#1) Solve for dy/dx: $y = \tan^3(x^3/9)$

A) $x^2 \tan^2(x^3/9)/3$

B) $x^2 \tan^2(x^3/9)\sec^2(x^3/9)$

C) $3x^2 \sec^2(x^3/9)$

D) $-3x^2 \tan^2(x^3/9)\csc^2(x^3/9)$

E) None of the above

$$y = \tan^3(x^3/9) \Rightarrow \frac{dy}{dx} = 3\tan^2(x^3/9)\sec^2(x^3/9)\cdot 3x^2/9 = \boxed{x^2\tan^2(x^3/9)\sec^2(x^3/9) = \frac{dy}{dx}}$$

#2) Solve for dy/dx: $y = \sqrt{3x^3\cos^3(3x^3)}$

A) $\dfrac{9x^3\cos^2(3x^3)\sin(3x^3)}{2\sqrt{3x^3\cos^3(3x^3)}}$

B) $\dfrac{9x^2[\cos^3(3x^3) - 9x^3\cos^2(3x^3)\sin(3x^3)]}{2\sqrt{3x^3\cos^3(3x^3)}}$

C) $\dfrac{9x^3\sin^2(3x^3)\cos(3x^3)}{2\sqrt{3x^3\cos^3(3x^3)}}$

D) $\dfrac{9x^2\sin^3(3x^3)}{2\sqrt{3x^3\cos^3(3x^3)}}$

E) None of the above

$$y = \sqrt{3x^3\cos^3(3x^3)} \Rightarrow \frac{dy}{dx} = \frac{1}{2}[3x^3\cos^3(3x^3)]^{-1/2}\left[-3x^3\cdot 3\cos^2(3x^3)\sin(3x^3)\cdot 9x^2 + \cos^3(3x^3)\cdot 9x^2\right]$$

$$= \boxed{\frac{9x^2[\cos^3(3x^3) - 9x^3\cos^2(3x^3)\sin(3x^3)]}{2\sqrt{3x^3\cos^3(3x^3)}} = \frac{dy}{dx}}$$

#3) Solve for dy/dx: $y^3x^2 = 1$

A) $-\frac{2}{3}x^{-5/3}$

B) $x^{-2/3}$

C) $6y^2x$

D) $x^2/3y^2$

E) None of the above

$$y^3x^2 = 1 \Leftrightarrow y^3 = x^{-2} \Leftrightarrow y = x^{-2/3} \Rightarrow \boxed{\frac{dy}{dx} = -\frac{2}{3}x^{-5/3}}$$

#4) Solve for dy/dx: $y^2 = \sin^2(y^2) + x$

A) $\dfrac{1}{2y - 4y\sin(y^2)\cos(y^2)}$

B) $2y - 4y\sin(y^2)\cos(y^2)$

C) $\dfrac{1}{2y - 4y\cos(y^2)}$

D) $\dfrac{1}{2} - 2\sin(y^2)\cos(y^2) + \dfrac{1}{2y}$

E) None of the above

$y^2 = \sin^2(y^2) \Rightarrow \frac{d}{dy}(y^2) = \frac{d}{dy}\left[\sin^2(y^2) + x\right] \Rightarrow 2y\frac{dy}{dx} = 2\sin(y^2)\cos(y^2)\cdot 2y\frac{dy}{dx} + 1$

$\Rightarrow \frac{dy}{dx}\left[2y - 4y\sin(y^2)\cos(y^2)\right] = 1 \Rightarrow \boxed{\frac{dy}{dx} = \frac{1}{2y - 4y\sin(y^2)\cos(y^2)}}$

#5) Solve for du/dv: $\cos(u) = \sin(v)$

A) $-\dfrac{\sin(v)}{\cos(u)}$

B) $\tan(u)$

C) $\cos(\sin(u))$

D) $-\dfrac{\cos(v)}{\sin(u)}$

E) None of the above

$\cos(u) = \sin(v) \Rightarrow \frac{d}{dv}\left[\cos(u)\right] = \frac{d}{dv}\left[\sin(v)\right]$

$\Rightarrow -\sin(u)\frac{du}{dv} = \cos(v) \Rightarrow \boxed{\frac{du}{dv} = -\frac{\cos(v)}{\sin(u)}}$

#6) Solve for dy/dx: $y = \sin^2(\cos(x^2y^2/4))$

A) $1 - x^2y\sin(\cos(x^2y^2/4))\cos(\cos(x^2y^2/4))\sin(x^2y^2/4)$

B) $x^2y\sin(\cos(x^2y^2/4))\cos(\cos(x^2y^2/4))\sin(x^2y^2/4)$

C) $\dfrac{1}{x^2y\sin(\cos(x^2y^2/4))\cos(\cos(x^2y^2/4))\sin(x^2y^2/4)} + 1$

D) $\dfrac{-xy^2\sin(\cos(x^2y^2/4))\cos(\cos(x^2y^2/4))\sin(x^2y^2/4)}{1 + x^2y\sin(\cos(x^2y^2/4))\cos(\cos(x^2y^2/4))\sin(x^2y^2/4)}$

E) None of the above

$y = \sin^2(\cos(x^2y^2/4))$

$\Rightarrow \dfrac{dy}{dx} = \dfrac{d}{dx}\left[\sin^2(\cos(x^2y^2/4))\right]$

$= -2\sin(\cos(x^2y^2/4))\cos(\cos(x^2y^2/4))\sin(x^2y^2/4)\,(1/4)\left(2xy^2 + x^2\cdot 2y\dfrac{dy}{dx}\right)$

$= -\sin(\cos(x^2y^2/4))\cos(\cos(x^2y^2/4))\sin(x^2y^2/4)\left(xy^2 + x^2y\dfrac{dy}{dx}\right)$

$\Rightarrow \dfrac{dy}{dx}\left(1 + x^2y\sin(\cos(x^2y^2/4))\cos(\cos(x^2y^2/4))\sin(x^2y^2/4)\right)$

$= -xy^2\sin(\cos(x^2y^2/4))\cos(\cos(x^2y^2/4))\sin(x^2y^2/4)$

$\Rightarrow \boxed{\dfrac{dy}{dx} = \dfrac{-xy^2\sin(\cos(x^2y^2/4))\cos(\cos(x^2y^2/4))\sin(x^2y^2/4)}{1 + x^2y\sin(\cos(x^2y^2/4))\cos(\cos(x^2y^2/4))\sin(x^2y^2/4)}}$

#7) Solve for dy/dx: $\sin^2(x) + xy = \cos^2(y)$

A) $\dfrac{\sin(2y) - y}{\sin(2x) - x}$

B) $\dfrac{-2\cos(x)\sin(x) - y}{2\cos(y)\sin(y) + x}$

C) $\dfrac{\sin^2(y) - y}{\cos^2(x) - x}$

D) $\dfrac{\sin(y) - y}{\cos(x) - x}$

E) None of the above

$$\sin^2(x) + xy = \cos^2(y) \Leftrightarrow xy - \cos^2(y) = -\sin^2(x) \Rightarrow \frac{d}{dx}\left[xy - \cos^2(y)\right] = \frac{d}{dx}\left[-\sin^2(x)\right]$$

$$\Rightarrow x\frac{dy}{dx} + y + 2\cos(y)\sin(y)\frac{dy}{dx} = -2\sin(x)\cos(x)$$

$$\Rightarrow \left(x + 2\cos(y)\sin(y)\right)\frac{dy}{dx} = -2\sin(x)\cos(x) - y \Rightarrow \boxed{\frac{dy}{dx} = \frac{-2\cos(x)\sin(x) - y}{2\cos(y)\sin(y) + x}}$$

#8) Solve for y'': $y' = \dfrac{2\cos(x)\sin(x) - y}{2\cos(y)\sin(y) + x}$

A) $\dfrac{\sin(2y) - y}{\sin(2x) - x}$

B) $\dfrac{[\sin(2y) - x][2\cos(2x) - y'] - [\sin(2x) - y][2\cos(2y)y' - 1]}{[\sin(2y) - x]^2}$

C) $\dfrac{[\sin(2x) - y][2\cos(2y)y' - 1]}{[\sin(2y) - x]^2}$

D) $\dfrac{\sin(y) - y}{\cos(x) - x} + \dfrac{[\sin(2y) - x][2\cos(2x) - y']}{[\sin(2y) - x]^2}$

E) None of the above

$$y' = \frac{2\cos(x)\sin(x) - y}{2\cos(y)\sin(y) + x} = \frac{\sin(2x) - y}{\sin(2y) - x}$$

$$\Rightarrow y'' = \frac{dy'}{dx} = \frac{d}{dx}\left[\frac{\sin(2x) - y}{\sin(2y) - x}\right] = \boxed{\frac{[\sin(2y) - x][2\cos(2x) - y'] - [\sin(2x) - y][2\cos(2y)y' - 1]}{[\sin(2y) - x]^2} = y''}$$

#9) Solve for dy/dx: $y = \arcsin(x^2)$

A) $\frac{x}{\sqrt{1-x^4}}$ B) $\frac{x^2}{\sqrt{1-x^2}}$ C) $\frac{1}{\sqrt{1-x^4}}$ D) $\frac{2x}{\sqrt{1-x^4}}$ E) None of the above

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$y = \arcsin(x^2) \Rightarrow \frac{dy}{dx} = \frac{d}{dx}\sin(\arcsin(x^2))$

We know, $\sin(\arcsin(x^2)) = x^2 \Rightarrow \frac{d}{dx}\sin(\arcsin(x^2)) = \frac{d}{dx}(x^2) = 2x$

$\Rightarrow \cos(\arcsin(x^2))\frac{d}{dx}\arcsin(x^2) = 2x$

$\Rightarrow \frac{dy}{dx} = \frac{d}{dx}\arcsin(x^2) = \frac{2x}{\cos(\arcsin(x^2))}$ but we can simplify $\cos(\arcsin(x^2))$

$\cos(\arcsin(x^2)) = \cos(\theta)$ where $\theta = \arcsin(x^2) \in [-\frac{\pi}{2}, \frac{\pi}{2}] \Rightarrow \sin(\theta) = x^2$ and $\cos(\theta) \geq 0$

$\cos^2(\theta) + \sin^2(\theta) \equiv 1 \Rightarrow \cos(\theta) = \sqrt{1-\sin^2(\theta)} = \sqrt{1-x^4} = \cos(\arcsin(x^2))$

Substituting, we have

$\frac{dy}{dx} = \frac{d}{dx}\arcsin(x^2) = \frac{2x}{\cos(\arcsin(x^2))} = \boxed{\frac{2x}{\sqrt{1-x^4}} = \frac{dy}{dx}}$

The following is the formula for the Taylor Series for a function $f(x)$ generated about the point $x=a$:

$$f(x) = f(a) + \frac{f'(a)}{1!}(x-a) + \frac{f''(a)}{2!}(x-a)^2 + \frac{f'''(a)}{3!}(x-a)^3 + \frac{f^{(4)}(a)}{4!}(x-a)^4 + \frac{f^{(5)}(a)}{5!}(x-a)^5 + \cdots$$

where $f^{(4)}(a)$ denotes the 4th derivative of the function f evaluated at the value a.

#10) Use the formula above to give the Taylor Series for $f(x) = \sqrt{x}$ about $a=1$.

A) $1 + \frac{(x-1)}{1!} + \frac{(x-1)^2}{2!} + \frac{(x-1)^3}{3\cdot 3!} + \frac{(x-1)^4}{3\cdot 5\cdot 4!} + \frac{(x-1)^5}{3\cdot 5\cdot 7\cdot 5!} + \cdots$

B) $1 + \frac{(x-1)}{1!} + \frac{(x-1)^2}{2\cdot 2!} + \frac{(x-1)^3}{3\cdot 3!} + \frac{(x-1)^4}{4\cdot 4!} + \frac{(x-1)^5}{5\cdot 5!} + \cdots$

C) $1 + \frac{(x-1)}{2\cdot 1!} - \frac{(x-1)^2}{2^2\cdot 2!} + \frac{3\cdot(x-1)^3}{2^3\cdot 3!} - \frac{3\cdot 5\cdot(x-1)^4}{2^4\cdot 4!} + \frac{3\cdot 5\cdot 7\cdot(x-1)^5}{2^5\cdot 5!} + \cdots$

D) $1 + \frac{(x-1)}{1!} + \frac{(x-1)^2}{2!} + \frac{3\cdot(x-1)^3}{3!} + \frac{3\cdot 5\cdot(x-1)^4}{4!} + \frac{3\cdot 5\cdot 7\cdot(x-1)^5}{5!} + \cdots$

E) None of the above

$f(x) = \sqrt{x} \Rightarrow f(1) = 1$

$f'(x) = x^{-1/2}/2 \Rightarrow f'(1) = 1/2$

$f''(x) = -x^{-3/2}/2^2 \Rightarrow f''(1) = -1/2^2$

$f'''(x) = (-1)^2\cdot 3x^{-5/2}/2^3 \Rightarrow f'''(1) = 3/2^3$

$f^{(4)}(x) = (-1)^3\cdot 3\cdot 5x^{-7/2}/2^4 \Rightarrow f^{(4)}(1) = -3\cdot 5/2^4$

$f^{(5)}(x) = (-1)^4\cdot 3\cdot 5\cdot 7\, x^{-9/2}/2^5 \Rightarrow f^{(5)}(1) = 3\cdot 5\cdot 7/2^5$

For $a=1$, the Taylor Series formula gives

$$f(x) = f(1) + \frac{f'(1)}{1!}(x-1) + \frac{f''(1)}{2!}(x-1)^2 + \frac{f'''(1)}{3!}(x-1)^3 + \frac{f^{(4)}(1)}{4!}(x-1)^4 + \frac{f^{(5)}(1)}{5!}(x-1)^5 + \cdots$$

$$= 1 + \frac{1/2}{1!}(x-1) + \frac{-1/2^2}{2!}(x-1)^2 + \frac{3/2^3}{3!}(x-1)^3 + \frac{-3\cdot 5/2^4}{4!}(x-1)^4 + \frac{3\cdot 5\cdot 7/2^5}{5!}(x-1)^5 + \cdots$$

$$= \boxed{1 + \frac{(x-1)}{2\cdot 1!} - \frac{(x-1)^2}{2^2\cdot 2!} + \frac{3\cdot(x-1)^3}{2^3\cdot 3!} - \frac{3\cdot 5\cdot(x-1)^4}{2^4\cdot 4!} + \frac{3\cdot 5\cdot 7\cdot(x-1)^5}{2^5\cdot 5!} + \cdots = \sqrt{x}}$$