1. (a) Compute the following indefinite integral:
$\int \left(x^2 + 1/x - e^{4x}\right) dx.$
Solution: $x^3/3 + \ln|x| - e^{4x}/4 + C.$

(b) Compute the following definite integral:
$\int_1^2 \left(\sqrt{2x+1} - x^{-2}\right) dx.$
Solution:

$$\int_1^2 \left(\sqrt{2x+1} - x^{-2}\right) dx = \left((2x+1)^{3/2}/3 + 1/x\right)_1^2 = \frac{5\sqrt{5} - 3\sqrt{3}}{3} - \frac{1}{2}.$$

2. (a) Suppose that the marginal revenue function for a company is 100 - x. Find the additional revenue received from doubling production if currently 10 units are being produced.
Solution: Additional revenue is $\int_{10}^{20} (100 - x)\, dx = 850$.

(b) Suppose that the marginal cost is $2x + 0.3x^2$, with fixed costs at 30. Find the total cost if 10 units are being produced.
Solution: Actual cost is $\int (2x + 0.3x^2)\, dx = x^2 + 0.1x^3 + C$. Since fixed cost is 30, we have $C = 30$. Hence total cost for 10 units is 230.

3. (a) Compute the area of the region between the curves $y = x + 1$ and $y = -x^2 - 1$ from $x = 0$ to $x = 1$.
Solution: Area is
$$\int_0^1 \Big((x+1) - (-x^2 - 1)\Big)\, dx = \frac{17}{6}.$$

(b) Compute the area enclosed by the curves $y = 1/2x^2 + 1$ and $y = -1/2x^2 + 3x - 1$.
Solution: The curves intersect at $x = 1$ and $x = 2$. The area is
$$\int_1^2 \Big((-1/2x^2 + 3x - 1) - (1/2x^2 + 1)\Big)\, dx = \frac{1}{6}.$$

4. Suppose that a lake is stocked with 100 fish. After 1 month, there are 150 fish in the lake. An ecological study predicts that the lake can support 600 fish. Use a logistic growth curve to estimate the number of fish in the lake after 1 year.
Solution: Let $f(t)$ be the fish population at end of t months. Then there exist k and B such that

$$f(t) = \frac{M}{1 + Be^{-Mkt}},$$

where $M = 600$. By assumption, $f(0) = M/(1 + B) = 100$. It follows that $B = 5$. In addition, since $f(1) = 150$, we have

$$600/\left(1 + Be^{-Mk}\right) = 150,$$

it follows that $e^{-Mk} = 3/5$, and hence

$$f(t) = \frac{600}{1 + 5\,(3/5)^t}.$$

Let $t = 12$. The fish population at the end of the year is

$$f(12) = \frac{600}{1 + 5\,(3/5)^{12}}.$$

5. (a) Find the average value of the function $f(x) = 1/x$ over the interval $-3 \leq x \leq -1$.
Solution: Average is $\frac{1}{(-1)-(-3)} \int_{-3}^{-1} 1/x dx = -1/2 \ln 3$.

(b) Use a Riemann sum to estimate the following sum for large enough values of n:

$$\sqrt{1+n} + \sqrt{2+n} + \sqrt{3+n} + \cdots + \sqrt{n+n}.$$

Solution: Define $x_j = 1 + jh$, where $h = 1/n$ and $j = 1, \cdots, n$. Then for each j, we have $\sqrt{j+n} = \sqrt{n}\sqrt{x_j}$. Sum is

$$\sqrt{n}\left(\sqrt{x_1} + \sqrt{x_2} + \sqrt{x_3} + \cdots + \sqrt{x_n}\right).$$

Since $x_1 = 1 + h$ and $x_n = 2$, $h\left(\sqrt{x_1} + \sqrt{x_2} + \sqrt{x_3} + \cdots + \sqrt{x_n}\right)$ is a Riemann sum approaching $\int_1^2 \sqrt{x} dx$, the sum is

$$\sqrt{n}\left(\sqrt{x_1} + \sqrt{x_2} + \sqrt{x_3} + \cdots + \sqrt{x_n}\right) = n^{3/2} h\left(\sqrt{x_1} + \sqrt{x_2} + \sqrt{x_3} + \cdots + \sqrt{x_n}\right) \approx n^{3/2} \int_1^2 \sqrt{x} dx.$$

6. If the demand equation for a monopolist is $p = 150 - 0.02x$ and the cost function is $C(x) = 10x + 300$, find the value of x that maximizes the profit.
   Solution: The Profit is

$$P(x) = x(150 - 0.02x) - 10x - 300,$$

   with $P'(x) = 140 - 0.04x$, and $P''(x) = -0.04 < 0$. Let $P'(x) = 0$, we obtain $x = 3500$. $P(3500)$ is the max since $P''(3500) < 0$.

7. (a) For $u = \sqrt{x}$ and $\frac{dy}{du} = u^2$, find $\frac{dy}{dx}$.
Solution:

$$\frac{dy}{dx} = \frac{dy}{du}\frac{du}{dx} = u^2 \frac{d}{dx}\left(\sqrt{x}\right) = \sqrt{x}/2.$$

(b) For $g(x) = x^2$ and $f'(x) = x\sqrt{x+1}$, find a formula for $\frac{d}{dx}f(g(x))$.
Solution: Since $g'(x) = 2x$, by the chain rule

$$\frac{d}{dx}f(g(x)) = f'(g(x))g'(x) = g(x)\sqrt{g(x)+1}(2x) = 2x^3\sqrt{x^2+1}.$$

8. Given function $f(x) = \ln(x^2 + 1)$.

   (a) Find the relative and absolute maximum/minimum of $f(x)$ if they exist.
   Solution: $f'(x) = 2x/(x^2+1)$ and $f''(x) = 2(1-x^2)/(x^2+1)^2$. Let $f'(x) = 0$ we have $x = 0$. Since $f''(0) > 0$, $x = 0$ is relative minimum. Since $f(0) = 0 \leq f(x)$ for any x, $x = 0$ is also an absolute minimum.

   (b) Sketch the graph of this function.