1. **(3 pts)** For which values of x does the graph of $f(x) = 2x^3 - 3x$ have horizontal tangents?

A) $x = 0$

B) $x = \frac{1}{\sqrt{2}}, \frac{-1}{\sqrt{2}}, 0$

C) $x = \frac{1}{\sqrt{2}}, \frac{-1}{\sqrt{2}}$

D) $x = \sqrt{2}, -\sqrt{2}$

2. **(3 pts)** Evaluate $\lim_{x \to \frac{\pi}{6}} \frac{1}{2\cos x - 1}$.

A) 0

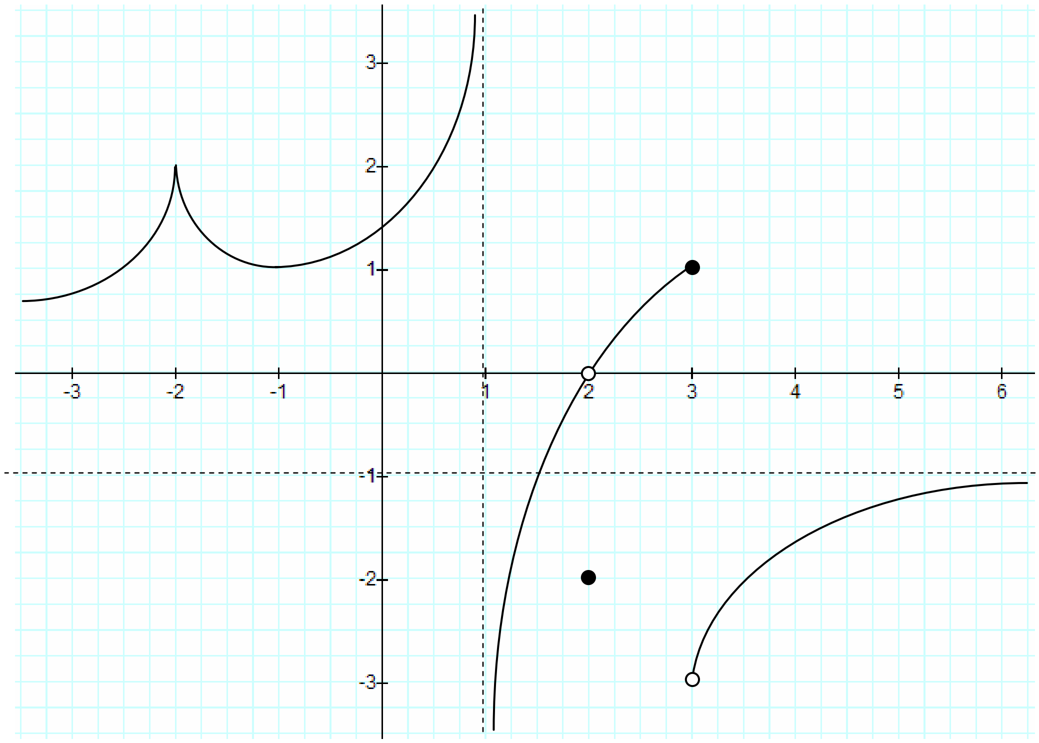
B) ∞

C) Does not exist

D) $\frac{1}{\sqrt{3} - 1}$

3. **(3 pts)** The graph of function $f(x)$ in the figure has a vertical asymptote at $x = 1$ and a horizontal asymptote at $y = -1$.

Use the graph to find all values of x where the function fails to be **continuous**.



A) $x = -2,\ x = 1,\ x = 2,\ x = 3$

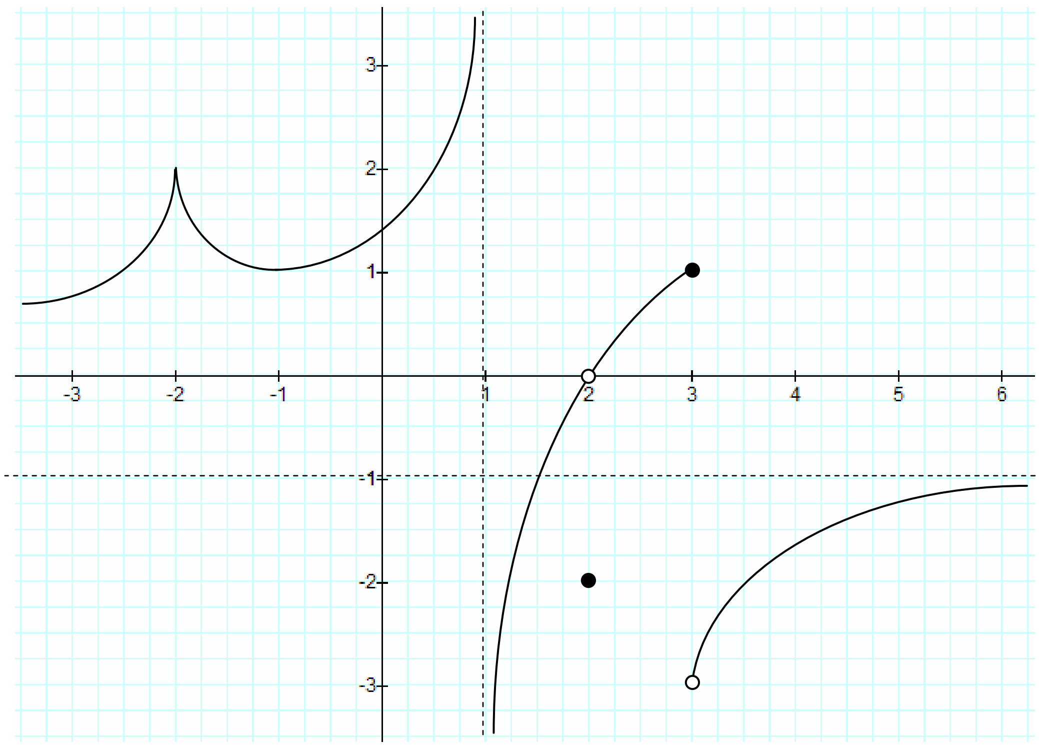
B) $x = 1,\ x = 2,\ x = 3$

C) $x = 2,\ x = 3$

D) $x = 3$

4. **(3 pts)** The graph of function $f(x)$ in the figure has a vertical asymptote at $x = 1$ and a horizontal asymptote at $y = -1$.

Use the graph to find all values of x at which the function is not **differentiable**.



A) $x = -2,\ x = 1,\ x = 2,\ x = 3$

B) $x = 1,\ x = 2,\ x = 3$

C) $x = 2,\ x = 3$

D) $x = 3$

5. **(3 pts)** Find all slant asymptotes of the function $g(x) = \dfrac{x^2 + 2x - 7}{x - 7}$.

A) $y = x - 5$

B) $y = x + 9$

C) $y = 1$

D) $y = 0$

6. **(3 pts)** Find the second derivative of $f(t) = \frac{7t^3}{3} + 7$.

A) $f''(t) = 7t^2$

B) $f''(t) = 7t$

C) $f''(t) = 14t + 7$

D) $f''(t) = 14t$

7. **(3 pts)** Evaluate $\lim_{h \to 0} \frac{2x+h}{x^3(x-h)}$.

A) -1

B) $\frac{2}{x^3}$

C) $\frac{2}{x^4}$

D) Does not exist

8. **(3 pts)** Evaluate $\lim_{x \to 13} \sqrt{3}$.

A) $\sqrt{13}$

B) 0

C) 13

D) $\sqrt{3}$

9. **(3 pts)** Find the intervals on which the function $f(\theta)=\dfrac{3\cos\theta}{\theta+9}$ is continuous.

A) $(-\infty,\ -9),\ (-9,\ \infty)$

B) $(-\infty,\ 9),\ (9,\ \infty)$

C) $(-\infty,\ \infty)$

D) $\left(-\infty,\ \dfrac{\pi}{2}\right),\ \left(\dfrac{\pi}{2},\ \infty\right)$

10. **(3 pts)** Solve for the angle θ in the equation $\sin^2\theta=\dfrac{1}{4}$, where $0\le\theta\le2\pi$.

A) $\theta=\dfrac{\pi}{6},\ \dfrac{5\pi}{6}$

B) $\theta=\dfrac{\pi}{4},\ \dfrac{3\pi}{4},\ \dfrac{5\pi}{4},\ \dfrac{7\pi}{4}$

C) $\theta=\dfrac{\pi}{6},\ \dfrac{5\pi}{6},\ \dfrac{7\pi}{6},\ \dfrac{11\pi}{6}$

D) $\theta=\dfrac{\pi}{3},\ \dfrac{2\pi}{3}$

11. **(3 pts)** It can be shown that $-x^2 \le f(x) \le 1 - \cos x$ for all values of x close to zero. Use these inequalities to find $\lim_{x \to 0} f(x)$.

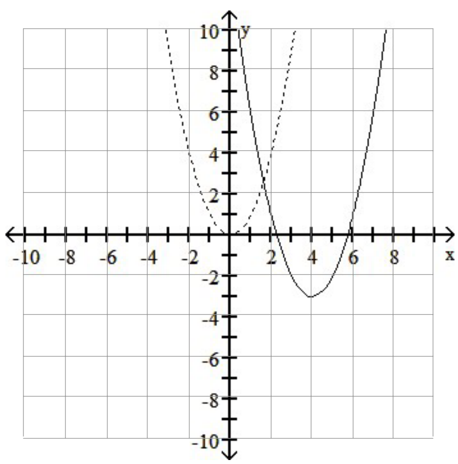
A) 0

B) 1

C) Does not exist

D) -1

12. **(3 pts)** The accompanying figure shows the graph of $y = x^2$ shifted to a new position. Write the equation for the new graph. Note – the graph of $y = x^2$ appears as a dotted line.



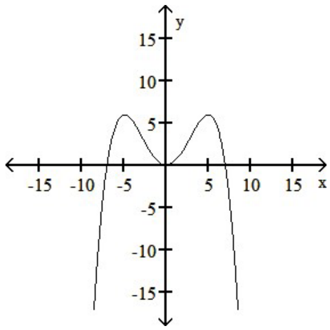
A) $y = (x-4)^2 - 3$

B) $y = (x-4)^2 + 3$

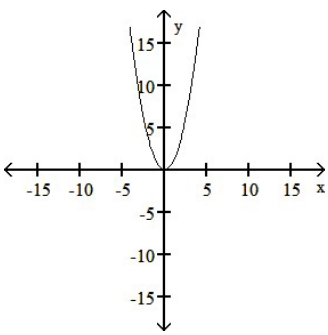
C) $y = (x+4)^2 - 3$

D) $y = (x+3)^2 - 4$

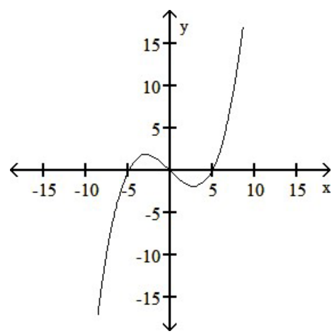
13. **(3 pts)** The graph of a function is given. Choose the answer that represents the graph of its derivative.



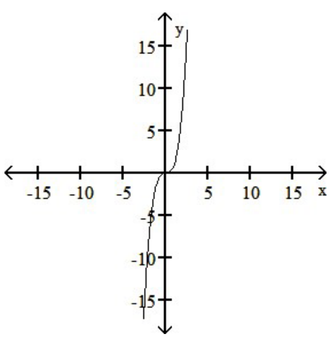
A)



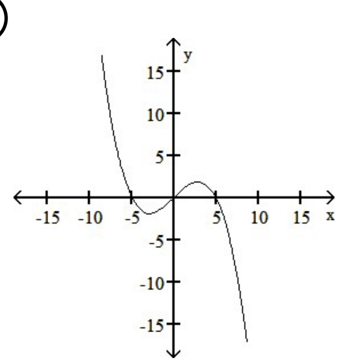
B)



C)



D)



14. **(3 pts)** Determine the value of the constant k for which the function $f(x)$ is continuous at $x = 8$.

$$f(x) = \begin{cases} x^2, & \text{if } x \le 8 \\ x + k, & \text{if } x > 8 \end{cases}$$

A) $k = 72$

B) $k = -8$

C) Impossible

D) $k = 56$

15. **(3 pts)** Let $f(x) = \dfrac{1}{x}$ and $g(x) = 3x^3$. Find $g(f(x))$.

A) $g(f(x)) = \dfrac{1}{3x^3}$

B) $g(f(x)) = \dfrac{3}{x^3}$

C) $g(f(x)) = \dfrac{1}{x^3}$

D) $g(f(x)) = 3x^2$

16. **(3 pts)** Evaluate $\lim\limits_{x \to 0^+} \dfrac{x^2 - 3x + 2}{x^3 - x}$.

A) ∞

B) 0

C) $-\infty$

D) -2

17. **(3 pts)** Use the table to evaluate $\frac{d}{dx}\left[2f(x)-4g(x)\right]\Big|_{x=1}$.

x	1	2	3	4
$f(x)$	2	3	–2	5
$f'(x)$	1	1	4	1
$g(x)$	6	4	2	4
$g'(x)$	3	–5	–3	7

A) –1

B) –20

C) 0

D) –10

18. **(3 pts)** Consider the position function $s(t)=5+\tan t$. Find the **average** velocity over the interval $\left[\frac{-\pi}{4}, \frac{\pi}{4}\right]$.

A) $v_{av}=\frac{-16}{11}$

B) $v_{av}=\frac{4}{\pi}$

C) $v_{av}=\frac{-4}{\pi}$

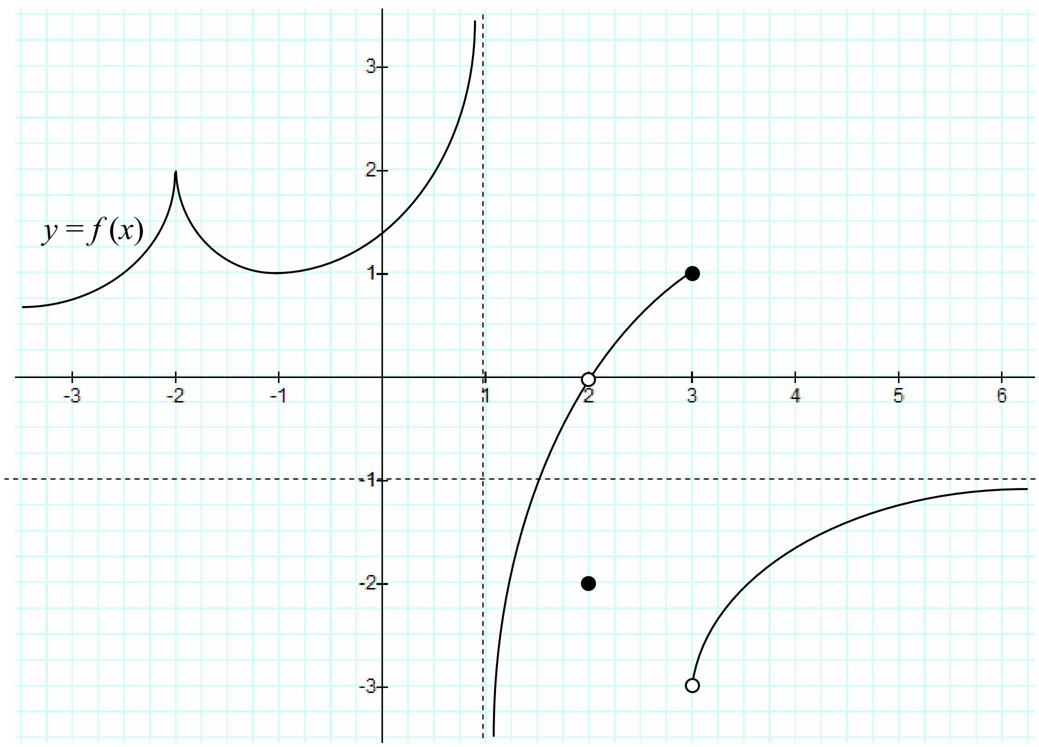
D) $v_{av}=0$

Free Response. The Free Response questions will count 45% of the total grade. Read each question carefully. In order to receive full credit you must show legible and logical (relevant) justification which supports your final answer. Give answers as exact answers. You are NOT permitted to use a calculator on any portion of this test.

1. **(8 pts.)** Use the graph to find each of the following limits, if it exists. **(1 pt. per part a – h)**

The graph of function $f(x)$ in the figure has a vertical asymptote at $x = 1$ and a horizontal asymptote at $y = -1$.

Infinite limits should be answered with "$= \infty$" or "$= -\infty$", whichever is appropriate.
If the limit does not exist (and cannot be answered as ∞ or $-\infty$), state "DNE."



a. $\lim_{x \to -2^-} f(x) = 2$

b. $\lim_{x \to 1^-} f(x) = \infty$

c. $\lim_{x \to 1^+} f(x) = -\infty$

d. $\lim_{x \to 2} f(x) = 0$

e. $\lim_{x \to 3^-} f(x) = 1$

f. $\lim_{x \to 3^+} f(x) = -3$

g. $\lim_{x \to 3} f(x)$ DNE

h. $\lim_{x \to \infty} f(x) = -1$

Work on Problem:	**Points**
Correctly states limits. (1 point each)	8 points
Notes: • Graded all or nothing. • No deduction for missing or inappropriate use of equals, but notate errors.	

2. **(12 pts.)** Find the limit of each of the following functions, if it exists.

Show all work to receive full credit.

If the limit does not exist (and cannot be answered as ∞ or $-\infty$), state "DNE."

a. **(4 pts.)** $\lim\limits_{x\to -5} \dfrac{x^2+10x+25}{x+5}$

$$\lim_{x\to -5} \frac{x^2+10x+25}{x+5} = \lim_{x\to -5} \frac{\cancel{(x+5)}(x+5)}{\cancel{(x+5)}} = \lim_{x\to -5}(x+5) = 0$$

Work on Problem:	Points
Recognizes implicitly or explicitly indeterminate form 0/0.	1 point
Factors numerator.	1 point
Cancels common factor.	1 point
Correctly evaluates limit.	1 point

Notes:

- Award one point total for recognizing 0/0 and the need to factor, but incorrect factoring leads to incorrect answer or leads to incorrect work to arrive at answer.
- Subtract 4 points for dividing by x in both numerator and denominator.
- Subtract 4 points for irrelevant work.
- Subtract ½ point for each type of notation error with a maximum of 1 point deduction for all notation errors. Types of notation errors include, but are not limited to, missing or incorrect use of equals, missing or incorrect use of parentheses, missing or incorrect limit notation such as: $\lim\limits_{x\to -5} = \ldots$ without function inside limit, inappropriate limit notation: $\lim\limits_{x\to -5}(x+5) = \lim\limits_{x\to -5}(-5+5)$, and the untrue statement: $\lim\limits_{x\to -5} \dfrac{x^2+10x+25}{x+5} = \dfrac{0}{0}$.
- Subtract ½ point for each minor algebra, arithmetic, and/or copy error.

b. **(4 pts.)** $\lim\limits_{x\to 0} \dfrac{x}{\sqrt{4-x}-2}$

$$\lim_{x\to 0} \frac{x}{\sqrt{4-x}-2} \cdot \frac{\sqrt{4-x}+2}{\sqrt{4-x}+2} = \lim_{x\to 0} \frac{x\left(\sqrt{4-x}+2\right)}{\cancel{4}-x-\cancel{4}} = \lim_{x\to 0} \frac{\cancel{x}\sqrt{4-x}+2}{-\cancel{x}} = \frac{\sqrt{4-0}+2}{-1} = -4$$

Work on Problem:	Points
Recognizes implicitly or explicitly indeterminate form 0/0.	1 point
Rationalizes the denominator by multiplying by the conjugate of the denominator over itself.	1 point
Cancels common factor.	1 point
Correctly evaluates limit.	1 point

Notes:

- Award one point total for recognizing 0/0 (I.F.) and the need to rationalize the denominator, but incorrect rationalizing leads to incorrect answer or leads to incorrect work to arrive at answer.
- Subtract 4 points for irrelevant work.
- Subtract ½ point for each type of notation error with a maximum of 1 point deduction for all notation errors. Types of notation errors include, but are not limited to, missing or incorrect use of equals, missing or incorrect use of parentheses, missing or incorrect limit notation such as: $\lim\limits_{x\to 0} = \ldots$ without function inside limit, and the untrue statement: $\lim\limits_{x\to 0} \dfrac{x}{\sqrt{4-x}-2} = \dfrac{0}{0}$.
- Subtract ½ point for each minor algebra, arithmetic, and/or copy error.

2.**(continued)** Find the limit of each of the following functions, if it exists.

Show all work to receive full credit.

If the limit does not exist (and cannot be answered as ∞ or $-\infty$), state "DNE."

c. **(4 pts.)** $\lim\limits_{x\to-\infty} \dfrac{7x+4}{\sqrt{3x^2+1}}$

There are 2 approaches to this problem. The guidelines apply to both methods.

Method I

$$\lim_{x\to-\infty} \frac{7x+4}{\sqrt{3x^2+1}} = \lim_{x\to-\infty} \frac{\dfrac{7x}{-x}+\dfrac{4}{-x}}{\sqrt{\dfrac{3x^2}{x^2}+\dfrac{1}{x^2}}} = \lim_{x\to-\infty} \frac{-7-\dfrac{4}{x}}{\sqrt{3+\dfrac{1}{x^2}}} = \frac{-7-0}{\sqrt{3+0}} = \frac{-7}{\sqrt{3}} \quad \text{OR} \quad \frac{-7\sqrt{3}}{3}$$

OR

Method II

$$\lim_{x\to-\infty} \frac{7x+4}{\sqrt{3x^2+1}} = \lim_{x\to-\infty} \frac{7x+4}{\sqrt{x^2\left(3+\dfrac{1}{x^2}\right)}} = \lim_{x\to-\infty} \frac{7x+4}{|x|\sqrt{\left(3+\dfrac{1}{x^2}\right)}} = \lim_{x\to-\infty} \frac{7x+4}{-x\sqrt{\left(3+\dfrac{1}{x^2}\right)}}$$

$$= \lim_{x\to-\infty} \frac{\dfrac{7x}{x}+\dfrac{4}{x}}{-\sqrt{3+\dfrac{1}{x^2}}} = \lim_{x\to-\infty} \frac{7+\dfrac{4}{x}}{-\sqrt{3+\dfrac{1}{x^2}}} = \frac{7+0}{-\sqrt{3+0}} = \frac{-7}{\sqrt{3}} \quad \text{OR} \quad \frac{-7\sqrt{3}}{3}$$

Work on Problem:	**Points**
Recognizes implicitly or explicitly indeterminate form ∞/∞.	1 point
Divides by x raised to the highest power of x in the denominator.	1 point
Simplifies expression.	1 point
Correctly evaluates limit.	1 point

Notes:

- Award one point total for recognizing ∞/∞ (I.F.) and the need to divide through by x raised to the highest power of x in the denominator, but incorrect algebra leads to incorrect answer or leads to incorrect work to arrive at answer.
- Subtract 2 points if just states the limit with no supporting work.
- Subtract 4 points for irrelevant work, such as rationalizing the denominator.
- Subtract ½ point for each type of notation error with a maximum of 1 point deduction for all notation errors. Types of notation errors include, but are not limited to, missing or incorrect use of equals, missing or incorrect use of parentheses, and missing or incorrect limit notation such as: $\lim\limits_{x\to 0} = \ldots$ without function inside limit.
- Subtract ½ point for each minor algebra, arithmetic, and/or copy error.

3. **(8 pts.)** Use the limit definition to find the derivative of $f(x)=\dfrac{8}{7+x}$.

(No credit given for using derivative theorems.)

$$f'(x)=\lim_{h\to 0}\frac{f(x+h)-f(x)}{h}=\lim_{h\to 0}\frac{\dfrac{8}{7+x+h}-\dfrac{8}{7+x}}{h}$$

$$=\lim_{h\to 0}\frac{\dfrac{8(7+x)}{(7+x+h)(7+x)}-\dfrac{8(7+x+h)}{(7+x+h)(7+x)}}{h}$$

$$=\lim_{h\to 0}\frac{\cancel{56}\cancel{+8x}\cancel{-56}\cancel{-8x}-8h}{h(7+x+h)(7+x)}$$

$$=\lim_{h\to 0}\frac{-8\cancel{h}}{\cancel{h}(7+x+h)(7+x)}$$

$$=\frac{-8}{(7+x+0)(7+x)}=\frac{-8}{(7+x)^2}$$

Work on Problem:	Points
Writes notation for derivative: $f'(x)$.	1 point
States the formula for the limit definition of the derivative. (Okay if implied.)	1 point
Substitutes $f(x+h)$ correctly.	1 point
Gets a common denominator between the two terms in the numerator. Equivalently, clears fractions in numerator and denominator by multiplying by common denominator (in numerator) over itself.	1 point
Distributes correctly.	1 point
Combines like terms.	1 point
Cancels common factor h.	1 point
Correctly evaluates limit. (No credit if this doesn't follow from work.)	1 point

Notes:

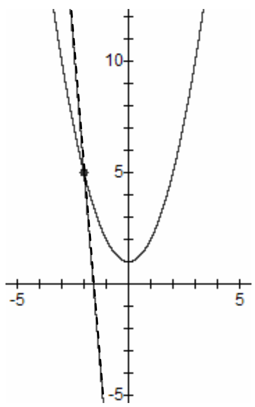
- Subtract 8 points for not using the limit definition of the derivative.
- Subtract 1 point for missing limit in the definition.
- Subtract ½ point for labeling as $m_{\tan}$.
- Subtract 6 points for substituting incorrectly to get $f'(x)=\lim\limits_{h\to 0}\dfrac{\dfrac{8}{7+x}+h-\dfrac{8}{7+x}}{h}=\lim\limits_{h\to 0}\dfrac{h}{h}=1$.
- Award a maximum of 3 points for early egregious error which leads to trivial limit problem.
- Subtract 8 points for irrelevant work.
- Subtract ½ point for each type of notation error (exclusive of errors in the definition of the derivative, such as omitting the limit) with a maximum of 1 point deduction for all notation errors.
  Types of notation errors include, but are not limited to, missing or incorrect use of equals, missing or incorrect use of parentheses, mislabeling the derivative as $f(x)$ instead of $f'(x)$, and missing or incorrect limit notation such as: $\lim\limits_{x\to 0}=\ldots$ without function inside limit.
- Subtract ½ point for each minor algebra, arithmetic, and/or copy error.

4. **(5 pts.)** Find the equation of the tangent line to $g(x) = x^2 + 1$ at $x = -2$.
Use the derivative theorems. You do not need to use the limit definition of the derivative.

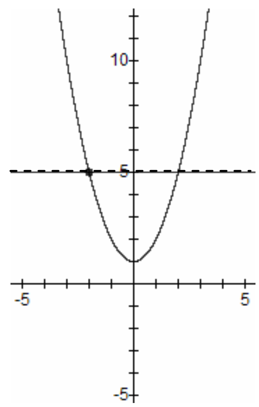
$g'(x) = 2x$
$g'(-2) = 2(-2) = -4$
The slope of the tangent line is -4.
$g(-2) = (-2)^2 + 1 = 5$
The point of tangency is $(-2, 5)$.
$y - 5 = -4(x - (-2))$
The equation of the tangent line is $y - 5 = -4(x + 2)$ or $y = -4x - 3$.

Circle the figure which best represents the graph of $g(x)$ and the tangent line to g at $x = -2$.

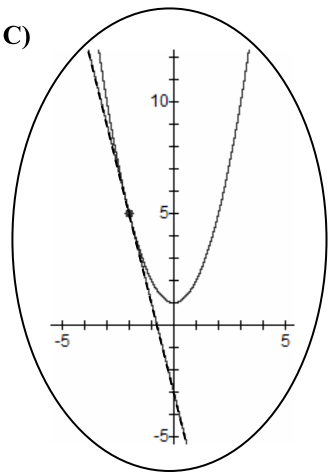
A)



B)



C)

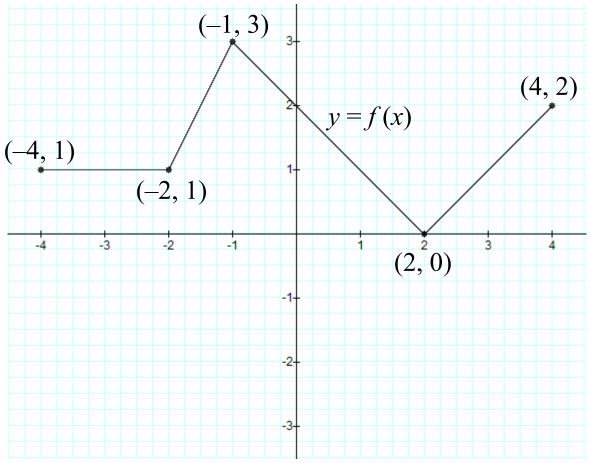


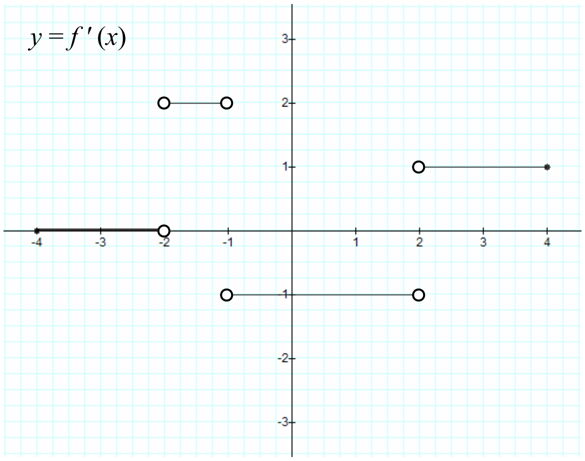
Work on Problem:	**Points**
Finds the derivative $g'(x)$.	1 point
Finds the slope of the tangent line $g'(-2)$.	1 point
Finds the y-coordinate of the point of tangency $g(-2)$.	1 point
States the equation of the tangent line to g at $x = -2$ in any form.	1 point
Circles the correct figure representing the graph of g and its tangent line at $x = -2$.	1 point

Notes:

- Subtract ½ point for each type of notation error with a maximum of 1 point deduction for all notation errors. Types of notation errors include, but are not limited to, missing or incorrect use of equals, missing or incorrect use of parentheses, mislabeling the derivative as $f(x)$ instead of $f'(x)$, and missing or incorrect limit notation such as: $\lim_{x \to 0} = \ldots$ without function inside limit.
- Subtract ½ point for each minor algebra, arithmetic, and/or copy error.

5. **(4 pts.)** The figure below is a graph of the function $y = f(x)$ on the closed interval $[-4, 4]$. The graph is made of line segments joined end to end. Graph the derivative of f.





Work on Problem:	Points
Correctly graphs the derivative. (1 point per slope of each line segment)	4 points

Notes:

- Subtract ½ point per line segment for a closed circle at an interior point up to a maximum of 1 point for the entire problem. Interior points should be open circles and endpoints (at $x = -4$ and $x = 4$) should be closed. However, no deduction for open circles or missing circles at the endpoints.
- Subtract ½ point if the graph extends beyond $x = -4$ or $x = 4$ or both.
- Subtract 1 point for connecting the line segments in the graph of the derivative with a vertical line.
- Subtract 2 points if the graph of the derivative is shifted in any direction.

6. **(8 pts.)** Consider the function $f(x)=\dfrac{x^2-4}{x^2-2x}$.

Grading Notes: (for each part of problem 6)	
• Subtract ½ point for each type of notation error with a maximum of 1 point deduction for all notation errors. Types of notation errors include, but are not limited to, missing or incorrect use of equals, missing or incorrect use of parentheses, and missing or incorrect limit notation such as: $\lim\limits_{x\to 0} = \ldots$ without function inside limit.	
• Subtract ½ point for each minor algebra, arithmetic, and/or copy error.	

a. **(3 pts.)** Show that the graph of $f(x)$ has a vertical asymptote at $x=0$ by evaluating both the left- and right-hand limits at $x=0$.

$$\lim_{x\to 0^-} f(x) = \lim_{x\to 0^-}\frac{x^2-4}{x^2-2x} = \lim_{x\to 0^-}\frac{\cancel{(x-2)}(x+2)}{x\cancel{(x-2)}} = \lim_{x\to 0^-}\frac{x+2}{x} = -\infty \qquad \text{SCRATCH: } \frac{2}{0^-}$$

$$\lim_{x\to 0^+} f(x) = \lim_{x\to 0^+}\frac{x^2-4}{x^2-2x} = \lim_{x\to 0^-}\frac{x+2}{x} = \infty \qquad \text{SCRATCH: } \frac{2}{0^+} \qquad \text{So, } x=0 \text{ is V.A.}$$

Work on Problem:	Points
Sets up the left- and right-hand limits at $x=0$. (½ point per limit)	1 point
Evaluates the left- and right-hand limits at $x=0$. (1 point per limit)	2 points
Notes:	
• No work is required.	

b. **(2 pts.)** The function $f(x)$ has a removable discontinuity at $x=2$.
Define $f(2)$ so that $f(x)$ is continuous at $x=2$. **Justify your answer using limits.**

$$\lim_{x\to 2} f(x) = \lim_{x\to 2}\frac{x^2-4}{x^2-2x} = \lim_{x\to 2}\frac{\cancel{(x-2)}(x+2)}{x\cancel{(x-2)}} = \lim_{x\to 2}\frac{x+2}{x} = \frac{2+2}{2} = 2 \qquad \text{So, } f(2)=2.$$

Work on Problem:	Points
Sets up the limit as $x \to 2$.	½ point
Evaluates the limit. (½ point for factoring, ½ point for canceling the common factor and evaluating)	1 point
States the value of $f(2)$.	½ point
Notes:	
•	

c. **(3 pts.)** Use limits to show that the graph of $f(x)$ has a horizontal asymptote at $y=1$.
Must show algebraic work, not just the answer.

$$\lim_{x\to\infty} f(x) = \lim_{x\to\infty}\frac{x^2-4}{x^2-2x} = \lim_{x\to\infty}\frac{\frac{x^2}{x^2}-\frac{4}{x^2}}{\frac{x^2}{x^2}-\frac{2x}{x^2}} = \lim_{x\to\infty}\frac{1-\frac{4}{x^2}}{1-\frac{2}{x}} = \frac{1-0}{1-0} = 1 \qquad \text{So, } y=1 \text{ is H.A.}$$

Work on Problem:	Points
Sets up the limit as $x \to \infty$ or as $x \to -\infty$.	1 point
Divides by x raised to the highest power of x in the denominator.	½ point
Simplifies expression.	½ point
Correctly evaluates limit.	1 point
Notes:	
• Subtract 1 point if just states the limit with no supporting work.	

Scantron **(1 pt.)**

Check to make sure your Scantron form meets the following criteria. If any of the items are NOT satisfied when your Scantron is handed in and/or when your Scantron is processed one point will be subtracted from your test total.

My Scantron:

- □ is bubbled with firm marks so that the form can be machine read;
- □ is not damaged and has no stray marks (the form can be machine read);
- □ has **18** bubbled in answers;
- □ has **MthSc 106** and my Section number written at the top;
- □ has my Instructor's name written at the top;
- □ has Test No. **1** written at the top;
- □ has Test Version **A** both written at the top and bubbled in below my CUID;
- □ and shows my correct CUID both written and bubbled in.