1. **(3 pts)** The function $y = \dfrac{3}{\sqrt{6x+3}}$ is a composite function $y = f(g(x))$.
Identify the inner function $u = g(x)$ and the outer function $y = f(u)$.

A) $u = g(x) = 6x+3,\ y = f(u) = \dfrac{3}{\sqrt{u}}$

B) $u = g(x) = 3,\ y = f(u) = \sqrt{6u+3}$

C) $u = g(x) = \sqrt{6x+3},\ y = f(u) = 3$

D) $u = g(x) = 6x+3,\ y = f(u) = \dfrac{3}{u}$

2. **(3 pts)** Does the graph of the function $y = 10x + 5\sin x$ have any horizontal tangents in the interval $0 \le x \le 2\pi$? If so, where?

A) Yes, at $x = \dfrac{2\pi}{3}$

B) Yes, at $x = \dfrac{2\pi}{3},\ x = \dfrac{4\pi}{3}$

C) Yes, at $x = \dfrac{\pi}{3},\ x = \dfrac{2\pi}{3}$

D) No

3. **(3 pts)** Use the table to evaluate $\frac{d}{dx}[f(x)g(x)]$ at $\boldsymbol{x=1}$.

$\boldsymbol{x}$	**1**	**2**	**3**	**4**
$\boldsymbol{f(x)}$	5	3	–2	5
$\boldsymbol{f'(x)}$	–5	1	4	1
$\boldsymbol{g(x)}$	7	4	2	4
$\boldsymbol{g'(x)}$	–3	–5	–3	7

A) –46

B) –50

C) 20

D) 50

4. **(4 pts)** If $x^2 - y^3 = 5$, then it can be shown that $\frac{dy}{dx} = \frac{2x}{3y^2}$. Find $\frac{d^2y}{dx^2}$.

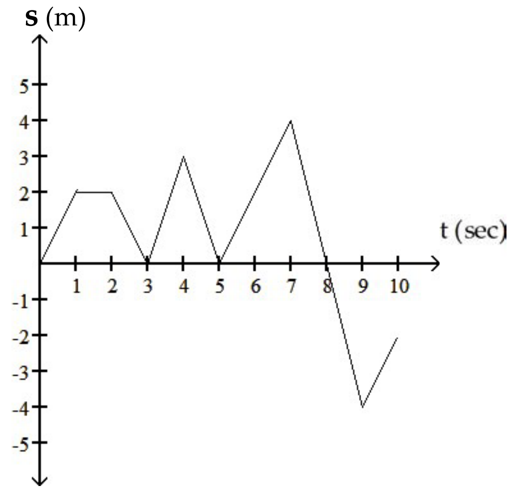
A) $\frac{d^2y}{dx^2} = \frac{6y^3 - 8x^2}{9y^5}$

B) $\frac{d^2y}{dx^2} = \frac{5y^3 - 8x^2}{9y^5}$

C) $\frac{d^2y}{dx^2} = \frac{6y^3 - 8x^2}{9y^3}$

D) $\frac{d^2y}{dx^2} = \frac{6y^3 - 8x^2}{9y^6}$

5. **(3 pts)** The figure shows the **position** $s(t)$ of a body moving along a coordinate line as a function of time t, where $0 \le t \le 10$. Use the figure to determine when the body is moving in the negative direction.



A) $8 < t \le 10$ sec

B) $2 < t < 3,\ 4 < t < 5,\ 7 < t < 9$ sec

C) $8 < t < 9$ sec

D) $2 < t < 3,\ 4 < t < 5,\ 7 < t < 8$ sec

6. **(3 pts)** Find the slope of the **normal** line to $2x^2 + y^2 = 11$ at the point $(1,\ 3)$.

A) $\frac{-2}{3}$

B) $\frac{-3}{2}$

C) $\frac{7}{6}$

D) $\frac{3}{2}$

7. **(3 pts)** Use the table to evaluate $\frac{d}{dx}\left[\frac{f(x)}{g(x)}\right]$ at $\boldsymbol{x = 2}$.

x	1	2	3	4
$f(x)$	5	3	–2	5
$f'(x)$	–5	1	4	1
$g(x)$	7	4	2	4
$g'(x)$	–3	–5	–3	7

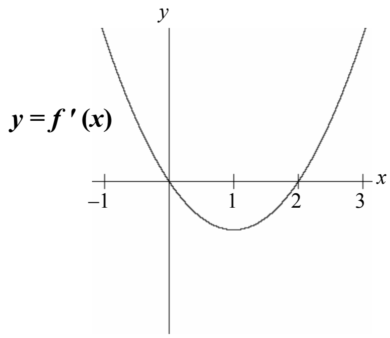
A) $\frac{19}{16}$

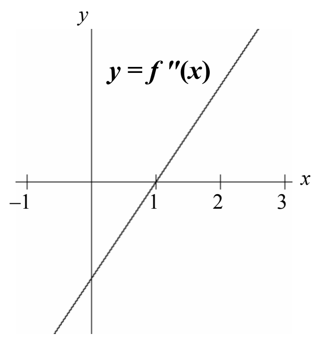
B) $\frac{-11}{16}$

C) $\frac{19}{25}$

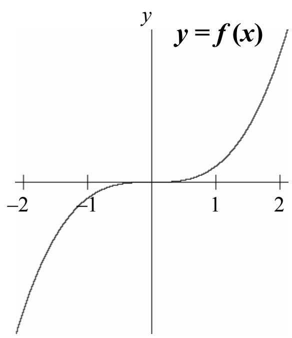
D) $\frac{-19}{25}$

8. **(3 pts)** Use the graphs of the first derivate of f, $y = f'(x)$, and the second derivative of f, $y = f''(x)$, to choose the correct graph of $y = f(x)$. Assume $f(0) = 0$.

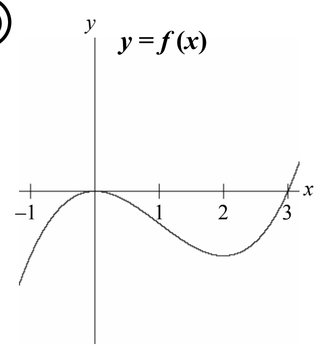




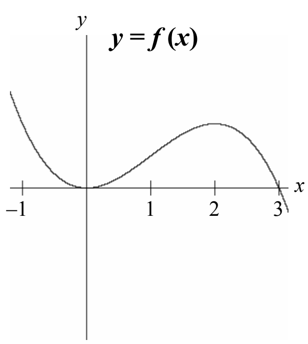
A)



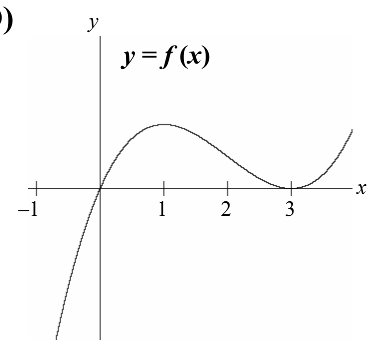
B)



C)



D)



9. **(3 pts)** Find the derivative of $y = \cos(\sin x)$.

A) $\dfrac{dy}{dx} = -\sin(\cos x)$

B) $\dfrac{dy}{dx} = \cos^2 x - \sin^2 x$

C) $\dfrac{dy}{dx} = -\sin(\sin x)\cos x$

D) $\dfrac{dy}{dx} = -\sin^2 x \cos x$

10. **(4 pts)** Two people standing together at a point begin walking at the same time and the same speed, with one walking north and the other east, so that their paths form a right angle. When the two people are both 5 m from the starting point, the triangular area formed by the positions of the two people and their starting point is changing at 4 m^2/s. How fast is each person walking when they are both 5 m from the starting point?

A) $\frac{2}{5}$ m/s

B) $\frac{4}{5}$ m/s

C) $\frac{25}{4}$ m/s

D) $\frac{8}{5}$ m/s

11. **(3 pts)** At time $t \geq 0$, the **velocity** of a body moving along the s-axis is $v(t) = t^2 - 7t + 6$. When is the body's **velocity** increasing?

A) $t < 6$

B) $t < \frac{7}{2}$

C) $t > 6$

D) $t > \frac{7}{2}$

12. **(3 pts)** Find the derivative of $r = 5 - \theta^7 \cos\theta$.

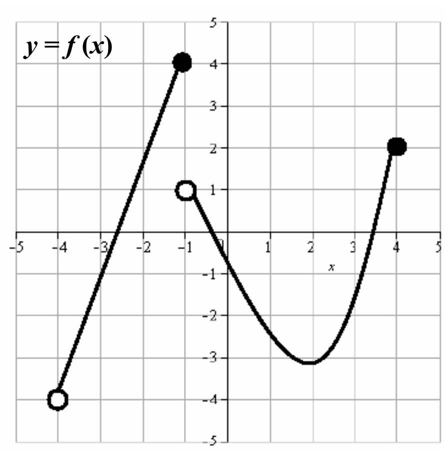
A) $\dfrac{dr}{d\theta} = 7\theta^6 \sin\theta$

B) $\dfrac{dr}{d\theta} = -7\theta^6 \cos\theta + \theta^7 \sin\theta$

C) $\dfrac{dr}{d\theta} = 7\theta^6 \cos\theta - \theta^7 \sin\theta$

D) $\dfrac{dr}{d\theta} = 7\theta^6 \sin\theta - \theta^7 \cos\theta$

13. **(3 pts)** Use the graph to identify the x-values at which local and absolute extreme values occur.



A) no local maximum, local minimum at $x = 2$;
no absolute maximum, no absolute minimum

B) no local maximum, local minimum at $x = 2$;
absolute maximum at $x = -1$, absolute minimum at $x = -4$

C) local maximum at $x = -1$, local minimum at $x = 2$;
absolute maximum at $x = -1$, no absolute minimum

D) local maximum at $x = -1$, local minimum at $x = 2$;
absolute maximum at $x = -1$, absolute minimum at $x = -4$

14. **(3 pts)** Evaluate $\lim_{x \to 0} \dfrac{2\sin 3x}{5x}$.

A) $\dfrac{3}{5}$

B) $\dfrac{2}{15}$

C) 0

D) $\dfrac{6}{5}$

15. **(3 pts)** Evaluate $\lim_{x \to \frac{\pi}{3}} \sqrt{81 + \sin(\pi \sec x)}$.

A) 9

B) 0

C) 1

D) $\sqrt{82}$

16. **(3 pts)** Use the table to evaluate $\frac{d}{dx}\left[g(x+f(x))\right]$ at $\boldsymbol{x=3}$.

x	3	4
$f(x)$	1	–3
$f'(x)$	6	2
$g(x)$	9	3
$g'(x)$	5	–5

A) –35

B) –30

C) 21

D) –5

17. **(3 pts)** Find the slope of the **tangent** line to $x^6y^6=64$ at the point $(2,\ 1)$.

A) –32

B) 2

C) $\frac{-1}{2}$

D) $\frac{-1}{4}$

Free Response. The Free Response questions will count 46% of the total grade. Read each question carefully. In order to receive full credit you must show legible and logical (relevant) justification which supports your final answer. Give answers as exact answers. You are NOT permitted to use a calculator on any portion of this test.

1. **(12 pts.)** Find the derivative of each of the following functions. **Do not simplify.**

a. **(4 pts.)** $y = \frac{x^2 + 2x - 2}{x^2 - 2}$

$$\frac{dy}{dx} = \frac{[2x+2]\left(x^2-2\right) - \left(x^2+2x-2\right)[2x]}{\left(x^2-2\right)^2}$$

$$\left\{ = \frac{\cancel{2x^3} \cancel{-4x} + 2x^2 - 4 \cancel{-2x^3} - 4x^2 + \cancel{4x}}{\left(x^2-2\right)^2} = \frac{-2x^2-4}{\left(x^2-2\right)^2} = \frac{-2\left(x^2+2\right)}{\left(x^2-2\right)^2} \right\}$$

OR

$$y = \frac{x^2+2x-2}{x^2-2} = \left(x^2+2x-2\right)\left(x^2-2\right)^{-1}$$

$$\frac{dy}{dx} = [2x+2]\left(x^2-2\right)^{-1} + \left(x^2+2x-2\right)\left[-\left(x^2-2\right)^{-2}(2x)\right]$$

Work on Problem:	**Points**
Applies the quotient rule (or rewrites the function as a product and applies the product rule). General form of the answer is required here. The individual terms are graded separately.	2 points
Finds the derivative of the numerator (or the first function).	1 point
Finds the derivative of the denominator (or the second function using the chain rule).	1 point

Notes:

- Subtract 2 points for not using the quotient rule or product rule.
- Subtract 1 point for reversing the terms in the numerator of the quotient rule.
- Subtract ½ point for incorrect simplification (even though simplification is not required).
- Subtract ½ point for each type of notation error with a maximum of 1 point deduction for all notation errors. Types of notation errors include, but are not limited to, missing or incorrect use of equals, missing or incorrect use of parentheses, and missing or incorrect derivative notation such as: $\frac{d}{dx} = \ldots$ without function inside derivative operator.
- Subtract ½ point for each minor algebra, arithmetic, and/or copy error.

1.**(continued)** Find the derivative of each of the following functions. **Do not simplify.**

b. **(4 pts.)** $g(t) = 7t(2t+3)^4$

$$g'(t) = [7](2t+3)^4 + 7t\left[4(2t+3)^3(2)\right]$$
$$\left\{= 7(2t+3)^3(2t+3+8t) = 7(2t+3)^3(10t+3)\right\}$$

Work on Problem:	Points
Applies the product rule. General form of the answer is required here. The individual terms are graded separately.	2 points
Finds the derivative of the first function.	1 point
Finds the derivative of the second function using the chain rule.	1 point

Notes:
- Subtract 2 points for not using the product rule unless the function has been expanded to take the derivative.
- Subtract ½ point for incorrect simplification (even though simplification is not required).
- Subtract ½ point for each type of notation error with a maximum of 1 point deduction for all notation errors. Types of notation errors include, but are not limited to, missing or incorrect use of equals, missing or incorrect use of parentheses, and missing or incorrect derivative notation such as: $\frac{d}{dx} = \ldots$ without function inside derivative operator.
- Subtract ½ point for each minor algebra, arithmetic, and/or copy error.

c. **(4 pts.)** $r(\theta) = (\sec\theta + \tan\theta)^{-3}$

$$r'(\theta) = -3(\sec\theta + \tan\theta)^{-4}(\sec\theta\tan\theta + \sec^2\theta)$$
$$\left\{ = \frac{-3\sec\theta\,\cancel{(\tan\theta + \sec\theta)}}{(\sec\theta+\tan\theta)^{\cancel{4}\,3}} = \frac{-3\sec\theta}{(\sec\theta+\tan\theta)^3} \right\}$$

Work on Problem:	Points
Applies the chain rule. General form of the answer is required here. The individual terms are graded separately.	2 points
Finds the derivative of the outside function.	1 point
Finds the derivative of the inside function.	1 point

Notes:
- Subtract 2 points for applying the chain rule incorrectly by substituting the derivative of the inside instead of multiplying by the derivative of the inside.
- Subtract ½ point for incorrect simplification (even though simplification is not required).
- Subtract ½ point for each type of notation error with a maximum of 1 point deduction for all notation errors. Types of notation errors include, but are not limited to, missing or incorrect use of equals, missing or incorrect use of parentheses, and missing or incorrect derivative notation such as: $\frac{d}{dx} = \ldots$ without function inside derivative operator.
- Subtract ½ point for each minor algebra, arithmetic, and/or copy error.

2. **(5 pts.)** Use implicit differentiation **(do not solve for y)** to find $\frac{dy}{dx}$ for the equation $x = \sec(20y)$.

$$\frac{d}{dx}(x) = \frac{d}{dx}\left[\sec(20y)\right]$$

$$1 = \sec(20y)\tan(20y)\left(20\frac{dy}{dx}\right)$$

$$\frac{dy}{dx} = \frac{1}{20\sec(20y)\tan(20y)} \qquad \left\{= \frac{1}{20}\cos(20y)\cot(20y)\right\}$$

Work on Problem:	Points
Attempts to use implicit differentiation to take the derivative of both sides with respect to x either implicitly or explicitly.	1 point
Finds the derivative of the left-hand side.	1 point
Finds the derivative of the right-hand side using the chain rule. (1 point for the derivative of the outside function and 1 point for the derivative of the inside function)	2 points
Isolates dy/dx in the final answer.	1 point

Notes:

- Attempt to follow mistakes through work making only one deduction for the original mistake.
- Subtract ½ point for incorrect simplification (even though simplification is not required).
- Subtract ½ point for each type of notation error with a maximum of 1 point deduction for all notation errors. Types of notation errors include, but are not limited to, missing or incorrect use of equals, missing or incorrect use of parentheses, missing or incorrect derivative notation such as: $\frac{d}{dx} = \ldots$ without function inside derivative operator and incorrectly having an equation inside the derivative operator $\frac{d}{dx}\left[x = \sec(20y)\right]$.
- Subtract ½ point for each minor algebra, arithmetic, and/or copy error.

3. **(7 pts.)** Determine the absolute extreme values of $f(x) = x^3 - 12x + 1$ on the interval [–3, 0]. State the location(s) of each absolute extreme.

$f'(x) = 3x^2 - 12 = 3(x^2 - 4) = 3(x-2)(x+2)$

($x = 2$ is not a crit. pt. because it is not in the given interval.)

$f'(x) = 0$ when $\cancel{x = 2}, x = -2$

$f'(x)$ always exists

x	$f(x) = x^3 - 12x + 1$
–3	$(-3)^3 - 12(-3) + 1 = -27 + 36 + 1 = 10$
–2	$(-2)^3 - 12(-2) + 1 = -8 + 24 + 1 = 17$ ← abs. max.
0	$(0)^3 - 12(0) + 1 = 1$ ← abs. min.

f has an absolute maximum value of <u>17</u> at x = <u>–2</u>.

f has an absolute minimum value of <u>1</u> at x = <u>0</u>.

Work on Problem:	Points
Finds the derivative of f. (1 point per correct term in derivative)	2 points
Sets the derivative equal to zero implicitly or explicitly.	1 point
Solves for x in the equation $f'(x) = 0$. (½ point for each value of x.)	1 point
Calculates the function value at the endpoints of the given interval and the one critical point.	1 point
Identifies the absolute extrema by filling in the blanks correctly. (½ point per value)	2 points

Notes:

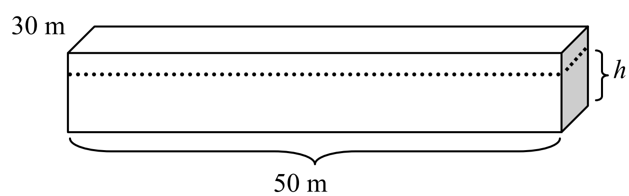
- Attempt to follow mistakes through work making only one deduction for the original mistake.
- Students should show all work when solving the equation $f'(x) = 0$ by finding both values of x and omitting the value $x = 2$ from subsequent calculations because it is not in the given interval.
- Subtract ½ point per extrema for reversing the values in the blanks.
- Subtract ½ point for each type of notation error with a maximum of 1 point deduction for all notation errors. Types of notation errors include, but are not limited to, missing or incorrect use of equals, missing or incorrect use of parentheses, missing or incorrect derivative notation such as: $\frac{d}{dx} = \ldots$ without function inside derivative operator and stating $f'(x) = 3x^2 - 12 = 0$ which incorrectly implies that the derivative is always zero.
- Subtract ½ point for each minor algebra, arithmetic, and/or copy error.

4. **(10 pts.)** A rectangular swimming pool measures 30 m wide and 50 m long and is the same depth everywhere. Suppose the pool is being filled through an inflow pipe that delivers water at a rate of 3 m^3/min.

a. **(8 pts)** How fast is the depth of the water in the pool increasing?

Instructions:
- Use t for time, h for the depth of the water in the pool, and V for the volume of water in the pool.
- Draw a picture and label it with known constants and given variables.
- Identify known rates and write an equation relating the variables.
- Answer the question including units.

Given $\frac{dV}{dt} = 3$ m^3/min. Find $\frac{dh}{dt}$.



$V = 30(50)h = 1500h$

$\frac{dV}{dt} = 1500\frac{dh}{dt}$

Since $\frac{dV}{dt} = 3$ m^3/min, $3 = 1500\frac{dh}{dt}$. So, $\frac{dh}{dt} = \frac{1}{500}$ m/min.

Work on Problem:	Points
Draws a picture and labels it appropriately.	1 point
Identifies explicitly or implicitly the given rate of change: *dV/dt*.	1 point
States the equation for the volume of water in the pool.	2 points
Takes the derivative of both sides of the volume formula with respect to time t to yield equation relating the variables and rates. (1 point per side)	2 points
Substitutes value of given rate to find *dh/dt*.	1 point
Correctly solves for *dh/dt* and provides units.	1 point

Notes:
- Subtract ½ point for missing or incorrect units.
- Attempt to follow mistakes through work making only one deduction for the original mistake.
- Subtract ½ point for each type of notation error with a maximum of 1 point deduction for all notation errors. Types of notation errors include, but are not limited to, missing or incorrect use of equals, missing or incorrect use of parentheses, missing or incorrect derivative notation such as: $\frac{d}{dt} = \ldots$ without function inside derivative operator and incorrectly having an equation inside the derivative operator $\frac{d}{dt}[V = 1500h]$.
- Subtract ½ point for each minor algebra, arithmetic, and/or copy error.

b. **(2 pts)** If the pool is 2 m deep everywhere, how long does it take to fill the pool?

The volume of water required to fill the pool is 30(50)(2) = 3000 m^3.

So, the amount of time to fill the pool is $\frac{V}{dV/dt} = \frac{3000}{3} = 1000$ min.

OR $\frac{h}{dh/dt} = \frac{2}{1/500}$ = 1000 min.

Work on Problem:	Points
Finds the amount of time to fill the pool and provides units.	2 points

Notes:
- Subtract ½ point for missing or incorrect units.
- Attempt to follow work from part a.

5. **(12 pts.)** Use the **first derivative** of f, $f'(x) = 12x^2(3-x)$, to complete the problem.

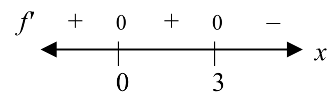
a. **(2 pts.)** State the intervals on which f is increasing and decreasing.

$f'(x) = 0$ when $x = 0, x = 3$

$f'(x)$ always exists

f is increasing on $(-\infty, 0), (0, 3)$. [OR $(-\infty, 3)$]

f is decreasing on $(3, \infty)$.



Work on Problem:	Points
States the intervals of increase.	1 point
States the intervals of decrease.	1 point

Notes:

- Because of the simplicity of this problem, no work is required for full credit.
- Subtract 1 point for missing or incorrect identification as intervals of increase or decrease.
- Subtract a maximum of ½ point for notation errors. Types of notation errors include using square brackets which indicate closed intervals instead of parentheses which indicate open intervals.
- Subtract ½ point for each minor algebra, arithmetic, and/or copy error.

b. **(2 pts.)** For which values of x does f have local extrema?
Identify each as a local maximum or local minimum.

f has a local maximum at $x = 3$.

Work on Problem:	Points
States the x-value of the local maximum and identifies it as such.	2 points

Notes:

- Because of the simplicity of this problem, no work is required for full credit.
- Attempt to follow work from part a.
- Subtract 1 point for missing or incorrect identification as local maximum.

5.**(continued)** Use the **first derivative** of f, $f'(x)=12x^2(3-x)$, to complete the problem.

c. **(4 pts.)** State the intervals on which f is concave up and concave down.

$f'(x)=12x^2(3-x)=36x^2-12x^3$

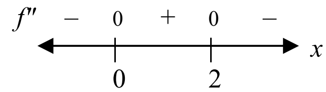
$f''(x)=72x-36x^2=36x(2-x)$

$f''(x)=0$ when $x=0, x=2$

$f''(x)$ always exists

f is concave up on $(0,2)$.

f is concave down on $(-\infty,0)$, $(2,\infty)$.



Work on Problem:	**Points**
Finds the second derivative of f.	1 point
Solves for x in the equation $f''(x)=0$. (½ point for each value of x.)	1 point
States the intervals of concave up.	1 point
States the intervals of concave down.	1 point

Notes:
- Subtract 1 point for missing or incorrect identification as intervals of concave up or concave down.
- Subtract a maximum of ½ point for notation errors. Types of notation errors include using square brackets which indicate closed intervals instead of parentheses which indicate open intervals.

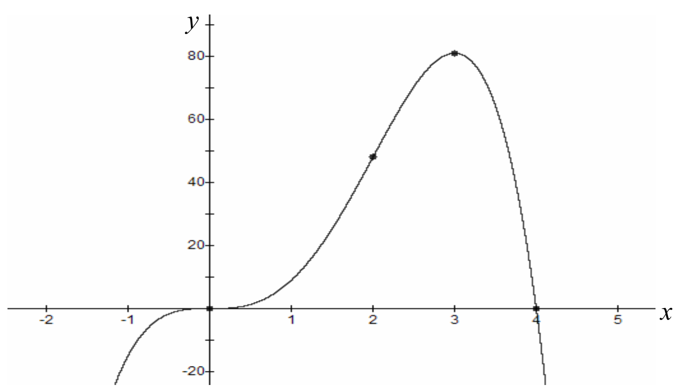
d. **(2 pts.)** For which values of x does f have inflection points?

f has inflection points at $x=0$ and $x=2$.

Work on Problem:	**Points**
States the x-value of the inflection points. (1 point per x-value)	2 points

Notes:
- Because of the simplicity of this problem, no work is required for full credit.
- Attempt to follow work from part c.

e. **(2 pts.)** Given the ordered pairs, (0, 0), (2, 48), (3, 81), and (4, 0), on the graph of f, sketch the graph of $y=f(x)$.



Work on Problem:	**Points**
Local maximum sketched as such and inflection points show change in concavity.	1 point
General shape follows correct curve.	1 point

Notes:
- Some attempt may be made to follow work from previous parts if mistakes are due to minor errors above.

Scantron **(1 pt.)**

Check to make sure your Scantron form meets the following criteria. If any of the items are NOT satisfied when your Scantron is handed in and/or when your Scantron is processed one point will be subtracted from your test total.

My Scantron:

- □ is bubbled with firm marks so that the form can be machine read;
- □ is not damaged and has no stray marks (the form can be machine read);
- □ has **17** bubbled in answers;
- □ has **MthSc 106** and my Section number written at the top;
- □ has my Instructor's name written at the top;
- □ has Test No. **2** written at the top;
- □ has Test Version **A** both written at the top and bubbled in below my CUID;
- □ and shows my correct CUID both written and bubbled in.