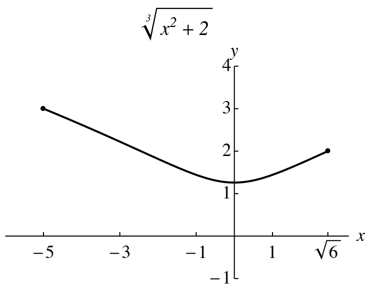
1. (a) $\lim\limits_{\theta\to 0} \dfrac{3\sin 5\theta}{4\theta} = \lim\limits_{\theta\to 0} \dfrac{3}{4}\cdot\dfrac{\sin 5\theta}{\theta} = \lim\limits_{\theta\to 0} \dfrac{3}{4}\cdot\dfrac{5}{1}\cdot\dfrac{\sin 5\theta}{5\theta} = \dfrac{3}{4}\cdot\dfrac{5}{1}\cdot 1 = \boxed{\dfrac{15}{4}}.$

(b) $\lim\limits_{\theta\to 0} \theta^2 \cot 3\theta \csc 2\theta = \lim\limits_{\theta\to 0} \dfrac{\theta^2}{1}\cdot\dfrac{\cos 3\theta}{\sin 3\theta}\cdot\dfrac{1}{\sin 2\theta} = \lim\limits_{\theta\to 0} \dfrac{\cos 3\theta}{1}\cdot\dfrac{\theta}{\sin 3\theta}\cdot\dfrac{\theta}{\sin 2\theta} =$

$\lim\limits_{\theta\to 0} \dfrac{\cos 3\theta}{1}\cdot\dfrac{1}{3}\cdot\dfrac{3\theta}{\sin 3\theta}\cdot\dfrac{1}{2}\cdot\dfrac{2\theta}{\sin 2\theta} = 1\cdot\dfrac{1}{3}\cdot 1\cdot\dfrac{1}{2}\cdot 1 = \boxed{\dfrac{1}{6}}.$

2.



$g(x) = \sqrt[3]{x^2+2}$ on $\left[-5, \sqrt{6}\right]$.

First we examine the endpoints.

At $x=-5$, $g(-5) = \sqrt[3]{(-5)^2+2} = \sqrt[3]{25+2} = \sqrt[3]{27} = 3$.

At $x=\sqrt{6}$, $g(\sqrt{6}) = \sqrt[3]{(\sqrt{6})^2+2} = \sqrt[3]{6+2} = \sqrt[3]{8} = 2$.

The endpoints are $(-5,3)$ and $\left(\sqrt{6},2\right)$.

Next we examine the critical points where g' may be undefined or zero.

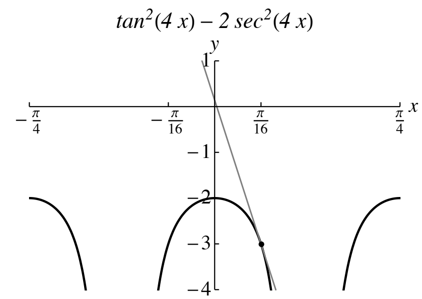
$$\begin{aligned} g(x) &= \sqrt[3]{x^2+2} \\ &= (x^2+2)^{1/3} \\ g'(x) &= \tfrac{1}{3}(x^2+2)^{-2/3}[x^2+2]' \\ &= \tfrac{1}{3}(x^2+2)^{-2/3}(2x) \\ &= \frac{2x}{3(x^2+2)^{2/3}}. \end{aligned}$$

The derivative g' is defined for all x because x^2+2 is always positive.

The derivative equals 0 when $x=0$. The corresponding y value is $g(0) = \sqrt[3]{0^2+2} = \sqrt[3]{2}$. The only critical point is $\left(0, \sqrt[3]{2}\right)$.

Therefore the absolute maximum value is $\boxed{3}$ at $x=-5$ and the absolute minimum value is $\boxed{\sqrt[3]{2}}$ at $x=0$.

3.



(a)

$$\begin{aligned} f(x) &= \tan^2 4x - 2\sec^2 4x \\ f'(x) &= 2\tan 4x\,[\tan 4x]' - 4\sec 4x\,[\sec 4x]' \\ &= 2\tan 4x\,\sec^2 4x\,[4x]' - 4\sec 4x\,\sec 4x\,\tan 4x\,[4x]' \\ &= 2\tan 4x\,\sec^2 4x\,(4) - 4\sec 4x\,\sec 4x\,\tan 4x\,(4) \\ &= 8\tan 4x\,\sec^2 4x - 16\tan 4x\,\sec^2 4x \\ &= \boxed{-8\tan 4x\,\sec^2 4x}. \end{aligned}$$

Alternate Solution: Use the identity $\tan^2\theta + 1 = \sec^2\theta$.

$$\begin{aligned} f(x) &= \tan^2 4x - 2\sec^2 4x \\ &= \tan^2 4x - 2(\tan^2 4x + 1) \\ &= -\tan^2 4x - 2 \\ f'(x) &= -2\tan 4x\,[\tan 4x]' \\ &= -2\tan 4x\,\sec^2 4x\,[4x]' \\ &= -2\tan 4x\,\sec^2 4x\,(4) \\ &= \boxed{-8\tan 4x\,\sec^2 4x}. \end{aligned}$$

(b) The tangent slope at $x = \frac{\pi}{16}$ is

$$\begin{aligned} f'\left(\tfrac{\pi}{16}\right) &= -8\tan\left(4\cdot\tfrac{\pi}{16}\right)\ \sec^2\left(4\cdot\tfrac{\pi}{16}\right) \\ &= -8\tan\left(\tfrac{\pi}{4}\right)\ \sec^2\left(\tfrac{\pi}{4}\right) \\ &= -8\cdot 1\cdot\frac{1}{\cos^2(\frac{\pi}{4})} \\ &= -8\cdot 1\cdot\frac{1}{1/2} \\ &= -8\cdot 1\cdot 2 = -16. \end{aligned}$$

The function value at $x = \frac{\pi}{16}$ is

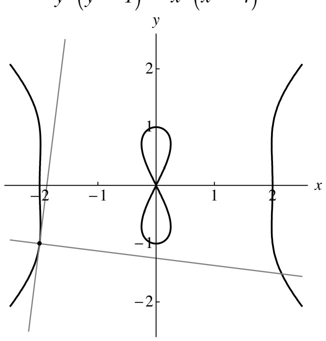
$$\begin{aligned} f(\tfrac{\pi}{16}) &= \tan^2(4\cdot\tfrac{\pi}{16}) - 2\sec^2(4\cdot\tfrac{\pi}{16}) \\ &= \tan^2(\tfrac{\pi}{4}) - 2\sec^2(\tfrac{\pi}{4}) \\ &= (1)^2 - \frac{2}{\cos^2(\frac{\pi}{4})} \\ &= (1)^2 - \frac{2}{1/2} \\ &= 1 - 4 = -3. \end{aligned}$$

Using the slope $m = -16$ and point $(\frac{\pi}{16}, -3)$, the equation of the tangent line at $x = \frac{\pi}{16}$ is

$$\begin{aligned} y &= y_1 + m(x - x_1) \\ y &= -3 - 16(x - \tfrac{\pi}{16}) \\ y &= -16x + \pi - 3. \end{aligned}$$

4.

$y^2\left(y^2 - 1\right) = x^2\left(x^2 - 4\right)$



(a)

$$\begin{aligned} y^2(y^2-1) &= x^2(x^2-4) \\ y^4 - y^2 &= x^4 - 4x^2 \\ 4y^3\frac{dy}{dx} - 2y\frac{dy}{dx} &= 4x^3 - 8x \\ \frac{dy}{dx}\left(4y^3 - 2y\right) &= 4x^3 - 8x \\ \frac{dy}{dx} &= \frac{4x^3-8x}{4y^3-2y} = \boxed{\frac{2x^3-4x}{2y^3-y}}. \end{aligned}$$

(b) The tangent slope is

$$\begin{aligned} \left.\frac{dy}{dx}\right|_{(-2,-1)} &= \frac{2(-2)^3 - 4(-2)}{2(-1)^3 - (-1)} \\ &= \frac{-16+8}{-2+1} = \frac{-8}{-1} = 8. \end{aligned}$$

The normal slope, which equals the negative reciprocal of the tangent slope, is $-1/8$. An equation for the normal line with slope $m = -1/8$ and point $(-2, -1)$ is

$$\begin{aligned} y &= y_1 + m(x - x_1) \\ y &= -1 - \tfrac{1}{8}(x+2) \\ y &= -\tfrac{1}{8}x - \tfrac{5}{4} \end{aligned}$$

5. (a) The function f must be continuous on $[a, b]$ and differentiable on (a, b) to satisfy the hypotheses of the Mean Value Theorem. Since f is a polynomial function, it is continuous and differentiable for all x.

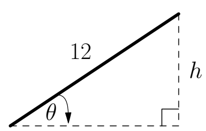
(b) We differentiate the function to get

$$\begin{aligned} f'(x) &= 6x \\ f'(c) &= 6c \end{aligned}$$

and substitute into the formula

$$\begin{aligned} f'(c) &= \frac{f(b)-f(a)}{b-a} \\ 6c &= \frac{3b^2-3a^2}{b-a} \\ 6c &= \frac{3(b^2-a^2)}{b-a} \\ 6c &= \frac{3(b+a)(b-a)}{b-a} \\ 6c &= 3(b+a) \\ c &= \boxed{\frac{a+b}{2}}. \end{aligned}$$

6.



(a) Since $\sin\theta = h/12$ and $\sin\left(\frac{\pi}{6}\right) = 1/2$, then

$$\begin{aligned} \frac{h}{12} &= \frac{1}{2} \\ h &= 6. \end{aligned}$$

When closed the bridge stands 3 meters above the water so the center of the bridge is $6+3 = \boxed{9\,\text{meters}}$ above the water when $\theta = \frac{\pi}{6}$.

(b) Given: $d\theta/dt = -\frac{\pi}{32}$. In the previous problem we found that when $\theta = \frac{\pi}{6}$, then $h = 6$.

We differentiate the sine function:

$$\begin{aligned} \sin\theta &= \frac{h}{12} \\ \cos\theta\,\frac{d\theta}{dt} &= \frac{1}{12}\cdot\frac{dh}{dt} \\ \left(\cos\frac{\pi}{6}\right)\cdot\left(-\frac{\pi}{32}\right) &= \frac{1}{12}\cdot\frac{dh}{dt} \\ \frac{\sqrt{3}}{2}\cdot\left(-\frac{\pi}{32}\right) &= \frac{1}{12}\cdot\frac{dh}{dt} \\ \frac{dh}{dt} &= -12\cdot\frac{\sqrt{3}}{2}\cdot\frac{\pi}{32} \\ &= \boxed{-\frac{3\sqrt{3}\pi}{16}\ \text{m/sec}}. \end{aligned}$$

(c) (This question can be answered without part (b).)

The bridge is closing at the constant rate of $\frac{\pi}{32}$ radians per second. (Note that we cannot use dh/dt because it is not a constant rate.) Starting when $\theta = \frac{\pi}{6}$, the number of seconds before the bridge is completely closed is

$$\frac{\frac{\pi}{6}}{\frac{\pi}{32}} = \frac{32}{6} = \frac{16}{3} = 5\frac{1}{3}\text{seconds.}$$

Bond is 170 meters away and traveling at the constant speed of 30 m/sec. The number of seconds before he gets to the bridge is

$$\frac{170}{30} = \frac{17}{3} = 5\frac{2}{3}\text{seconds.}$$

$\boxed{\text{No}}$, Bond will not get there in time. The bridge will be closed when he arrives. He will need to leap over the bridge or dive under it.

Extra Credit

Let s represent the distance from any point (x, y) to the origin. We are given that $dx/dt = 3$. We wish to find ds/dt when $x = 1$.

We use the distance formula $s = \sqrt{(x_1-x_2)^2 + (y_1-y_2)^2}$. The distance from any point (x, y) to the origin is $s = \sqrt{x^2+y^2}$.

When $x = 1$, $y = (1+1)^{3/2} = 2\sqrt{2}$ and $s = \sqrt{x^2+y^2} = \sqrt{1^2+(2\sqrt{2})^2} = 3$.

Next substitute $y = (x+1)^{3/2}$ into the distance formula.

$$\begin{aligned} s &= \sqrt{x^2 + y^2} \\ s^2 &= x^2 + y^2 \\ s^2 &= x^2 + ((x+1)^{3/2})^2 \\ s^2 &= x^2 + (x+1)^3 \end{aligned}$$

Then differentiate and solve.

$$\begin{aligned} s^2 &= x^2 + (x+1)^3 \\ 2s\frac{ds}{dt} &= 2x\frac{dx}{dt} + 3(x+1)^2\frac{dx}{dt} \\ 2s\frac{ds}{dt} &= \left(2x + 3(x+1)^2\right)\frac{dx}{dt} \\ 2(3)\frac{ds}{dt} &= (2(1) + 3(1+1)^2)(3) \\ 6\frac{ds}{dt} &= (2+12)(3) \\ \frac{ds}{dt} &= \frac{42}{6} = \boxed{7\,\text{units/sec}}. \end{aligned}$$