & \vdots & 4\cr
                 3 & \ddots & 5}\right)
$$
```

$$\begin{matrix}1 & 2 & 3\\ 2 & 3 & 4\\ 3 & 4 & 5\end{matrix} \qquad \begin{pmatrix}1 & \cdots & 3\\ 2 & \vdots & 4\\ 3 & \ddots & 5\end{pmatrix}$$

11.4.7 Array

```
$$
  \begin{array}{lcll}
      \Psi(x,t) &=& A({\rm e}^{{\rm i}kx}-{\rm e}^{-{\rm i}kx})
                    {\rm e}^{-{\rm i}\omega t}&\\
                &=& D\sin kx{\rm e}^{-{\rm i}\omega t}, & D=2{\rm i}A
  \end{array}
$$
```

$$\begin{array}{lcll}\Psi(x,t) &=& A(\mathrm{e}^{\mathrm{i}kx}-\mathrm{e}^{-\mathrm{i}kx})\mathrm{e}^{-\mathrm{i}\omega t}\\ &=& D\sin kx\mathrm{e}^{-\mathrm{i}\omega t}, & D=2\mathrm{i}A\end{array}$$

11.4.8 Cases

```
$$
  \psi(x)=\cases{A{\rm e}^{{\rm i}kx}+B{\rm e}^{{-\rm i}kx},
                                & for $x=0$\cr
            D{\rm e}^-{\kappa x}, & for $x=0$.}
$$
```

$$\psi(x)=\begin{cases}A\mathrm{e}^{\mathrm{i}kx}+B\mathrm{e}^{-\mathrm{i}kx}, & \text{for } x=0\\ D\mathrm{e}^{-\kappa x}, & \text{for } x=0.\end{cases}$$

11.4.9 Stackrel

```
$$
   a\stackrel{def}{=} \alpha + \beta\quad
   \stackrel{thermo}{\longrightarrow}
$$
```

$$a \stackrel{def}{=} \alpha+\beta \quad \stackrel{thermo}{\longrightarrow}$$

11.4.10 Atop

```
$$
   \sum_{k=1 \atop k=0} \qquad
\sum_{123 \atop{234 \atop {890 \atop 456}}}
$$
```

$$\sum_{\substack{k=1\\k=0}} \qquad \sum_{\substack{123\\234\\890\\456}}$$

11.4.11 Square root

```
$$
   \sqrt[n]{\frac{x^n-y^n}{1+u^{2n}}}
$$
```

$$\sqrt[n]{\frac{x^n-y^n}{1+u^{2n}}}$$

11.4.12 Choose

```
$$
   {123 \choose 456}\qquad {x^n-y^n \choose 1+u^{2n}}
$$
```

$$\binom{123}{456} \qquad \binom{x^n - y^n}{1 + u^{2n}}$$

11.5 Definitions for Theorems

We should define \newtheorem{*thm*}{*Theorem*} etc in preamble.

```
\newtheorem{thm}{Theorem}
\begin{thm}
This is body matter for testing this environment.
\end{thm}
```

Theorem 1 *This is body matter for testing this environment.*

```
\newtheorem{rmk}{Remark}[section]
\begin{rmk}
This is body matter for testing this environment.
\end{rmk}
```

Remark 11.5.1 This is body matter for testing this environment.

```
\newtheorem{col}{Corollary}
\begin{col}[Richard, 1987]
This is body matter for testing this environment.
\end{col}
```

Corollary 1 (Richard, 1987) *This is body matter for testing this environment.*

```
\newtheorem{lem}{Lemma}[thm]
\begin{lem}
This is body matter for testing this environment.
\end{lem}
```

Lemma 1.1 *This is body matter for testing this environment.*

```
\newtheorem{exa}{Example}[lem]
\begin{exa}
This is body matter for testing this environment.
\end{exa}
```

Example 1.1.1 *This is body matter for testing this environment.*

11.6 AMS-LaTeX[2]

Following are some of the component parts of the amsmath package, available individually and can be used separately in a \usepackage command:

amsbsy defines the amsmath \boldsymbol and (poor man's bold) \pmb commands.

amscd defines some command for easing the generation of commutative diagrams.

amsfonts defines the \frak and \Bbb commands and set up the fonts msam (extra math symbols A), msbm (extra math symbols B, and blackboard bold), eufm (Euler Fraktur), extra sizes of cmmib (bold math italic and bold lowercase Greek), and cmbsy (bold math symbols and bold script), for use in mathematics.

[2] CTAN: /tex-archive/macros/latex/packages/amslatex

amssymb defines the names of all the math symbols available with the $\mathcal{AMS}$ fonts collection.

amstext defines the amsmath `\text` command.

11.6.1 Align environment

Align environment is used for two or more equations when vertical alignment is desired (usually binary relations such as equal signs are aligned).

```
\begin{align}
 F_{\rm fer}(k)  =& -\frac{16 x_0 ^3 t}{3\pi }\bigg( \sum_{l=1}^\infty
                    -\frac{\nu^5}{t^4 (x_0^2-l-\frac{1}{4})^3}\bigg[S
                     \bigg(\frac{\sqrt{x_0^2+l^2}}{t};2 \bigg)
                    + 2S\bigg(\frac{\nu}{t};2 \bigg)\bigg] \bigg)\\
 F_{\rm red}(t) =& -\frac{16 x_0 ^3 t}{3\pi }\sum_{l=1}^\infty
                    \bigg\{ \frac{1}{2\nu (x_0^2+l^2)^2} \nonumber\\
                & -\frac{\nu^5}{t^4 (x_0^2-l-\frac{1}{4})^3}\bigg[S
                    \bigg( \frac{\sqrt{x_0^2+l^2}}{t};2 \bigg)
                    +2S\bigg(\frac{\nu}{t};2 \bigg)\bigg] \nonumber\\
                & +V(x_e ,x_{\alpha})  -g \delta (x_e - x_{\alpha}) \bigg\}.
\end{align}
```

$$F_{\rm fer}(k) = -\frac{16x_0^3 t}{3\pi}\left(\sum_{l=1}^{\infty} -\frac{\nu^5}{t^4(x_0^2 - l - \frac{1}{4})^3}\left[S\left(\frac{\sqrt{x_0^2+l^2}}{t};2\right) + 2S\left(\frac{\nu}{t};2\right)\right]\right) \tag{5}$$

$$\begin{aligned} F_{\rm red}(t) = -\frac{16x_0^3 t}{3\pi}\sum_{l=1}^{\infty}\Bigg\{ & \frac{1}{2\nu(x_0^2+l^2)^2} \\ & -\frac{\nu^5}{t^4(x_0^2 - l - \frac{1}{4})^3}\left[S\left(\frac{\sqrt{x_0^2+l^2}}{t};2\right) + 2S\left(\frac{\nu}{t};2\right)\right] \\ & + V(x_e, x_\alpha) - g\delta(x_e - x_\alpha)\Bigg\}. \end{aligned} \tag{6}$$

11.6.2 Gather environment

Gather environment is used for two or more equations, but when there is no alignment desired among them each one is centered separately between the left and right margins.

```
\begin{gather}
 \frac{\int_0^\infty\varepsilon\exp(-\beta\varepsilon)\,{\rm d}
 \varepsilon}{\int_0^\infty\exp(-\beta\varepsilon)\,{\rm d}\varepsilon}
 \frac{\int_0^\infty\varepsilon\exp(-\beta\varepsilon)\,{\rm d}\varepsilon}
 {\int_0^\infty\exp(-\beta\varepsilon)}\\
 \int_0^\infty\exp(-\beta\varepsilon)\,{\rm d}\exp(-\beta\varepsilon)\nonumber\\
 \frac{\int_0^\infty\varepsilon\exp(-\beta\varepsilon)\,{\rm d}\varepsilon}
 {\int_0^\infty\exp(-\beta\varepsilon)}\\
 \int_0^\infty\exp(-\beta\varepsilon)\,{\rm d}\exp(-\beta\varepsilon)
\end{gather}
```

$$\frac{\int_0^\infty \varepsilon\exp(-\beta\varepsilon)\,\mathrm{d}\varepsilon}{\int_0^\infty \exp(-\beta\varepsilon)\,\mathrm{d}\varepsilon}\frac{\int_0^\infty \varepsilon\exp(-\beta\varepsilon)\,\mathrm{d}\varepsilon}{\int_0^\infty \exp(-\beta\varepsilon)} \tag{7}$$

$$\int_0^\infty \exp(-\beta\varepsilon)\,\mathrm{d}\exp(-\beta\varepsilon) \tag{8}$$

$$\frac{\int_0^\infty \varepsilon\exp(-\beta\varepsilon)\,\mathrm{d}\varepsilon}{\int_0^\infty \exp(-\beta\varepsilon)} \tag{9}$$

$$\int_0^\infty \exp(-\beta\varepsilon)\,\mathrm{d}\exp(-\beta\varepsilon) \tag{10}$$

11.6.3 Alignat environment

The align environment takes up the whole width of a display. If you want to have several "align"-type structures side by side, you can use an alignat environment. It has one required argument, for specifying the number of "align" structures. For an argument of n, the number of ampersand characters per line is $2n - 1$ (one ampersand for alignment within each align structure, and ampersands to separate the align structures from one another).

```
\begin{alignat}{2}
  L_1 & = R_1 &\qquad L_2 & = R_2\\
  L_3 & = R_3 &\qquad L_4 & = R_4
\end{alignat}
```

$$L_1 = R_1 \qquad L_2 = R_2 \tag{11}$$

$$L_3 = R_3 \qquad L_4 = R_4 \tag{12}$$

11.6.4 Alignment Environments as Parts of Displays

There are some other equation alignment environments that do not constitute an entire display. They are self-contained units that can be used inside other formulae, or set side by side. The environment names are: aligned, gathered and alignedat. These environments take an optional argument to specify their vertical positioning with respect to the material on either side. The default alignment is centered ([c]), and its effect is seen in the following example.

```
\begin{equation*}
\begin{aligned}
  x^2 + y^2 &= 1\\
          x &= \sqrt{1-y^2}
\end{aligned}
\qquad
\begin{gathered}
  (a+b)^2 = a^2 + 2ab + b^2 \\
  (a+b) \cdot (a-b) = a^2 - b^2
\end{gathered}
\end{equation*}
```

$$\begin{aligned} x^2 + y^2 &= 1 \\ x &= \sqrt{1-y^2} \end{aligned} \qquad \begin{gathered} (a+b)^2 = a^2 + 2ab + b^2 \\ (a+b) \cdot (a-b) = a^2 - b^2 \end{gathered}$$

The same mathematics can now be typeset using vertical alignments for the environments.

```
\begin{equation*}
\begin{aligned}[b]
  x^2 + y^2 &= 1\\
          x &= \sqrt{1-y^2}
\end{aligned}
\qquad
\begin{gathered}[t]
  (a+b)^2 = a^2 + 2ab + b^2 \\
  (a+b) \cdot (a-b) = a^2 - b^2
\end{gathered}
\end{equation*}
```

$$\begin{aligned} x^2 + y^2 &= 1 \\ x &= \sqrt{1-y^2} \end{aligned} \qquad \begin{gathered} (a+b)^2 = a^2 + 2ab + b^2 \\ (a+b) \cdot (a-b) = a^2 - b^2 \end{gathered}$$

11.6.5 Multline environment

The multline environment is a variation of the equation environment used for equations that do not fit on a single line. The first line of a multline will be at the left margin and the last line at the right margin except for an indentation on both sides whose amount is equal to multline-gap.

```
\begin{multline}
  {\int_0^\infty\varepsilon\exp(-\beta\varepsilon)\,{\rm d}
  \varepsilon}{\int_0^\infty\exp(-\beta\varepsilon)\,{\rm d}
  \varepsilon}{\int_0^\infty\varepsilon\exp(-\beta\varepsilon)\,
  {\rm d}\varepsilon}{\int_0^\infty\exp(-\beta\varepsilon)}\\
  {\int_0^\infty\varepsilon\exp(-\beta\varepsilon)\,{\rm d}
  \varepsilon}{\int_0^\infty\exp(-\beta\varepsilon)\,{\rm d}
  \varepsilon}{\int_0^\infty\varepsilon}
  {\int_0^\infty\exp(-\beta\varepsilon)}
\end{multline}
```

$$\int_0^\infty \varepsilon\exp(-\beta\varepsilon)\,\mathrm{d}\varepsilon \int_0^\infty \exp(-\beta\varepsilon)\,\mathrm{d}\varepsilon \int_0^\infty \varepsilon\exp(-\beta\varepsilon)\,\mathrm{d}\varepsilon \int_0^\infty \exp(-\beta\varepsilon) \int_0^\infty \varepsilon\exp(-\beta\varepsilon)\,\mathrm{d}\varepsilon \int_0^\infty \exp(-\beta\varepsilon)\,\mathrm{d}\varepsilon \int_0^\infty \varepsilon \int_0^\infty \exp(-\beta\varepsilon) \tag{13}$$

11.6.6 Split environment

The split environment is for single equations that are too long to fit on a single line and hence must be split into multiple lines. Unlike multline, however, the split environment provides for alignment among the split lines.

```
\begin{equation}
   \begin{split}
     (a+b)^4 & = (a+b)^2(a+b)^2\\
             & = (a^2+2ab+b^2)(a^2+2ab+b^2)\\
             & = a^4+4a^3b+6a^2b^2+4ab^3+b^4
   \end{split}
\end{equation}
```

$$\begin{aligned}(a+b)^4 &= (a+b)^2(a+b)^2\\ &= (a^2+2ab+b^2)(a^2+2ab+b^2)\\ &= a^4+4a^3b+6a^2b^2+4ab^3+b^4\end{aligned} \tag{14}$$

11.6.7 Cases

```
\begin{equation}
   P_{r-j}=
   \begin{cases}
     0 & \text{if $r-j$ is odd},\\
     r!\,(-1)^{(r-j)/2} & \text{if $r-j$ is even}.
   \end{cases}
\end{equation}
```

$$P_{r-j} = \begin{cases} 0 & \text{if } r-j \text{ is odd},\\ r!\,(-1)^{(r-j)/2} & \text{if } r-j \text{ is even}.\end{cases} \tag{15}$$

11.6.8 Matrix

```
\begin{gather*}
\begin{matrix}   0 & 1\\ 1 & 0   \end{matrix}\qquad
\begin{pmatrix} 0 & -i\\ i & 0 \end{pmatrix}\qquad
\begin{bmatrix} a & b\\ c & d   \end{bmatrix}\qquad
\begin{vmatrix} 0 & 1\\ -1 & 0 \end{vmatrix}\qquad
\begin{Vmatrix} f & g\\ e & v   \end{Vmatrix}
\end{gather*}
```

$$\begin{matrix} 0 & 1 \\ 1 & 0 \end{matrix} \qquad \begin{pmatrix} 0 & -i \\ i & 0 \end{pmatrix} \qquad \begin{bmatrix} a & b \\ c & d \end{bmatrix} \qquad \begin{vmatrix} 0 & 1 \\ -1 & 0 \end{vmatrix} \qquad \begin{Vmatrix} f & g \\ e & v \end{Vmatrix}$$

11.6.9 substack environment

```
\begin{equation*}
  \sum_{\substack{0\leq i\leq m\\ 0>j>n}}
\end{equation*}
```

$$\sum_{\substack{0\leq i\leq m\\ 0>j>n}}$$

```
\begin{equation*}
   \sum^{\substack{0\leq i\leq m\\ 0>j>n}}
\end{equation*}
```

$$\sum^{\substack{0\leq i\leq m\\ 0>j>n}}$$

11.6.10 Commutative Diagram[3]

```
\begin{equation*}
\begin{CD}
   S_\Lambda^{\mathcal{W}}\otimes T @j> T\\
   @VVV @VV{{\rm End}P}V\\
   (S\otimes T)/I  @=   (Z\otimes T)/J
\end{CD}
\end{equation*}
```

$$\begin{CD} S_\Lambda^{\mathcal{W}}\otimes T @>{j}>> T \\ @VVV @VV{\mathrm{End}P}V \\ (S\otimes T)/I @= (Z\otimes T)/J \end{CD}$$

```
\begin{equation*}
\begin{CD}
 S_\Lambda^{\mathcal{W}}\otimes T @j>  T_{XF}          @xyz> T\\
 @V{{Out}p}VV                     &      &               @AA{{\rm End}P}A\\
 (S\otimes T)/I                   @=     X_{\mathcal{F}} @fg>  (Z\otimes T)/J
\end{CD}
\end{equation*}
```

$$\begin{CD} S_\Lambda^{\mathcal{W}}\otimes T @>{j}>> T_{XF} @>{xyz}>> T \\ @V{Outp}VV @. @. @AA{\mathrm{End}P}A \\ (S\otimes T)/I @= X_{\mathcal{F}} @>{fg}>> (Z\otimes T)/J \end{CD}$$

[3] amscd.sty package should be loaded.

11.6.11 Binom

```
\begin{equation*}
   \binom{x}{y}
\end{equation*}
```

$$\binom{x}{y}$$

11.6.12 $\mathcal{AMS}$ symbols

\iint $\iint$ \iiint $\iiint$ \iiiint $\iiiint$

11.7 Mathematical Symbols

11.7.1 Lowercase Greek letters

α	\alpha	θ	\theta	o	o	τ	\tau
β	\beta	ϑ	\vartheta	π	\pi	υ	\upsilon
γ	\gamma	ι	\iota	ϖ	\varpi	ϕ	\phi
δ	\delta	κ	\kappa	ρ	\rho	φ	\varphi
ϵ	\epsilon	λ	\lambda	ϱ	\varrho	χ	\chi
ε	\varepsilon	μ	\mu	σ	\sigma	ψ	\psi
ζ	\zeta	ν	\nu	ς	\varsigma	ω	\omega
η	\eta	ξ	\xi				

11.7.2 Uppercase Greek letters

Γ	\Gamma	Λ	\Lambda	Σ	\Sigma	Ψ	\Psi
Δ	\Delta	Ξ	\Xi	Υ	\Upsilon	Ω	\Omega
Θ	\Theta	Π	\Pi	Φ	\Phi		

11.7.3 Math mode accents

$\hat{a}$	\hat{a}	$\acute{a}$	\acute{a}	$\bar{a}$	\bar{a}	$\dot{a}$	\dot{a}	$\breve{a}$	\breve{a}
$\check{a}$	\check{a}	$\grave{a}$	\grave{a}	$\vec{a}$	\vec{a}	$\ddot{a}$	\ddot{a}	$\tilde{a}$	\tilde{a}

11.7.4 Binary Operation Symbols

$\pm$	\pm	$\cap$	\cap	$\diamond$	\diamond	$\oplus$	\oplus
$\mp$	\mp	$\cup$	\cup	$\triangle$	\bigtriangleup	$\ominus$	\ominus
$\times$	\times	$\uplus$	\uplus	∇	\bigtriangledown	$\otimes$	\otimes
$\div$	\div	$\sqcap$	\sqcap	$\triangleleft$	\triangleleft	$\oslash$	\oslash
$*$	\ast	$\sqcup$	\sqcup	$\triangleright$	\triangleright	$\odot$	\odot
$\star$	\star	$\vee$	\vee	$\lhd$[a]	\lhd	$\bigcirc$	\bigcirc
$\circ$	\circ	$\wedge$	\wedge	$\rhd$[a]	\rhd	$\dagger$	\dagger
$\bullet$	\bullet	$\setminus$	\setminus	$\unlhd$[a]	\unlhd	$\ddagger$	\ddagger
$\cdot$	\cdot	$\wr$	\wr	$\unrhd$[a]	\unrhd	$\amalg$	\amalg

[a] Not predefined in NFSS. Use the latexsym or amssymb package.

11.7.5 Relation symbols

$\leq$	\leq	$\geq$	\geq	$\equiv$	\equiv	$\models$	\models
$\prec$	\prec	$\succ$	\succ	$\sim$	\sim	$\perp$	\perp
$\preceq$	\preceq	$\succeq$	\succeq	$\simeq$	\simeq	$\mid$	\mid
$\ll$	\ll	$\gg$	\gg	$\asymp$	\asymp	$\parallel$	\parallel
$\subset$	\subset	$\supset$	\supset	$\approx$	\approx	$\bowtie$	\bowtie
$\subseteq$	\subseteq	$\supseteq$	\supseteq	$\cong$	\cong	$\bowtie$	\Join
$\sqsubset$	\sqsubset	$\sqsupset$	\sqsupset	$\neq$	\neq	$\frown$	\smile
$\sqsubseteq$	\sqsubseteq	$\sqsupseteq$	\sqsupseteq	$\doteq$	\doteq	$\smile$	\frown
$\in$	\in	$\ni$	\ni	$\notin$	\notin	$\propto$	\propto
$\vdash$	\vdash	$\dashv$	\dashv				

11.7.6 Arrow symbols

$\leftarrow$	\leftarrow	$\longleftarrow$	\longleftarrow	$\uparrow$	\uparrow
$\Leftarrow$	\Leftarrow	$\Longleftarrow$	\Longleftarrow	$\Uparrow$	\Uparrow
$\rightarrow$	\rightarrow	$\longrightarrow$	\longrightarrow	$\downarrow$	\downarrow
$\Rightarrow$	\Rightarrow	$\Longrightarrow$	\Longrightarrow	$\Downarrow$	\Downarrow
$\leftrightarrow$	\leftrightarrow	$\longleftrightarrow$	\longleftrightarrow	$\updownarrow$	\updownarrow
$\Leftrightarrow$	\Leftrightarrow	$\Longleftrightarrow$	\Longleftrightarrow	$\Updownarrow$	\Updownarrow
$\mapsto$	\mapsto	$\longmapsto$	\longmapsto	$\nearrow$	\nearrow
$\hookleftarrow$	\hookleftarrow	$\hookrightarrow$	\hookrightarrow	$\searrow$	\searrow
$\leftharpoonup$	\leftharpoonup	$\rightharpoonup$	\rightharpoonup	$\swarrow$	\swarrow
$\leftharpoondown$	\leftharpoondown	$\rightharpoondown$	\rightharpoondown	$\nwarrow$	\nwarrow
$\rightleftharpoons$	\rightleftharpoons	$\leadsto$	\leadsto		

11.7.7 Miscellaneous symbols

$\ldots$	\ldots	$\imath$	\imath	$\Im$	\Im	$\aleph$	\aleph
$\prime$	\prime	$\flat$	\flat	$\ddots$	\ddots	$\emptyset$	\emptyset
$\exists$	\exists	$\clubsuit$	\clubsuit	$\hbar$	\hbar	$\triangle$	\triangle
$\diamond$	\Diamond[a]	$\Re$	\Re	$\Box$	\Box[a]	$\neq$	\neq
$\top$	\top	$\vdots$	\vdots	ℓ	\ell	$\wp$	\wp
$\bot$	\bot	∞	\infty	$\sharp$	\sharp	$\spadesuit$	\spadesuit
$\mho$	\mho	$\surd$	\surd	$\heartsuit$	\heartsuit	∂	\partial
$\cdots$	\cdots	$\jmath$	\jmath	$\angle$	\angle		
$\forall$	\forall	$\natural$	\natural	∇	\nabla	$\diamondsuit$	\diamondsuit

[a] Not predefined in NFSS. Use the latexsym or amssymb package.

11.7.8 Variable-sized symbols

$\sum$	\sum	$\prod$	\prod	$\coprod$	\coprod	$\int$	\int	$\oint$	\oint
$\bigcap$	\bigcap	$\bigcup$	\bigcup	$\bigsqcup$	\bigsqcup	$\bigvee$	\bigvee	$\bigwedge$	\bigwedge
$\bigodot$	\bigodot	$\bigotimes$	\bigotimes	$\bigoplus$	\bigoplus	$\biguplus$	\biguplus		

11.7.9 Delimiters

$\uparrow$	\uparrow	$\}$	\}	$\lceil$	\lceil
$\{$	\{	$\rfloor$	\rfloor	/	/
$\lfloor$	\lfloor	$\rangle$	\rangle	$\Downarrow$	\Downarrow
$\langle$	\langle	$\|$	\\|	$\Updownarrow$	\Updownarrow
$\|$	|	$\downarrow$	\downarrow	$\rceil$	\rceil
$\Uparrow$	\Uparrow	$\updownarrow$	\updownarrow	\	\backslash

11.7.10 LaTeX math constructs

$\widetilde{abc}$	\widetilde{abc}	$\widehat{abc}$	\widehat{abc}
$\overleftarrow{abc}$	\overleftarrow{abc}	$\overrightarrow{abc}$	\overrightarrow{abc}
$\overline{abc}$	\overline{abc}	$\underline{abc}$	\underline{abc}
$\overbrace{abc}$	\overbrace{abc}	$\underbrace{abc}$	\underbrace{abc}
$\sqrt{abc}$	\sqrt{abc}	$\sqrt[n]{abc}$	\sqrt[n]{abc}
f'	f'	$\frac{abc}{xyz}$	\frac{abc}{xyz}

11.7.11 AMS Greek and Hebrew (available with amssymb package)

F	\digamma	$\varkappa$	\varkappa	$\beth$	\beth	$\daleth$	\daleth	$\gimel$	\gimel

11.7.12 AMS delimiters (available with amssymb package)

$\ulcorner$	\ulcorner	$\urcorner$	\urcorner	$\llcorner$	\llcorner	$\lrcorner$	\lrcorner

11.7.13 AMS miscellaneous (available with amssymb package)

$\hbar$	\hbar	$\hslash$	\hslash	$\vartriangle$	\vartriangle
$\triangledown$	\triangledown	$\square$	\square	$\lozenge$	\lozenge
Ⓢ	\circledS	$\angle$	\angle	$\measuredangle$	\measuredangle
$\nexists$	\nexists	$\mho$	\mho	$\Finv$	\Finv
$\Game$	\Game	$\Bbbk$	\Bbbk	$\backprime$	\backprime
$\varnothing$	\varnothing	$\blacktriangle$	\blacktriangle	$\blacktriangledown$	\blacktriangledown
$\blacksquare$	\blacksquare	$\blacklozenge$	\blacklozenge	$\bigstar$	\bigstar
$\sphericalangle$	\sphericalangle	$\complement$	\complement	$\eth$	\eth
$\diagup$	\diagup	$\diagdown$	\diagdown		

[a] Not defined in old releases of the amssymb package; define with the \DeclareMathSymbol command.

11.7.14 AMS negated arrows (available with amssymb package)

$\nleftarrow$	\nleftarrow	$\nrightarrow$	\nrightarrow	$\nLeftarrow$	\nLeftarrow
$\nRightarrow$	\nRightarrow	$\nleftrightarrow$	\nleftrightarrow	$\nLeftrightarrow$	\nLeftrightarrow

11.7.15 $\mathcal{AMS}$ binary relations (available with amssymb package)

≦	\leqq	⩽	\leqslant	⪕	\eqslantless
≲	\lesssim	≶	\lessapprox	≊	\approxeq
⋖	\lessdot	⋘	\lll	≶	\lessgtr
⋚	\lesseqgtr	⪋	\lesseqqgtr	≒	\doteqdot
≓	\risingdotseq	≒	\fallingdotseq	∽	\backsim
⋍	\backsimeq	⫅	\subseteqq	⋐	\Subset
⊏	\sqsubset	≼	\preccurlyeq	⋞	\curlyeqprec
≾	\precsim	⪷	\precapprox	◁	\vartriangleleft
⊴	\trianglelefteq	⊨	\vDash	⊪	\Vvdash
⌣	\smallsmile	⌢	\smallfrown	≏	\bumpeq
≎	\Bumpeq	≧	\geqq	⩾	\geqslant
⪖	\eqslantgtr	≳	\gtrsim	⪆	\gtrapprox
⋗	\gtrdot	⋙	\ggg	≷	\gtrless
⋛	\gtreqless	⪌	\gtreqqless	≖	\eqcirc
≗	\circeq	≜	\triangleq	∼	\thicksim
≈	\thickapprox	⫆	\supseteqq	⋑	\Supset
⊐	\sqsupset	≽	\succcurlyeq	⋟	\curlyeqsucc
≿	\succsim	⪸	\succapprox	▷	\vartriangleright
⊵	\trianglerighteq	⊩	\Vdash	∣	\shortmid
∥	\shortparallel	≬	\between	⋔	\pitchfork
∝	\varpropto	◀	\blacktriangleleft	∴	\therefore
϶	\backepsilon	▶	\blacktriangleright	∵	\because

11.7.16 $\mathcal{AMS}$ binary operators (available with amssymb package)

∔	\dotplus	∖	\smallsetminus	⋒	\Cap
⋓	\Cup	⟑	\barwedge	⩒	\veebar
⩛	\doublebarwedge	⊟	\boxminus	⊠	\boxtimes
⊡	\boxdot	⊞	\boxplus	⋇	\divideontimes
⋉	\ltimes	⋊	\rtimes	⋋	\leftthreetimes
⋌	\rightthreetimes	⋏	\curlywedge	⋎	\curlyvee
⊖	\circleddash	⊛	\circledast	◎	\circledcirc
⋅	\centerdot	⊺	\intercal		

11.7.17 $\mathcal{AMS}$ negated binary relations (available with amssymb package)

≮	\nless	≰	\nleq	⪇	\nleqslant
≰	\nleqq	≨	\lneq	≨	\lneqq
≨	\lvertneqq	⋦	\lnsim	⪉	\lnapprox
⊀	\nprec	⪯̸	\npreceq	⋨	\precnsim
⪉	\precnapprox	≁	\nsim	∤	\nshortmid
∤	\nmid	⊬	\nvdash	⊭	\nvDash
⋪	\ntriangleleft	⋬	\ntrianglelefteq	⊈	\nsubseteq
⊊	\subsetneq	⫋	\varsubsetneq	⫋	\subsetneqq
⫋	\varsubsetneqq	≯	\ngtr	≱	\ngeq
⩾̸	\ngeqslant	≱	\ngeqq	≩	\gneq
≩	\gneqq	≩	\gvertneqq	⋧	\gnsim
⪊	\gnapprox	⊁	\nsucc	⪰̸	\nsucceq
⋧	\succnsim	⪺	\succnapprox	≆	\ncong
∦	\nshortparallel	∦	\nparallel	⊯	\nvDash
⊯	\nVDash	⋫	\ntriangleright	⋭	\ntrianglerighteq
⊉	\nsupseteq	⊉	\nsupseteqq	⊋	\supsetneq
⫌	\varsupsetneq	⫌	\supsetneqq	⫌	\varsupsetneqq

11.7.18 $\mathcal{AMS}$ arrows (available with amssymb package)

⇢	\dashrightarrow	⇠	\dashleftarrow	⇇	\leftleftarrows
⇆	\leftrightarrows	⇚	\Lleftarrow	↞	\twoheadleftarrow
↢	\leftarrowtail	↫	\looparrowleft	⇋	\leftrightharpoons
↶	\curvearrowleft	↺	\circlearrowleft	↰	\Lsh
⇈	\upuparrows	↿	\upharpoonleft	⇃	\downharpoonleft
⊸	\multimap	↭	\leftrightsquigarrow	⇉	\rightrightarrows
⇄	\rightleftarrows	⇉	\rightrightarrows	⇄	\rightleftarrows
↠	\twoheadrightarrow	↣	\rightarrowtail	↬	\looparrowright
⇌	\rightleftharpoons	↷	\curvearrowright	↻	\circlearrowright
↱	\Rsh	⇊	\downdownarrows	↾	\upharpoonright
⇂	\downharpoonright	⇝	\rightsquigarrow		

11.7.19 Log-like symbols

arccos	\arccos	arcsin	\arcsin	arctan	\arctan	arg	\arg
cos	\cos	cosh	\cosh	cot	\cot	coth	\coth
csc	\csc	deg	\deg	det	\det	dim	\dim
exp	\exp	gcd	\gcd	hom	\hom	inf	\inf
ker	\ker	lg	\lg	lim	\lim	lim inf	\liminf
lim sup	\limsup	ln	\ln	log	\log	max	\max
min	\min	Pr	\Pr	sec	\sec	sin	\sin
sinh	\sinh	sup	\sup	tan	\tan	tanh	\tanh

11.7.20 Double accents in math (available with amssymb package)

$\acute{\acute{A}}$	\Acute{\Acute{A}}	$\bar{\bar{A}}$	\Bar{\Bar{A}}
$\breve{\breve{A}}$	\Breve{\Breve{A}}	$\check{\check{A}}$	\Check{\Check{A}}
$\ddot{\ddot{A}}$	\Ddot{\Ddot{A}}	$\dot{\dot{A}}$	\Dot{\Dot{A}}
$\grave{\grave{A}}$	\Grave{\Grave{A}}	$\hat{\hat{A}}$	\Hat{\Hat{A}}
$\tilde{\tilde{A}}$	\Tilde{\Tilde{A}}	$\vec{\vec{A}}$	\Vec{\Vec{A}}

11.7.21 Other Styles

11.7.21.1 Caligraphic letters

$$\mathcal{A}\mathcal{B}\mathcal{C}\mathcal{D}\mathcal{E}\mathcal{F}\mathcal{G}\mathcal{H}\mathcal{I}\mathcal{J}\mathcal{K}\mathcal{L}\mathcal{M}\mathcal{N}\mathcal{O}\mathcal{P}\mathcal{Q}\mathcal{R}\mathcal{S}\mathcal{T}\mathcal{U}\mathcal{V}\mathcal{W}\mathcal{X}\mathcal{Y}\mathcal{Z}$$

```
use \mathcal{}
```

11.7.21.2 Mathbb letters

$$\mathbb{A}\mathbb{B}\mathbb{C}\mathbb{D}\mathbb{E}\mathbb{F}\mathbb{G}\mathbb{H}\mathbb{I}\mathbb{J}\mathbb{K}\mathbb{L}\mathbb{M}\mathbb{N}\mathbb{O}\mathbb{P}\mathbb{Q}\mathbb{R}\mathbb{S}\mathbb{T}\mathbb{U}\mathbb{V}\mathbb{W}\mathbb{X}\mathbb{Y}\mathbb{Z}$$

```
use \mathbb{}
```

11.7.21.3 Mathfrak letters

$$\mathfrak{A}\mathfrak{B}\mathfrak{C}\mathfrak{D}\mathfrak{E}\mathfrak{F}\mathfrak{G}\mathfrak{H}\mathfrak{I}\mathfrak{J}\mathfrak{K}\mathfrak{L}\mathfrak{M}\mathfrak{N}\mathfrak{O}\mathfrak{P}\mathfrak{Q}\mathfrak{R}\mathfrak{S}\mathfrak{T}\mathfrak{U}\mathfrak{V}\mathfrak{W}\mathfrak{X}\mathfrak{Y}\mathfrak{Z}$$

```
use \mathfrak{} with amssymb package
```

11.7.21.4 Math bold italic letters

$$\boldsymbol{A}\boldsymbol{B}\boldsymbol{C}\boldsymbol{D}\boldsymbol{E}\boldsymbol{F}\boldsymbol{G}\boldsymbol{H}\boldsymbol{I}\boldsymbol{J}\boldsymbol{K}\boldsymbol{L}\boldsymbol{M}\boldsymbol{N}\boldsymbol{O}\boldsymbol{P}\boldsymbol{Q}\boldsymbol{R}\boldsymbol{S}\boldsymbol{T}\boldsymbol{U}\boldsymbol{V}\boldsymbol{W}\boldsymbol{X}\boldsymbol{Y}\boldsymbol{Z}$$

```
use \mathbi{}
```

11.7.21.5 Math Sans serif letters

$$\mathsf{A\,B\,C\,D\,E\,F\,G\,H\,I\,J\,K\,L\,M\,N\,O\,P\,Q\,R\,S\,T\,U\,V\,W\,X\,Y\,Z}$$

use \mathsf{}

11.7.21.6 Math bold letters

$$\mathbf{A\,B\,C\,D\,E\,F\,G\,H\,I\,J\,K\,L\,M\,N\,O\,P\,Q\,R\,S\,T\,U\,V\,W\,X\,Y\,Z}$$

use \mathbf{}

11.7.22 Accents–Symbols

ó	\'{o}	ö	\"{o}	ô	\^{o}
ò	\‘{o}	õ	\~{o}	ō	\={o}
ȯ	\.{o}	ŏ	\u{o}	ő	\H{o}
o͡o	\t{oo}	o̧	\c{o}	ọ	\d{o}
o̲	\b{o}	Å	\AA	å	\aa
ß	\ss	ı	\i	ȷ	\j
ø	\o	s͡	\t s	š	\v s
Ø	\O	¶	\P	§	\S
ṣ	\d s	s̊	\r s	s̋	\H s

11.8 Accents and Foreign Letters

11.8.1 Printing command characters

The characters # $ ~ _ ^ % { } are interpreted as commands. If they are to be printed as text, the character \ must precede them:

$ = \$ & = \& % = \% # = \# _ = _ { = \{ } = \}

11.8.2 The special characters

These special characters do not exist on the computer keyboard. They can however be generated by special commands as follows:

§= \S †= \dag ‡= \ddag ¶= \P ©= \copyright £= \pounds

11.8.3 Foreign letters

Special letters that exist in European languages other than English can also be generated with TEX. These are:

œ= \oe	Œ= \OE	æ= \ae	Æ= \AE	å= \aa	Å= \AA	¡ = !‘
ø= \o	Ø= \O	ł= \l	Ł= \L	ß= \ss	SS= \SS	¿ = ?‘

11.8.4 Accents

ò = \‘o	ó = \'o	ô = \^o	ö = \"o	õ = \~o
ō = \=o	ȯ = \.o	ŏ = \u o	ǒ = \v o	ő = \H o
o͡o = \t{oo}	o̧ = \c o	ọ = \d o	o̲ = \b o	o̊ = \r o

The last command, \r, is new to LATEX 2ε. The *o* above is given merely as an example: any letter may be used. With *i* and *j* it should be pointed out that the dot must first be removed. This is carried out by prefixing these letters with \. The command \i yield ı.