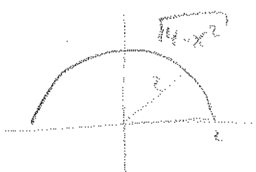
1. (30 points) Compute the following integrals and/or derivatives:

a. $\int (x^5 - 3x^2 + 2)\, dx$ $= \boxed{\frac{1}{6}x^6 - x^3 + 2x + C}$

b. $\int_0^3 2x^2\, dx$ $= \frac{2}{3}x^3 \Big|_0^3 = \frac{2}{3}(3)^3 = 2 \cdot 3^2 = \boxed{18}$

c. $\int \sec^2(3x)\, dx$ $= \boxed{\frac{1}{3}\tan(3x) + C}$

d. $\int_0^2 5\sqrt{4 - x^2}\, dx$ $= 5\int_0^2 \sqrt{4-x^2}\, dx = 5\left[\frac{1}{4} \text{ Area circle rad } 2\right]$



$= 5\left(\frac{1}{4}\pi(2)^2\right) = \boxed{5\pi}$

e. $\int 20\sin^4(2x)\cos(2x)\, dx$

Let $u = \sin(2x)$
$du = 2\cos(2x)\, dx$
$\frac{1}{2}du = \cos(2x)\, dx$

$\|$

$\int 20u^4 \frac{1}{2}\, du = \int 10u^4\, du = 2u^5 + C = \boxed{2\sin^5(2x) + C}$

2. (30 points) Compute the following integrals and/or derivatives:

a. $\int_0^{\pi^2} \frac{\sin(\sqrt{x})}{\sqrt{x}}\, dx$

let $u = \sqrt{x}$

$du = \frac{1}{2}\frac{1}{\sqrt{x}}\, dx$

$2\,du = \frac{1}{\sqrt{x}}\, dx$

$x = 0 \Rightarrow u = 0$

$x = \pi^2 \Rightarrow u = \sqrt{\pi^2} = \pi$

$$\int_0^{\pi} 2\sin u\, du = -2\cos u \Big|_0^{\pi} = -2\left[\cos\pi - \cos 0\right]$$

$$= -2\left[-1 - 1\right] = (-2)(-2) = \boxed{4}$$

b. $\int_0^{\pi/2} (f(\sin x)\cos x + x)\, dx$, given that $\int_0^1 f(x)\, dx = \frac{1}{2}$

$$= \int_0^{\pi/2} f(\sin x)\cos x\, dx + \int_0^{\pi/2} x\, dx$$

let $u = \sin x$

$du = \cos x\, dx$

$x = 0 \Rightarrow u = \sin 0 = 0$

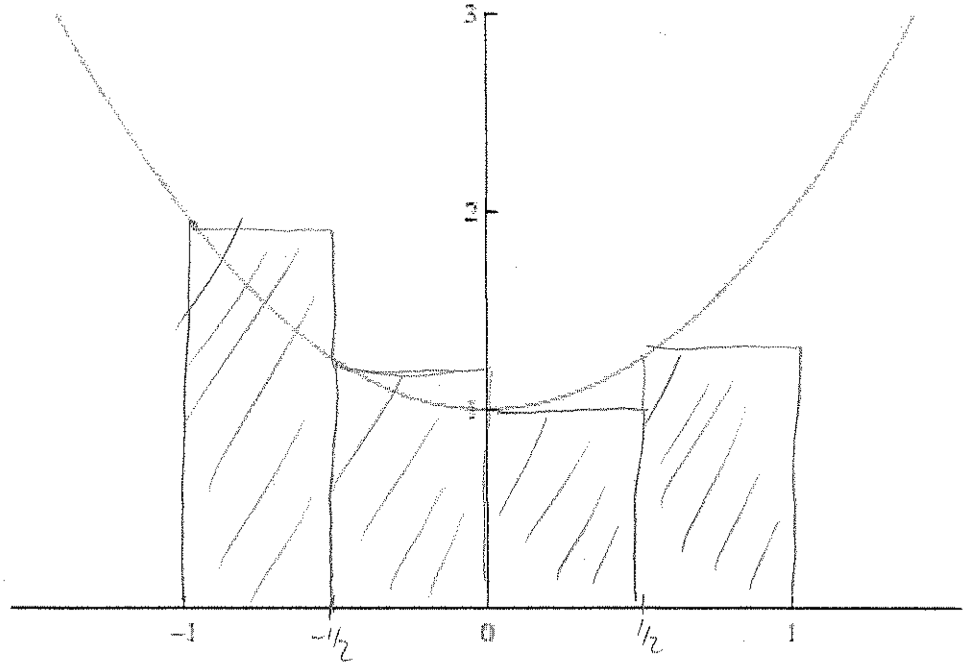
$x = \pi/2 \Rightarrow u = \sin \pi/2 = 1$

$$= \int_0^1 f(u)\, du + \left[\frac{1}{2}x^2\Big|_0^{\pi/2}\right] = \frac{1}{2} + \frac{1}{2}(\pi/2)^2 = \boxed{\frac{1}{2} + \frac{\pi^2}{8}}$$

c. $\frac{d}{dx}\int_{\pi}^{x} \tan(t)\, dt$ $= \boxed{\tan x}$

d. $\frac{d}{dx}\int_{-2x}^{x^3} \frac{d\theta}{\sqrt{1 - 2\sin^2\theta}}$ $= \boxed{\frac{3x^2}{\sqrt{1 - 2\sin^2(x^3)}} - \frac{-2}{\sqrt{1 - 2\sin^2(-2x)}}}$

3. (10 points) Estimate the integral $\int_{-1}^{1} (x^2 + 1)\, dx$ using 4 rectangles and left-endpoints. Draw your estimation on the graph below.



Let $f = x^2+1$, $\quad \Delta x = \frac{1 - -1}{4} = \frac{1}{2}$

$$\int_{-1}^{1} (x^2+1)\, dx \approx \left[f(-1) + f(-1/2) + f(0) + f(1/2) \right] \Delta x$$

$$= \left[(-1)^2+1 + (-1/2)^2+1 + 0^2+1 + (1/2)^2+1 \right] \frac{1}{2}$$

$$= \left[4 + 1 + \frac{1}{2} \right] \frac{1}{2} = \boxed{\frac{11}{4}}$$

4. (10 points) Use the **definition of the definite integral** to compute $\int_0^3 2x^2\,dx$.

$$\int_a^b f(x)\,dx = \lim_{n\to\infty} \sum_{k=1}^{n} f(x_k)\Delta x, \quad \text{where } \Delta x = \frac{b-a}{n}, \quad x_k = a + k\Delta x$$

Here, $\Delta x = \frac{3}{n}$, $x_k = 0 + k\frac{3}{n} = \frac{3k}{n}$.

$$\sum_{k=1}^{n} f(x_k)\Delta x = \sum_{k=1}^{n} 2\left(\frac{3k}{n}\right)^2 \frac{3}{n} = \sum_{k=1}^{n} \frac{2\cdot 3^3}{n^3} k^2$$

$$= \frac{2\cdot 3^3}{n^3} \sum_{k=1}^{n} k^2 = \frac{2\cdot 3^3}{n^3} \frac{n(n+1)(2n+1)}{6}$$

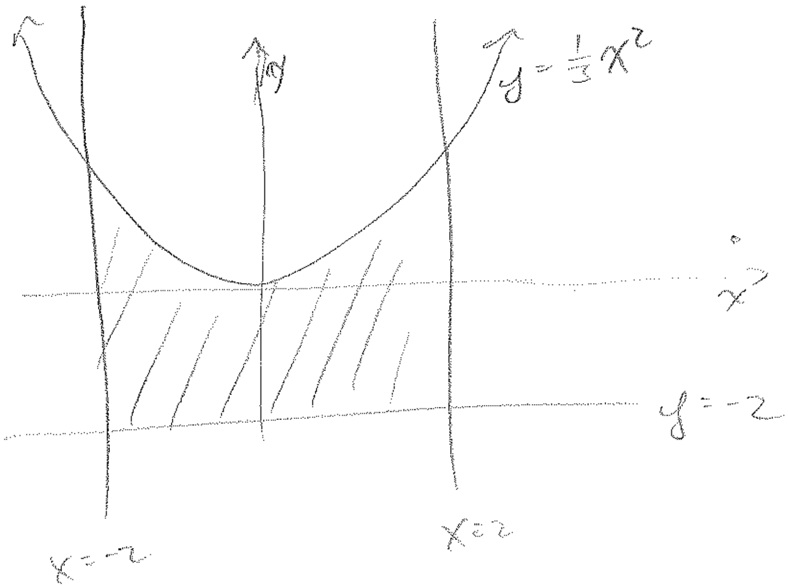
$$\int_0^3 2x^2\,dx = \lim_{n\to\infty} \sum_{k=1}^{n} 2\left(\frac{3k}{n}\right)^2 \frac{3}{n} = \lim_{n\to\infty} \frac{2\cdot 3^3}{6} \frac{n(n+1)(2n+1)}{n^3}$$

$$\frac{\text{order } 3}{\text{order } 3} \quad \text{ratio of leading coeffs} \quad \frac{2}{1}$$

$$= \frac{2\cdot 3^3}{6}\cdot 2$$

$$= 2\cdot 3^2 = \boxed{18}$$

5. (10 points) Find the area between the curves $y = -2$, $y = \frac{1}{3}x^2$, $x = -2$, $x = 2$.



$$A = \int_{-2}^{2} \left(\frac{1}{3}x^2 - (-2)\right) dx = \int_{-2}^{2} \frac{1}{3}x^2 + 2 \; dx$$

$$= 2\int_0^2 \left(\frac{1}{3}x^2 + 2\right) dx = 2\left[\frac{1}{9}x^3 + 2x \Big|_0^2\right]$$

$$= 2\left(\frac{2^3}{9} + 2\cdot 2\right) = \frac{16}{9} + 8 = \frac{88}{9}$$

6. (10 points) Find the area bounded between the curves $y = x^3 + 6x$ and $y = 5x^2$. You may leave your answer in the form of a definite integral.

Where do curves intersect?

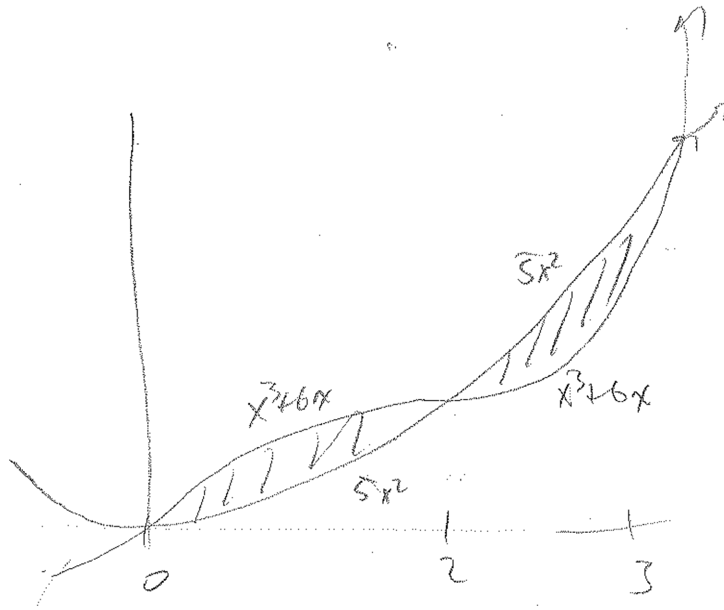
$$x^3 + 6x = 5x^2$$

$$x^3 - 5x^2 + 6x = 0$$

$$x(x^2 - 5x + 6) = 0$$

$$x(x-2)(x-3) = 0$$

$$x = 0, 2, 3$$



Check: Which curve is on top/bottom by plugging in sample values.

$(1)^3 + 6(1) - 5(1)^2 = 7 > 0 \Rightarrow x^3+6x$ on top on $[0,2]$

or

$(1)(1-2)(1-3) = + - - = +$

Sim, plug in 2.5

$(2.5)(2.5-2)(2.5-3) = + + - = -$

$\Rightarrow 5x^2$ on top on $[2,3]$

$$\boxed{\text{Area} = \int_0^2 x^3+6x-5x^2 \, dx + \int_2^3 5x^2 - (x^3+6x) \, dx}$$