1. (a) The statement $\lim_{x \to x_0} f(x) = L$ is true if for every $\epsilon > 0$ there exists a number $\delta > 0$ such that for all x

$$0 < |x - x_0| < \delta \implies |f(x) - L| < \epsilon.$$

(b) $x_0 = \frac{3}{2}, f(x) = \frac{1}{2}(x-3), L = -\frac{3}{4}, \epsilon = \frac{1}{8}$.

$$\begin{aligned} |f(x) - L| &< \epsilon \\ \left|\frac{1}{2}(x-3) + \frac{3}{4}\right| &< \frac{1}{8} \\ -\frac{1}{8} < \frac{1}{2}(x-3) + \frac{3}{4} &< \frac{1}{8} \\ -\frac{7}{8} < \frac{1}{2}(x-3) &< -\frac{5}{8} \\ -\frac{7}{4} < x - 3 &< -\frac{5}{4} \\ \frac{5}{4} < x &< \frac{7}{4} \end{aligned}$$

Now center the interval about $x_0 = \frac{3}{2}$. The distance from x_0 to either endpoint is $\frac{1}{4}$ so choose $\boxed{\delta = \frac{1}{4}.}$ Then

$$0 < \left|x - \frac{3}{2}\right| < \frac{1}{4} \implies \left|\frac{1}{2}(x-3) + \frac{3}{4}\right| < \frac{1}{8}.$$

2. (a) $\lim_{x \to -5^-} \frac{x^2 - 4x - 5}{x^2 + 4x - 5} = \lim_{x \to -5^-} \frac{(x-5)(x+1)}{(x+5)(x-1)} = \boxed{\infty}$.
(The expression is (neg · neg) / (neg · neg) = pos.)

(b) $\lim_{\theta \to 2\pi^-} -\frac{2\csc\theta}{\sec\theta} = \lim_{\theta \to 2\pi^-} \frac{(-2/\sin\theta)}{1/\cos\theta} = \lim_{\theta \to 2\pi^-} \frac{-2\cos\theta}{\sin\theta} = \boxed{\infty}$.
(The expression is (neg · pos) / neg = pos.)

(Alternatively $\lim_{\theta \to 2\pi^-} -2\cot\theta = \infty$.)

Note that when θ approaches 2π from the left side, $\sin\theta$ has negative values, and $\lim_{\theta \to 2\pi^-} \cos\theta = 1$, which is positive.

3. (a) Let $y = \frac{3}{1-x}$. Then

$$\begin{aligned} \frac{dy}{dx} &= \lim_{h \to 0} \frac{f(x+h) - f(x)}{h} \\ &= \lim_{h \to 0} \frac{\frac{3}{1-(x+h)} - \frac{3}{1-x}}{h} \\ &= \lim_{h \to 0} \frac{1}{h}\left(\frac{3}{1-x-h} - \frac{3}{1-x}\right) \\ &= \lim_{h \to 0} \frac{1}{h}\left(\frac{3(1-x)}{(1-x-h)(1-x)} - \frac{3(1-x-h)}{(1-x)(1-x-h)}\right) \\ &= \lim_{h \to 0} \frac{1}{h}\left(\frac{3-3x-(3-3x-3h)}{(1-x)(1-x-h)}\right) \\ &= \lim_{h \to 0} \frac{1}{h}\left(\frac{3-3x-3+3x+3h}{(1-x)(1-x-h)}\right) \\ &= \lim_{h \to 0} \frac{1}{h}\left(\frac{3h}{(1-x)(1-x-h)}\right) \\ &= \lim_{h \to 0} \frac{3}{(1-x)(1-x-h)} \\ &= \boxed{\frac{3}{(1-x)^2}}. \end{aligned}$$

(b) Quotient Rule: $\left(\frac{u}{v}\right)' = \frac{vu' - uv'}{v^2}$.

For $y = \frac{3}{1-x}$, $u = 3$ and $v = 1 - x$. Then

$$\begin{aligned} \frac{dy}{dx} &= \frac{(1-x)(3)' - (3)(1-x)'}{(1-x)^2} \\ &= \frac{(1-x)(0) - (3)(-1)}{(1-x)^2} \\ &= \boxed{\frac{3}{(1-x)^2}}. \end{aligned}$$

(c) The curve has no horizontal tangents because $\frac{3}{(1-x)^2} = 0$ has no solutions. The derivative does not equal 0 for any value of x.

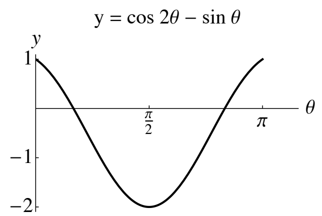
4. The Intermediate Value Theorem for continuous functions states that if function f is continuous on a closed interval $[a, b]$, then f takes on every value between $f(a)$ and $f(b)$.

The function $f(\theta) = \cos 2\theta - \sin\theta$ is a continuous function because $\cos 2\theta$ and $\sin\theta$ are both continuous for all θ.

Let $f(\theta) = \cos 2\theta - \sin\theta$. Then

$$f(0) = \cos 0 - \sin 0 = 1 - 0 = 1,$$
$$f\left(\tfrac{\pi}{2}\right) = \cos\pi - \sin\tfrac{\pi}{2} = -1 - 1 = -2,$$
$$f(\pi) = \cos 2\pi - \sin\pi = 1 - 0 = 1.$$

By the IVT, since f takes on every value from 1 to -2 for $0 \le \theta \le \frac{\pi}{2}$, then $f(\theta) = 0$ for at least one value of θ in $[0, \frac{\pi}{2}]$. Note that there is a second solution in $[\frac{\pi}{2}, \pi]$.



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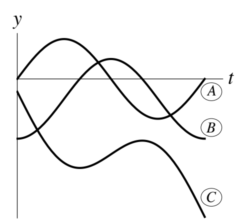


We examine the *slope* of each function to determine the *values* of its derivative.

The graph of C has neg-pos-neg slope, which corresponds to the neg-pos-neg values of B, so B is the derivative of C.

The graph of B has pos-neg slope, which corresponds to the pos-neg values of A, so A is the derivative of B.

The graph of A has pos-neg-pos slope, which does not correspond to the values of B or C.

We conclude that C is the position function, B is the velocity function, and A is the acceleration function.

6. (a)

$$f(x) = \begin{cases} ax^2 - 6, & x \le 4 \\ -4\sqrt{x}, & x > 4. \end{cases}$$

For the LH function, $f(4) = a\,(4^2) - 6 = 16a - 6$.
For the RH function, $\lim_{x\to 4} -4\sqrt{x} = -4\sqrt{4} = -8$.

Function f is continuous if the LH and RH values are equal:
$16a - 6 = -8 \implies 16a = -2 \implies \boxed{a = -\frac{1}{8}}$.

(b) Function f is differentiable at $x = 4$ if the LH and RH derivatives are equal.

LH derivative: $m_1 = 2ax = 2\left(-\frac{1}{8}\right)(4) = -1$.

RH derivative: $m_2 = -\frac{2}{\sqrt{x}} = -\frac{2}{\sqrt{4}} = -\frac{2}{2} = -1$.

Since the LH and RH derivatives both equal -1, f is differentiable at $x = 4$.

7. (a) Moving to the right: $(2,3), (5,6)$

(b) Slowing down: $(1,2), (2\frac{1}{2},3), (4,4\frac{1}{2}), (5\frac{1}{2},6)$

(c) Positive acceleration: $(1,2\frac{1}{2}), (4,4\frac{1}{2}), (5,5\frac{1}{2})$

(d) Maximum speed: $t = 4$

8. (a)

$$s(t) = 81 - 16t^2$$
$$v(t) = -32t$$
$$\text{speed} = 32t$$

Speed reaches 48 ft/sec when

$$32t = 48$$
$$t = \frac{48}{32} = \boxed{\frac{3}{2}\text{ sec}}.$$

Height at $t = 3/2$ is

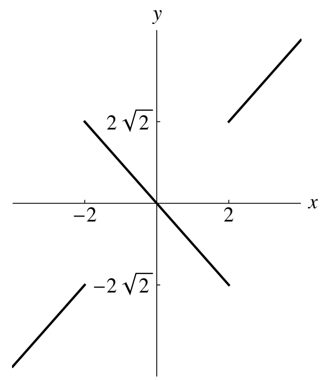
$$s\left(\frac{3}{2}\right) = 81 - 16\left(\frac{3}{2}\right)^2$$
$$= 81 - 16\left(\frac{9}{4}\right) = 81 - 36$$
$$= \boxed{45\text{ ft}}.$$

(b) Draco hits the ground when

$$\begin{aligned} s(t) &= 0 \\ 81 - 16t^2 &= 0 \\ 16t^2 &= 81 \\ t^2 &= \frac{81}{16} \\ t &= \boxed{\frac{9}{4}\,\text{sec}}. \end{aligned}$$

His velocity at the moment of impact is

$$v\left(\frac{9}{4}\right) = -32\left(\frac{9}{4}\right) = \boxed{-72\,\text{ft/sec}}.$$



Extra Credit

Note that $\dfrac{x}{|x|} = 1$ for $x > 0$ and $\dfrac{x}{|x|} = -1$ for $x < 0$.

Similarly $\dfrac{x^2-4}{|x^2-4|} = 1$ when $x^2 - 4 > 0$ and $\dfrac{x^2-4}{|x^2-4|} = -1$ when $x^2 - 4 < 0$.

We examine the LH and RH limits at $x = -2$ and $x = 2$ to determine whether there is a removable discontinuity or jump discontinuity at those points.

$$\begin{aligned} \lim_{x\to-2^-} \frac{\sqrt{2}x(x^2-4)}{|x^2-4|} &= \lim_{x\to-2^-} \sqrt{2}x = -2\sqrt{2} \\ \lim_{x\to-2^+} \frac{\sqrt{2}x(x^2-4)}{|x^2-4|} &= \lim_{x\to-2^+} -\sqrt{2}x = 2\sqrt{2} \\ \lim_{x\to2^-} \frac{\sqrt{2}x(x^2-4)}{|x^2-4|} &= \lim_{x\to2^-} -\sqrt{2}x = -2\sqrt{2} \\ \lim_{x\to2^+} \frac{\sqrt{2}x(x^2-4)}{|x^2-4|} &= \lim_{x\to2^+} \sqrt{2}x = 2\sqrt{2} \end{aligned}$$

Because there are jump discontinuities at $x = -2$ and $x = 2$, the function f cannot be extended to be continuous.

Note that $f(x)$ can be rewritten as

$$f(x) = \begin{cases} \sqrt{2}x, & x < -2 \\ -\sqrt{2}x, & -2 < x < 2 \\ \sqrt{2}x, & x > 2 \end{cases}$$