#1) Which of the following is completely true?

A) $\boxed{\lim_{x\to\pi/4}\left[\sin(x)\cos(x)\right] = -\lim_{x\to-\pi/4}\left[\sin(x)\cos(x)\right]}$

B) $\lim_{x\to 0^-}\left[1/x\right] = \lim_{x\to 0}\left[1/x^2\right]$

C) $\lim_{x\to\pi/2}\left[\tan(x+\pi)\right] = \lim_{x\to-\pi/2}\left[\tan(x-\pi)\right]$

D) All of the above

E) None of the above

A) True. $\lim_{x\to\pi/4}\left[\sin(x)\cos(x)\right] = \sin(\pi/4)\cos(\pi/4) = (\sqrt{2}/2)(\sqrt{2}/2) = 1/2$ and

$-\lim_{x\to-\pi/4}\left[\sin(x)\cos(x)\right] = -\sin(-\pi/4)\cos(-\pi/4) = -(-\sqrt{2}/2)(\sqrt{2}/2) = 1/2$

B) False. $\lim_{x\to 0^-}\left[1/x\right] = -\infty$ but $\lim_{x\to 0}\left[1/x^2\right] = \infty$

C) False. $\lim_{x\to\pi/2}\left[\tan(x+\pi)\right]$ does not exist since

$$\lim_{x\to\pi/2^-}\left[\tan(x+\pi)\right] = -\infty \neq \lim_{x\to\pi/2^+}\left[\tan(x+\pi)\right] = \infty$$

and $\lim_{x\to-\pi/2}\left[\tan(x-\pi)\right]$ does not exist either since

$$\lim_{x\to-\pi/2^-}\left[\tan(x-\pi)\right] = -\infty \neq \lim_{x\to-\pi/2^+}\left[\tan(x-\pi)\right] = \infty$$

#2) If the limit of g(r) is 2 as r approaches infinity, which of the following are completely true?

A) $\boxed{\lim_{r\to\infty}\left[g^{2/3}(r)\right] = \left[\lim_{r\to\infty} g(r)\right]^{2/3}}$

B) $\lim_{r\to-\infty}\left[g(r)\right] = -2$

C) $\lim_{r\to\infty}\left[g(3r)\right] = 6$

D) All of the above

E) None of the above

A) True. Because $\lim_{r\to\infty} g(r)$ exists, the Power Rule Limit Law says

$$\lim_{r\to\infty}\left[g^{2/3}(r)\right] = \left[\lim_{r\to\infty} g(r)\right]^{2/3}$$

B) False. Just because we know $\lim_{r\to\infty}\left[g(r)\right] = 2$, this does not tell us anything about the behavior of $g(r)$ as $r \to -\infty$. Specifically, we don't know what $\lim_{r\to-\infty}\left[g(r)\right]$ is.

C) False. Since $\lim_{r\to\infty}\left[g(r)\right] = 2$ and since $\lim_{r\to\infty} 3r = \lim_{r\to\infty} r = \infty$

we know $\lim_{r\to\infty}\left[g(3r)\right] = \lim_{r\to\infty}\left[g(r)\right] = 2 \neq 6$

#3) Which of the following is completely true?

A) $\lim_{x\to\infty}[1/(3x^2+2x+1)] = \lim_{x\to-\infty}[3x^2+2x+1]$

B) $\lim_{x\to 0}[1/(3x^2+2x+1)] = \lim_{x\to\infty}[1/(3x^2+2x+1)]$

C) $\lim_{x\to 0^+}[1/3x] = \lim_{x\to 0^-}[1/3x]$

D) All of the above

E) None of the above

A) False. $\lim_{x\to\infty}[1/(3x^2+2x+1)] = 0$ but

$\lim_{x\to-\infty}[3x^2+2x+1] = \infty$

B) False. $\lim_{x\to 0}[1/(3x^2+2x+1)] = 1$ but

$\lim_{x\to\infty}[1/(3x^2+2x+1)] = 0$

C) False. $\lim_{x\to 0^+}[1/3x] = \infty$ but $\lim_{x\to 0^-}[1/3x] = -\infty$

#4) Which of the following is completely true?

A) $\lim_{x\to 1}[(1-\sqrt{x})/(1-x)] = 1/\lim_{x\to 1}[(1-x)/(1-\sqrt{x})]$

B) $\lim_{x\to a}[(\sqrt{a}-\sqrt{x})/(a-x)] = \lim_{x\to a}[(a-x)/(\sqrt{a}-\sqrt{x})]/a$

C) $\lim_{x\to 0}[\sin(x)/x] = \lim_{x\to 0}[\sin^2(x)/x]$

D) All of the above

E) None of the above

A) True. $\lim_{x\to 1}[(1-\sqrt{x})/(1-x)] = \lim_{x\to 1}\left[(1-\sqrt{x})/[(1-\sqrt{x})(1+\sqrt{x})]\right]$

$= \lim_{x\to 1}[1/(1+\sqrt{x})] = 1/2$ and

$1/\lim_{x\to 1}[(1-x)/(1-\sqrt{x})] = 1/\lim_{x\to 1}\left[[(1-\sqrt{x})(1+\sqrt{x})]/(1-\sqrt{x})\right]$

$= 1/\lim_{x\to 1}[1+\sqrt{x}] = 1/2$

B) False. $\lim_{x\to a}[(\sqrt{a}-\sqrt{x})/(a-x)] = \lim_{x\to a}\left[(\sqrt{a}-\sqrt{x})/[(\sqrt{a}-\sqrt{x})(\sqrt{a}+\sqrt{x})]\right]$

$= \lim_{x\to a}[1/(\sqrt{a}+\sqrt{x})] = 1/2\sqrt{a}$ but

$\lim_{x\to a}[(a-x)/(\sqrt{a}-\sqrt{x})]/a = \lim_{x\to a}\left[[(\sqrt{a}-\sqrt{x})(\sqrt{a}+\sqrt{x})]/(\sqrt{a}-\sqrt{x})\right]/a$

$= \lim_{x\to a}[\sqrt{a}+\sqrt{x}]/a = 2\sqrt{a}/a = 2/\sqrt{a}$

C) False. $\lim_{x\to 0}[\sin(x)/x] = \lim_{x\to 0}\left[\frac{1}{x}\left(x-\frac{x^3}{3!}+\frac{x^5}{5!}-\frac{x^7}{7!}+\cdots\right)\right]$

$= \lim_{x\to 0}\left[1-\frac{x^2}{3!}+\frac{x^4}{5!}-\frac{x^6}{7!}+\cdots\right] = 1$ but

$\lim_{x\to 0}[\sin^2(x)/x] = \lim_{x\to 0}\left[\sin(x)\cdot\frac{\sin(x)}{x}\right]$

$= \lim_{x\to 0}[\sin(x)]\cdot\lim_{x\to 0}\left[\frac{\sin(x)}{x}\right] = 0\cdot 1 = 0$

#5) Which of the following is completely true?

A) $\lim_{x\to\infty} [(5x^2-3x+1)/(3x^2-5x+1)] = \lim_{x\to-\infty} [(5x^2-5x+5)/(3x^2-3x+3)]$

B) $\lim_{x\to\infty} [(-3x+1)/(-5x+1)] = \lim_{x\to-\infty} [(-3x+1)/(-5x+1)]$

C) $\lim_{x\to\infty} [(-3x+1)/(3x^2-5x+1)] = \lim_{x\to-\infty} [(-5x+5)/(3x^2-3x+3)]$

D) All of the above

E) None of the above

A) True. $\lim_{x\to\infty} [(5x^2-3x+1)/(3x^2-5x+1)] = \lim_{x\to\infty} \left[(5-\cancel{3/x}^{\,0}+\cancel{1/x^2}^{\,0})/(3-\cancel{5/x}^{\,0}+\cancel{1/x^2}^{\,0})\right] = 5/3$

and

$\lim_{x\to-\infty} [(5x^2-5x+5)/(3x^2-3x+3)] = \lim_{x\to\infty} \left[(5-\cancel{5/x}^{\,0}+\cancel{5/x^2}^{\,0})/(3-\cancel{3/x}^{\,0}+\cancel{3/x^2}^{\,0})\right] = 5/3$

B) True. $\lim_{x\to\infty} [(-3x+1)/(-5x+1)] = \lim_{x\to\infty} \left[(-3+\cancel{1/x}^{\,0})/(-5+\cancel{1/x}^{\,0})\right] = 3/5$

and

$\lim_{x\to-\infty} [(-3x+1)/(-5x+1)] = \lim_{x\to-\infty} \left[(-3+\cancel{1/x}^{\,0})/(-5+\cancel{1/x}^{\,0})\right] = 3/5$

C) True. $\lim_{x\to\infty} [(-3x+1)/(3x^2-5x+1)] = \lim_{x\to\infty} \left[(-\cancel{3/x}^{\,0}+\cancel{1/x^2}^{\,0})/(3-\cancel{5/x}^{\,0}+\cancel{1/x^2}^{\,0})\right] = \frac{0}{3} = 0$

and

$\lim_{x\to-\infty} [(-5x+5)/(3x^2-3x+3)] = \lim_{x\to-\infty} \left[(-\cancel{5/x}^{\,0}+\cancel{5/x^2}^{\,0})/(3-\cancel{3/x}^{\,0}+\cancel{3/x^2}^{\,0})\right] = \frac{0}{3} = 0$

#6) Which of the following is completely true?

A) If a function f is continuous at a point c then $\lim_{x \to c} f(x) = f(c)$

B) If $\lim_{x \to c} f(x) = f(c)$ then f is continuous at the point c.

C) If $\lim_{x \to c} f(x) = f(c)$ and $\lim_{x \to c} g(x) = g(c)$ then $f \circ g$ is continuous at the point c.

D) All of the above

E) None of the above

A) True. B) True.

A function f is continuous at a point c if and only if $\lim_{x \to c} f(x) = f(c)$

C) True. Compositions of continuous functions are also continuous.

#7) Which of the following is completely true?

A) $\lim_{x\to -2}[(x+2)/(\sqrt{x^2+5}-3)] = \lim_{x\to 2}[(\sqrt{x^2+12}-4)/(x-2)]$

B) $\lim_{x\to -1}[(x^2+3x+2)/(x^2-x-2)] = \lim_{x\to 1}[(x^2+x-2)/(x^2-1)]$

C) $\lim_{x\to 0}[(5x^3+8x^2)/(3x^4-16x^2)] = \lim_{x\to 2}[(x-2)/(x^2-4)]$

D) All of the above

$\boxed{\text{E) None of the above}}$

A) False. $\lim_{x\to -2}[(x+2)/(\sqrt{x^2+5}-3)] = \lim_{x\to -2}\left[\left(\frac{x+2}{\sqrt{x^2+5}-3}\right)\left(\frac{\sqrt{x^2+5}+3}{\sqrt{x^2+5}+3}\right)\right]$

$= \lim_{x\to -2}\left[\frac{(x+2)(\sqrt{x^2+5}+3)}{x^2+5-3+3}\right] = \frac{0}{9} = 0$ but

$\lim_{x\to 2}[(\sqrt{x^2+12}-4)/(x-2)] = \lim_{x\to 2}\left[\left(\frac{\sqrt{x^2+12}-4}{x-2}\right)\left(\frac{\sqrt{x^2+12}+4}{\sqrt{x^2+12}+4}\right)\right]$

$= \lim_{x\to 2}\left[\frac{x^2-4}{(x-2)(\sqrt{x^2+12}+4)}\right] = \lim_{x\to 2}\left[\frac{(x-2)(x+2)}{(x-2)(\sqrt{x^2+12}+4)}\right]$

$= \lim_{x\to 2}\left[\frac{x+2}{\sqrt{x^2+12}+4}\right] = \frac{4}{8} = \frac{1}{2}$

B) False. $\lim_{x\to -1}[(x^2+3x+2)/(x^2-x-2)] = \lim_{x\to -1}\left[\frac{(x+2)(x+1)}{(x-2)(x+1)}\right] = \lim_{x\to -1}\left[\frac{(x+2)}{(x-2)}\right] = -\frac{1}{3}$

but

$\lim_{x\to 1}[(x^2+x-2)/(x^2-1)] = \lim_{x\to 1}\left[\frac{(x-1)(x+2)}{(x-1)(x+1)}\right] = \lim_{x\to 1}\left[\frac{(x+2)}{(x+1)}\right] = \frac{3}{2}$

C) False. $\lim_{x\to 0}[(5x^3+8x^2)/(3x^4-16x^2)] = \lim_{x\to 0}\left[\frac{x^2(5x+8)}{x^2(3x^2-16)}\right] = \lim_{x\to 0}\left[\frac{(5x+8)}{(3x^2-16)}\right] = -\frac{8}{16} = -\frac{1}{2}$

but

$\lim_{x\to 2}[(x-2)/(x^2-4)] = \lim_{x\to 2}\left[\frac{(x-2)}{(x-2)(x+2)}\right] = \lim_{x\to 2}\left[\frac{1}{x+2}\right] = \frac{1}{4}$

#8) Which of the following is completely true?

A) $\lim_{x\to 0}\left[x^2\sin(1/3x^2)\right]=1/3$

B) $\boxed{\lim_{x\to 1}\left[(x-1)^2\sin[1/(x-1)]\right]=0}$

C) $\lim_{x\to\infty}\left[\sin(x)/x\right]=1$

D) All of the above

E) None of the above

A) False. $-x^2 \le x^2\sin(1/3x^2) \le x^2$

$$\Rightarrow \lim_{x\to 0}[-x^2] \le \lim_{x\to 0}[x^2\sin(1/3x^2)] \le \lim_{x\to 0}[x^2]$$

$$\Rightarrow 0 \le \lim_{x\to 0}[x^2\sin(1/3x^2)] \le 0$$

$$\Rightarrow \lim_{x\to 0}[x^2\sin(1/3x^2)] = 0 \ne 1/3$$

B) True. $-(x-1)^2 \le (x-1)^2\sin[1/(x-1)] \le (x-1)^2$

$$\Rightarrow \lim_{x\to 1}[-(x-1)^2] \le \lim_{x\to 1}[(x-1)^2\sin[1/(x-1)]] \le \lim_{x\to 1}[(x-1)^2]$$

$$\Rightarrow 0 \le \lim_{x\to 1}[(x-1)^2\sin[1/(x-1)]] \le 0$$

$$\Rightarrow \lim_{x\to 1}[(x-1)^2\sin[1/(x-1)]] = 0$$

C) False. $-1 \le \sin(x) \le 1$

$$\Rightarrow -1/x \le \sin(x)/x \le 1/x$$

$$\Rightarrow \lim_{x\to\infty}[-1/x] \le \lim_{x\to\infty}[\sin(x)/x] \le \lim_{x\to\infty}[1/x]$$

$$\Rightarrow 0 \le \lim_{x\to\infty}[\sin(x)/x] \le 0$$

$$\Rightarrow \lim_{x\to\infty}[\sin(x)/x] = 0 \ne 1$$

#9) Which of the following is completely true? Note: $\lim_{x\to 0}[\sin(x)/x] = 1$

A) $\lim_{x\to 0}[\sin(\sqrt{3}x)/\sqrt{3}x] = \lim_{x\to 0}[\sin(\sqrt{2}x)/\sqrt{2}x]$

B) $\lim_{x\to 0}[x/\sqrt{2}\sin(x)] = \sqrt{\lim_{x\to 0}[x^2/\sin(2x^2)]}$

C) $\lim_{x\to 0}\left[\dfrac{\sin(3x)\cot(5x)}{x\cot(4x)}\right] = \lim_{x\to 0}\left[\dfrac{x\tan(3x)\cot(x/4)}{\sin(5x)}\right]$

D) All of the above

E) None of the above

A) True. $\lim_{x\to 0}[\sin(\sqrt{3}x)/\sqrt{3}x] = \lim_{\theta\to 0}\left[\dfrac{\sin(\theta)}{\theta}\right] = 1$ where $\theta = \sqrt{3}x$ and $\lim_{\theta\to 0} \equiv \lim_{x\to 0}$

and

$\lim_{x\to 0}[\sin(\sqrt{2}x)/\sqrt{2}x] = \lim_{\alpha\to 0}\left[\dfrac{\sin(\alpha)}{\alpha}\right] = 1$ where $\alpha = \sqrt{2}x$ and $\lim_{\alpha\to 0} \equiv \lim_{x\to 0}$

B) True. $\lim_{x\to 0}[x/\sqrt{2}\sin(x)] = \lim_{x\to 0}\left[\dfrac{1}{\sqrt{2}}\cdot\dfrac{x}{\sin(x)}\right] = \dfrac{1}{\sqrt{2}}\cdot\lim_{x\to 0}\left[\dfrac{x}{\sin(x)}\right] = \dfrac{1}{\sqrt{2}}\cdot\lim_{x\to 0}\left[\left(\dfrac{\sin(x)}{x}\right)^{-1}\right]$

$= \dfrac{1}{\sqrt{2}}\cdot\left(\lim_{x\to 0}\left[\dfrac{\sin(x)}{x}\right]\right)^{-1} = \dfrac{1}{\sqrt{2}}(1)^{-1} = \dfrac{1}{\sqrt{2}}$ and

$\sqrt{\lim_{x\to 0}[x^2/\sin(2x^2)]} = \sqrt{\lim_{x\to 0}\left[\dfrac{2}{2}\cdot\dfrac{x^2}{\sin(2x^2)}\right]} = \sqrt{\lim_{x\to 0}\left[\dfrac{1}{2}\cdot\dfrac{2x^2}{\sin(2x^2)}\right]}$

$= \sqrt{\lim_{\theta\to 0}\left[\dfrac{1}{2}\cdot\dfrac{\theta}{\sin(\theta)}\right]}$ where $\theta = 2x^2$ and $\lim_{\theta\to 0} \equiv \lim_{x\to 0}$

$= \sqrt{\dfrac{1}{2}\cdot\lim_{\theta\to 0}\left[\dfrac{\theta}{\sin(\theta)}\right]} = \sqrt{\dfrac{1}{2}\cdot\lim_{\theta\to 0}\left[\left(\dfrac{\sin(\theta)}{\theta}\right)^{-1}\right]} = \dfrac{1}{\sqrt{2}}\cdot\sqrt{\left(\lim_{\theta\to 0}\left[\dfrac{\sin(\theta)}{\theta}\right]\right)^{-1}}$

$= \dfrac{1}{\sqrt{2}}\cdot\sqrt{(1)^{-1}} = \dfrac{1}{\sqrt{2}}\cdot\sqrt{1} = \dfrac{1}{\sqrt{2}}$

C) True. $\lim_{x\to 0}\left[\frac{\sin(3x)\cot(5x)}{x\cot(4x)}\right] = \lim_{x\to 0}\left[\left(\frac{3}{3}\right)\left(\frac{5x}{5x}\right)\left(\frac{4x}{4x}\right)\left(\frac{\sin(3x)}{x}\right)\left(\frac{\cos(5x)}{\sin(5x)}\right)\left(\frac{\sin(4x)}{\cos(4x)}\right)\right]$

$$= \lim_{x\to 0}\left[\left(\frac{3\cdot 4x}{5x}\right)\left(\frac{\sin(3x)}{3x}\right)\left(\frac{5x}{\sin(5x)}\right)\left(\frac{\sin(4x)}{4x}\right)\left(\frac{\cos(5x)}{\cos(4x)}\right)\right]$$

$$= \lim_{x\to 0}\left[\left(\frac{12}{5}\right)\right]\cdot\lim_{x\to 0}\left[\frac{\sin(3x)}{3x}\right]\cdot\lim_{x\to 0}\left[\frac{5x}{\sin(5x)}\right]\cdot\lim_{x\to 0}\left[\frac{\sin(4x)}{4x}\right]\lim_{x\to 0}\left[\frac{\cos(5x)}{\cos(4x)}\right]$$

$$= \frac{12}{5}\cdot\lim_{3x\to 0}\left[\frac{\sin(3x)}{3x}\right]\cdot\lim_{5x\to 0}\left[\left(\frac{\sin(5x)}{5x}\right)^{-1}\right]\cdot\lim_{4x\to 0}\left[\frac{\sin(4x)}{4x}\right]\cdot\left[\frac{\cos(0)}{\cos(0)}\right]$$

since $\lim_{kx\to 0} \equiv \lim_{x\to 0}$

$$= \frac{12}{5}\cdot(1)\cdot\left(\lim_{5x\to 0}\left[\frac{\sin(5x)}{5x}\right]\right)^{-1}\cdot(1)\cdot\left(\frac{1}{1}\right)$$

$$= \frac{12}{5}\cdot(1)^{-1} = \frac{12}{5}$$

and similarly,

$$\lim_{x\to 0}\left[\frac{x\tan(3x)\cot(x/4)}{\sin(5x)}\right] = \lim_{x\to 0}\left[\frac{5}{5}\cdot\frac{3x}{3x}\cdot\frac{x/4}{x/4}\cdot\frac{x}{\sin(5x)}\cdot\frac{\sin(3x)}{\cos(3x)}\cdot\frac{\cos(x/4)}{\sin(x/4)}\right]$$

$$= \lim_{x\to 0}\left[\frac{3x}{5(x/4)}\cdot\frac{5x}{\sin(5x)}\cdot\frac{\sin(3x)}{3x}\cdot\frac{x/4}{\sin(x/4)}\cdot\frac{\cos(x/4)}{\cos(3x)}\right]$$

$$= \lim_{x\to 0}\left[\frac{12}{5}\right]\cdot\lim_{5x\to 0}\left[\left(\frac{\sin(5x)}{5x}\right)^{-1}\right]\cdot\lim_{3x\to 0}\left[\frac{\sin(3x)}{3x}\right]\cdot\lim_{x/4\to 0}\left[\left(\frac{\sin(x/4)}{x/4}\right)^{-1}\right]\cdot\lim_{x\to 0}\left[\frac{\cos(x/4)}{\cos(3x)}\right]$$

$$= \frac{12}{5}\cdot\left(\lim_{5x\to 0}\left[\frac{\sin(5x)}{5x}\right]\right)^{-1}\cdot(1)\cdot\left(\lim_{x/4\to 0}\left[\frac{\sin(x/4)}{x/4}\right]\right)^{-1}\cdot\left(\frac{\cos(0)}{\cos(0)}\right)$$

$$= \frac{12}{5}\cdot(1)^{-1}\cdot(1)^{-1}\cdot\left(\frac{1}{1}\right) = \frac{12}{5}$$

#10) Which of the following is completely true?

A) $\lim_{x \to \pi/6} [\sec^2(2x) + \csc^2(2x)] = \lim_{x \to \pi/6} [\sec^2(2x)\csc^2(2x)]$

B) $\lim_{x \to a} [\cos(2x)] = \lim_{x \to a} [\cos^2(x)] - \lim_{x \to a} [\sin^2(x)] \quad \forall a \in \mathbb{R}$

C) $\lim_{x \to a} [\cos^2(3x)] + \lim_{x \to a} [\sin^2(3x)] = 1 \quad \forall a \in \mathbb{R}$

D) $\boxed{\text{All of the above}}$

E) None of the above

A) True. $\lim_{x \to \pi/6} [\sec^2(2x) + \csc^2(2x)] = \lim_{x \to \pi/6} \left[\frac{1}{\cos^2(2x)} + \frac{1}{\sin^2(2x)}\right]$

$$= \lim_{x \to \pi/6} \left[\frac{1}{\cos^2(2x)} \cdot \frac{\sin^2(2x)}{\sin^2(2x)} + \frac{1}{\sin^2(2x)} \cdot \frac{\cos^2(2x)}{\cos^2(2x)}\right]$$

$$= \lim_{x \to \pi/6} \left[\frac{\sin^2(2x) + \cos^2(2x)}{\cos^2(2x)\sin^2(2x)}\right] = \lim_{x \to \pi/6} \left[\frac{1}{\cos^2(2x)\sin^2(2x)}\right]$$

$$= \lim_{x \to \pi/6} \left[\frac{1}{\cos^2(2x)} \cdot \frac{1}{\sin^2(2x)}\right] = \lim_{x \to \pi/6} [\sec^2(2x)\csc^2(2x)]$$

B) True. $\lim_{x \to a} [\cos(2x)] = \lim_{x \to a} [\cos^2(x) - \sin^2(x)]$ (Double Angle Identity for Cosine)

$$= \lim_{x \to a} [\cos^2(x)] - \lim_{x \to a} [\sin^2(x)]$$

C) True. $\lim_{x \to a} [\cos^2(3x)] + \lim_{x \to a} [\sin^2(3x)] = \lim_{x \to a} [\cos^2(3x) + \sin^2(3x)]$

$$= \lim_{x \to a} [1] = 1$$