Exam I Version A Solutions

1. **C** Set x and y components equal and solve for t. $t = 4$ at $(2, -3)$, $t = 0$ at $(0, 1)$, $t = 1$ at $(1, 0)$, but $-1 = \sqrt{t}$ has no solution, so the point not on the curve is $(\mathbf{-1}, \mathbf{0})$.

2. **D** $\dfrac{\cos x}{1+\sin x} \cdot \dfrac{(1-\sin x)}{(1-\sin x)} = \dfrac{\cos x(1-\sin x)}{1-\sin^2 x} = \dfrac{1-\sin x}{\cos x} = \sec \mathbf{x} - \tan x$.

3. **A** Since $-1 \le \cos x \le 1$, $\dfrac{-1}{x} \le \dfrac{\cos x}{x} \le \dfrac{1}{x}$. Since $\pm\dfrac{1}{x} \to 0$ as $x \to \infty$, by the Squeeze Theorem, the limit is **0**.

4. **A** As we approach 4 from the right along the curve, the y-value approaches **4**.

5. **B** Let $f(x) = x^3 - x^2 + x$. f is continuous since it is a polynomial, and $f(2) = 6$, $f(3) = 21$, so $f(2) < 10 < f(3)$. Therefore, **by the Intermediate Value Theorem, there is a solution to** $\mathbf{f(x) = 10}$ **on** $[\mathbf{2}, \mathbf{3}]$.

6. **E** $\mathbf{a} = (6\mathbf{i} + 3\mathbf{j}) - (-3\mathbf{i} + \mathbf{j}) = 9\mathbf{i} + 2\mathbf{j}$. To form a unit vector $\hat{\mathbf{a}}$, multiply by the reciprocal of the magnitude: $\hat{\mathbf{a}} = \dfrac{1}{\sqrt{9^2+2^2}}(9\mathbf{i} + 2\mathbf{j}) = \dfrac{\mathbf{9}}{\sqrt{\mathbf{85}}}\mathbf{i} + \dfrac{\mathbf{2}}{\sqrt{\mathbf{85}}}\mathbf{j}$.

7. **B** $\lim_{x\to 5^-} f(x) = 6 - 5 = 1$. $\lim_{x\to 5^+} f(x) = -8 + 2(5) = 2$, and $f(5) = 1$. Therefore, since $\lim_{x\to 5^-} f(x) = f(5) \neq \lim_{x\to 5^+} f(x)$, **f is continuous only from the left.**

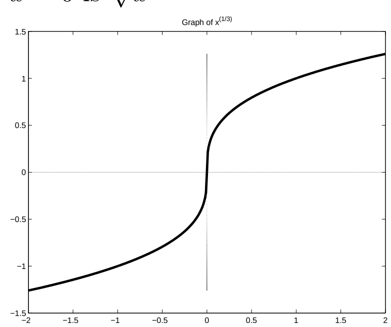
8. **E** Factor x from numerator and denominator: $\lim_{x\to\infty} \dfrac{x(4+\frac{9}{x})}{x(3-\frac{8}{x})} = \dfrac{4}{3}$. Similarly, $\lim_{x\to -\infty} \dfrac{x(4+\frac{9}{x})}{x(3-\frac{8}{x})} = \dfrac{\mathbf{4}}{\mathbf{3}}$.

9. **A** The angle between the gravity force (weight) and the motion of the block is 30°, so the work done is $W = |\mathbf{F}||\mathbf{D}|\cos 30° = (30)(20)\left(\dfrac{\sqrt{3}}{2}\right) = \mathbf{300\sqrt{3}}$ **ft-lbs**.

10. **B** $f'(3) = \lim_{h\to 0} \dfrac{f(3+h)-f(3)}{h} = \lim_{h\to 0} h^2 - 4h + 7 = 7$. The equation of the line whose slope is 7 and which passes through the point $(3, 4)$ is $y - 4 = 7(x-3)$, or $\mathbf{y = 7x - 17}$.

11. **C** $\lim_{x\to 3} \dfrac{x(x-3)}{x-3} = \lim_{x\to 3} x = \mathbf{3}$.

12. **E** (a) is both continuous and differentiable everywhere; (c) and (d) are not continuous at $x = 0$. From the graph of (e), we see that the function continuous but not differentiable at $x = 0$ is $\sqrt[3]{x}$



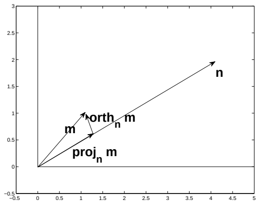
13. $f'(1) = \lim_{x\to 1} \dfrac{f(x)-f(1)}{x-1} = \lim_{h\to 0} \dfrac{\sqrt{3+2x}-\sqrt{5}}{x-1} \cdot \dfrac{\sqrt{3+2x}+\sqrt{5}}{\sqrt{3+2x}+\sqrt{5}}$
$= \lim_{x\to 1} \dfrac{3+2x-5}{(x-1)(\sqrt{3+2x}+\sqrt{5})}$
$= \lim_{x\to 1} \dfrac{2(x-1)}{(x-1)(\sqrt{3+2x}+\sqrt{5})} = \lim_{x\to 1} \dfrac{2}{\sqrt{3+2x}+\sqrt{5}} = \dfrac{\mathbf{2}}{\mathbf{2\sqrt{5}}} = \dfrac{\mathbf{1}}{\sqrt{\mathbf{5}}}$.

14. .

   (a) $\text{proj}_{\mathbf{n}}\mathbf{m} = \dfrac{\mathbf{m}\cdot\mathbf{n}}{|\mathbf{n}|^2}\mathbf{n} = \dfrac{4+2}{(4^2+2^2)}\langle 4, 2\rangle = \left\langle \dfrac{6}{5}, \dfrac{3}{5} \right\rangle$, so $\text{orth}_{\mathbf{n}}\mathbf{m} = \langle 1, 1\rangle - \left\langle \dfrac{6}{5}, \dfrac{3}{5} \right\rangle = \left\langle -\dfrac{1}{5}, \dfrac{2}{5} \right\rangle$.

   (b) The point $(0, 0)$ is on the line, so let $\mathbf{b} = \langle 1, 1\rangle - \langle 0, 0\rangle = \langle 1, 1\rangle$. The vector $\mathbf{v} = \langle 4, 2\rangle$ is in the direction of the line, so $\mathbf{v}^{\perp} = \langle -2, 4\rangle$ is orthogonal to the line. The distance is found

by $|\text{comp}_{\mathbf{v}^\perp}\mathbf{b}| = \dfrac{-2+4}{\sqrt{(-2)^2+4^2}} = \dfrac{\mathbf{2}}{\sqrt{\mathbf{20}}}$
(NOTE that this is the magnitude of the vector in (a) above!)



15. Find a common denominator:
$\displaystyle\lim_{x\to 2}\frac{\frac{1}{x}\cdot\frac{2}{2}-\frac{1}{2}\cdot\frac{x}{x}}{x-2} = \lim_{x\to 2}\frac{1}{x-2}\cdot\frac{2-x}{2x}$
$\displaystyle= \lim_{x\to 2} -\frac{1}{2x} = -\frac{\mathbf{1}}{\mathbf{4}}.$

16. $\mathbf{F_1} = \langle 8\cos 45°, 8\sin 45°\rangle = \langle 4\sqrt{2}, 4\sqrt{2}\rangle$. $\mathbf{F_2} = \langle 14\cos 60°, -14\sin 60°\rangle = \langle 7, -7\sqrt{3}\rangle$. The resultant force is $\mathbf{F_1}+\mathbf{F_2} = \langle 4\sqrt{2}+7, 4\sqrt{2}-7\sqrt{3}\rangle$. Therefore, the magnitude of the resultant force is $\sqrt{(\mathbf{4}\sqrt{\mathbf{2}}+\mathbf{7})^\mathbf{2}+(\mathbf{4}\sqrt{\mathbf{2}}-\mathbf{7}\sqrt{\mathbf{3}})^\mathbf{2}}$.

17. We use $\mathbf{r}(t) = \mathbf{r_0}+t\mathbf{v}$ to find the parametric equations. $\mathbf{r_0}$ corresponds to the point at $t=1$: $\mathbf{r}(1) = 5\mathbf{i}+5\mathbf{j}$. $\mathbf{v} = \mathbf{r}'(1) = \displaystyle\lim_{t\to 1}\frac{((5t)\mathbf{i}+(8-3t^2)\mathbf{j})-(5\mathbf{i}+5\mathbf{j})}{t-1} = \lim_{t\to 1}\frac{(5t-5)\mathbf{i}+(3-3t^2)\mathbf{j}}{t-1} = \lim_{t\to 1}\left(\frac{5t-5}{t-1}\right)\mathbf{i}+\left(\frac{3-3t^2}{t-1}\right)\mathbf{j} = \left(\lim_{t\to 1}\frac{5(t-1)}{t-1}\right)\mathbf{i} + \left(\lim_{t\to 1}\frac{3(1+t)(1-t)}{t-1}\right)\mathbf{j} = 5\mathbf{i}-6\mathbf{j}$. Therefore, the vector equation of the tangent line is $\mathbf{r}(t) = (5\mathbf{i}+5\mathbf{j})+t(5\mathbf{i}-6\mathbf{j})$, so the parametric equations are $\mathbf{r(t)} = (\mathbf{5}+\mathbf{5t})\mathbf{i}+(\mathbf{5}-\mathbf{6t})\mathbf{j}$, or $\mathbf{x}=\mathbf{5}+\mathbf{5t}$, $\mathbf{y}=\mathbf{5}-\mathbf{6t}$.