

1. For each of the functions $f(x)$ given below, find $f'(x)$.

(a) **4 points** $f(x) = x^5 + 5x^4 + 3x^2 + 9$

Solution:

$$f'(x) = 5x^4 + 20x^3 + 6x$$

(b) **4 points** $f(x) = x^4 e^x$

Solution: This requires the product rule. Recall that the derivative of e^x is e^x .

$$f'(x) = 4x^3 e^x + x^4 e^x$$

(c) **4 points** $f(x) = \frac{3x^2 + 2}{x^3 + 2 \tan x}$

Solution: Using the quotient rule,

$$\frac{6x(x^3 + 2 \tan x) - (3x^2 + 2)(x^2 + 2 \sec^2 x)}{(x^3 + 2 \tan x)^2}$$

There is little point in trying to simplify this.

2. Compute each of the following derivatives as indicated:

(a) **4 points** $\frac{d}{d\theta} \left[\cos \left(\frac{\pi}{180} \theta \right) \right]$

Solution: This is just the derivative of the $\cos \theta$, when θ is in degrees. Using the chain rule, we get

$$-\frac{\pi}{180} \sin \left(\frac{\pi}{180} \theta \right)$$

(b) **4 points** $\frac{d}{du} [\sin(3u) \sin(4u)]$

Solution: Use the product rule to get

$$\left(\frac{d}{du} \sin(3u) \right) \sin(4u) + \sin(3u) \left(\frac{d}{du} \sin(4u) \right)$$

and then use the chain rule to get the answer, which is

$$3 \cos(3u) \sin(4u) + 4 \sin(3u) \cos(4u).$$

(c) 4 points $\frac{d}{dt} \left[\frac{t}{7} - \frac{7}{t} \right]$

Solution: If you rewrite this as $\frac{1}{7}t - 7t^{-1}$, it is clear the derivative is $\frac{1}{7} + 7t^{-2}$

3. 8 points Write a limit that represents the slope of the graph

$$y = \begin{cases} 6 + x \ln |x| & x \neq 0 \\ 6 & x = 0 \end{cases}$$

at $x = 0$. You **do not need to evaluate the limit**.

Solution: To do this, we need to remember the definition of the derivative, which is $f'(a) = \lim_{h \rightarrow 0} \frac{f(a+h) - f(a)}{h}$. In the current case, $a = 0$, and notice that $f(0) = 6$, so we have

$$\lim_{h \rightarrow 0} \frac{f(h) - f(0)}{h} = \lim_{h \rightarrow 0} \frac{(6 + h \ln |h|) - 6}{h}$$

This simplifies to

$$\lim_{h \rightarrow 0} \frac{h \ln |h|}{h} = \lim_{h \rightarrow 0} \ln |h| = -\infty,$$

although it wasn't required for you to do this.

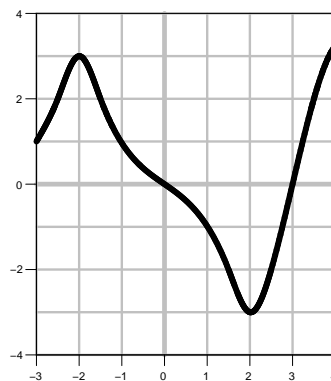
4. At right is the graph of **the derivative** f' of a function.

- (a) 4 points List all values of x with $-3 \leq x \leq 4$ where $f(x)$ has a local minimum.

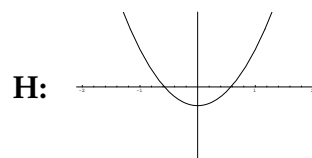
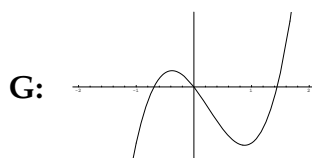
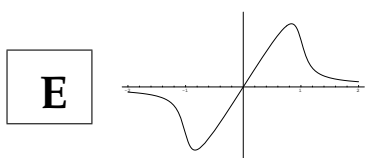
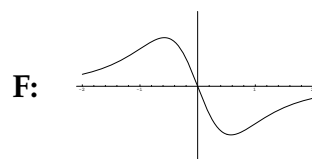
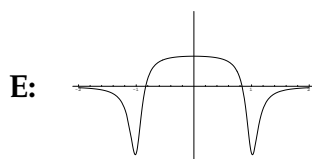
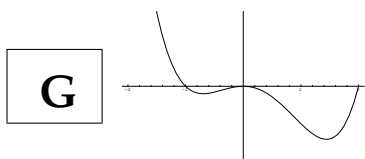
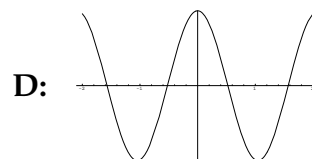
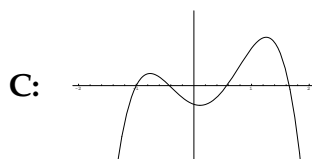
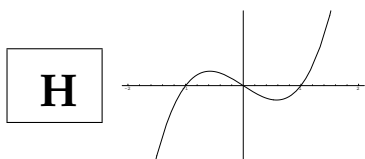
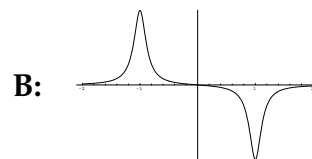
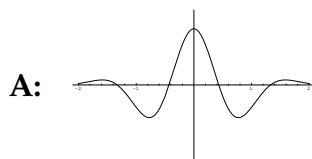
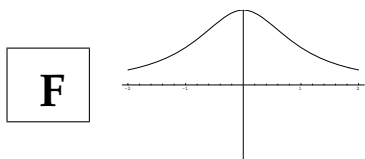
Solution: A local minimum for $f(x)$ will occur where $f'(x)$ changes from negative to positive. This happens at $x = 3$.

- (b) 4 points At $x = -1$, is $f(x)$ concave up, concave down, or neither?

Solution: We know that a function is concave up when its second derivative is positive, and concave down when f'' is negative. The graph shows $f'(x)$, which is decreasing near $x = -1$. That means the derivative of $f'(x)$ is negative near $x = -1$, so $f''(-1) < 0$. Hence $f(x)$ is concave down at $x = -1$.



5. 16 points For each of the 4 functions graphed in the left column, find the corresponding derivative function among any of the 8 choices on the right (not just on the same row) and put its letter in the corresponding box.



6. Let $f(x) = x e^{-2x}$.

(a) **3 points** Calculate $f'(x)$

Solution: We use the product rule and the chain rule:

$$f'(x) = e^{-2x} - 2x e^{-2x}$$

(b) **3 points** Calculate $f''(x)$?

Solution: Taking the derivative of the above gives

$$f''(x) = -2e^{-2x} - 2e^{-2x} + 4x e^{-2x}$$

which simplifies to

$$4x e^{-2x} - 4e^{-2x}$$

(c) **4 points** For what values of x is $f(x)$ increasing?

Solution: To answer this, we need to know when $f'(x) > 0$, that is, where

$$e^{-2x} - 2x e^{-2x} > 0$$

Factoring out the exponential term gives $e^{-2x}(1 - 2x) > 0$, and since e^{-2x} is always positive, we only need ask where $1 - 2x > 0$. This happens for

$$x < \frac{1}{2}.$$

(d) **4 points** For what values of x is $f(x)$ concave down?

Solution: We need to know when $f''(x) < 0$, so factor $f''(x)$ as

$$4e^{-2x}(2x - 1).$$

As before, we can ignore the exponential term, since it is always positive, and we see that $f''(x) < 0$ when $x < 1/2$.

7. 10 points Write the equation of the line tangent to the curve

$$y = 3x^4 - 5x + \sqrt{x} \quad \text{at } x = 1$$

Solution: To write the equation of a line, we need a point and a slope. Since the line is tangent to the curve at $x = 1$, it contains the point $(1, f(1)) = (1, -1)$.

To get the slope, we calculate $f'(1)$. Taking the derivative gives

$$f'(x) = 12x^3 - 5 + \frac{1}{2}x^{-1/2},$$

so $f'(1) = 12 - 5 + \frac{1}{2} = \frac{15}{2}$. Hence the line is

$$y + 1 = \frac{15}{2}(x - 1), \quad \text{or, equivalently,} \quad y = \frac{15}{2}x - \frac{17}{2}$$

8. 10 points A ladder 14 feet long rests against a vertical wall. Let θ be the angle between the top of the ladder and the wall, and let ℓ be the distance from the bottom of the ladder to the wall. If the bottom of the ladder slides away from the wall, how fast does ℓ change with respect to θ when $\theta = \frac{\pi}{6}$?

Solution: Since the ladder forms a right triangle with the wall, we have $\ell = 14 \sin \theta$. The rate of change of ℓ with respect to θ is $\frac{d\ell}{d\theta}$, which is $14 \cos \theta$. We want its value when $\theta = \frac{\pi}{6}$, so that is

$$14 \cos \left(\frac{\pi}{6} \right) = 14 \cdot \frac{\sqrt{3}}{2} = 7\sqrt{3}$$

