

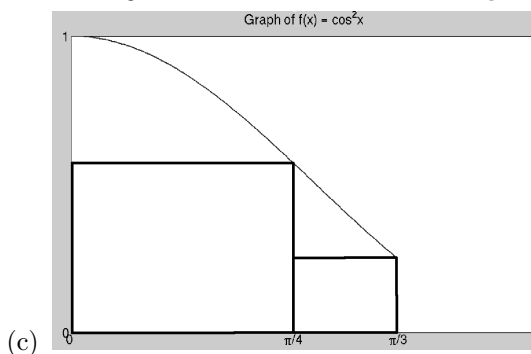
1. **D** Since the numerator and denominator both approach 0, use L'Hospital's Rule:
 
$$\lim_{x \rightarrow 0} \frac{e^x - \cos x - 2x}{x^2 - 2x} = \lim_{x \rightarrow 0} \frac{e^x + \sin x - 2}{2x - 2} = \frac{1}{2}$$
2. **E** Use properties of logarithms:  $\ln(x^2 + x) = \ln(x+4)$ , so  $x^2 + x = x + 4$ ,  $x^2 - 4 = 0$ , which yields  $x = 2$  or  $x = -2$ . Since the domain of the left-hand side of the original equation is  $x > 0$ , the only solution is  $\mathbf{x = 2}$ .
3. **A**  $f'$  positive means  $f$  is increasing, and  $f'$  decreasing means  $f''$  is negative, so  $f$  is concave down. The only graph increasing and concave down is graph **A**.
4. **B** The original function  $f$  is decreasing when  $f'$  is negative, which occurs when  $x \in (\mathbf{a, c}) \cup (\mathbf{e, \infty})$ .
5. **B**  $f$  has a critical value when  $f' = 0$ , namely when  $x = a, c, e$ . Using the signs of the derivative  $f'$ , we find that  $f$  is increasing for  $x \in (-\infty, a) \cup (c, e)$  and decreasing for  $x \in (a, c) \cup (e, \infty)$ . Therefore,  $f$  has a local minimum only when  $\mathbf{x = c}$ .
6. **E** Let  $y = \log_4 \left( \frac{1}{8} \right)$ . Then  $4^y = \frac{1}{8}$ , or  $2^{2y} = 2^{-3}$ , which means  $2y = -3$  and  $y = -\frac{3}{2}$ . (Alternatively,  $\log_4 \left( \frac{1}{8} \right) = \frac{\ln 1/8}{\ln 4} = \frac{-3 \ln 2}{2 \ln 2}$  and cancel)
7. **A** Exponential growth means  $y = Ce^{kt}$ , where  $y$  is the population after  $t$  minutes. When  $t = 0$ ,  $y = 200$ , so  $y = 200e^{kt}$ . When  $t = 30$ ,  $y = 600$ , so  $3 = e^{k(30)}$ ,  $30k = \ln 3$ , or  $k = \frac{\ln 3}{30}$ . When  $t = 45$ ,  $y = 200e^{\ln 3/30 \cdot 45} = 200e^{3/2 \ln 3} = 200(3)^{3/2} = \mathbf{600\sqrt{3}}$  bacteria (ignoring appropriate rounding).
8. **A** Let  $y = \tan^{-1} 4$ . Then  $\tan y = \frac{4}{1}$ , so  $y$  is an acute angle of a right triangle with opposite side 4, adjacent side 1, hence hypotenuse  $\sqrt{17}$ . Therefore,  $\cos y = \frac{1}{\sqrt{17}}$ .
9. **C**  $f'(x) = 3x^2 - 3 = 0$  when  $x = 1$  or  $x = -1$ . Since  $f$  is continuous on a closed, bounded interval, it attains its absolute maximum and minimum at either a critical value or an endpoint. Since  $f(-1) = 3$ ,  $f(1) = -1$ ,  $f(3) = 19$ , the **minimum value is -1 and the maximum value is 19**.
10. **E** An antiderivative of  $f(x) = \ln x$  means its derivative is  $\ln x$ . Using the product rule,  $\frac{d}{dx}(x \ln x - x) = x \cdot \frac{1}{x} + 1 \cdot \ln x - 1 = \ln x$ , so the correct antiderivative is  $\mathbf{x \ln x - x}$ .
11. **B** The limit given is the definition of the derivative, so the correct answer is  $\frac{d}{dx}(5^x) = (\ln 5)\mathbf{5^x}$ .
12. **E**  $a(t) = v'(t) = 3t + 2$ , so  $v(t) = \frac{3}{2}t^2 + 2t + C$ . Since  $v(0) = 0$ ,  $C = 0$  and  $v(t) = \frac{3}{2}t^2 + 2t$ . Thus,  $v(2) = \frac{3}{2}(2^2) + 2(2) = \mathbf{10 \text{ ft/sec}}$ .
13. .
  - (a)  $f'(x) = x \cdot \frac{1}{1+x^2} + 1 \cdot \arctan x - \frac{1}{2} \cdot \frac{2x}{1+x^2} = \frac{x}{1+x^2} + \arctan x - \frac{x}{1+x^2} = \arctan \mathbf{x}$ .
  - (b)  $g'(x) = \frac{3 + 4e^{4x}}{3x + 2 + e^{4x}}$ , so  $g'(0) = \frac{3 + 4}{2 + 1} = \frac{\mathbf{7}}{3}$
14. .
  - (a)  $f'(x) = 0$  when  $x = 2$ .  $f'(x) < 0$  when  $x < 2$  and  $f'(x) > 0$  when  $x > 2$ , so  $f$  is **increasing on  $(2, \infty)$  and decreasing on  $(-\infty, 2)$** .
  - (b) Based on the above answer,  $f$  has a **local minimum at  $\mathbf{x = 2}$** .
  - (c)  $f''(x) = (x-2) \cdot 3e^{3x} + 1 \cdot e^{3x} = e^{3x}(3x-5)$ .  $f''(x) = 0$  when  $x = \frac{5}{3}$ .  $f''(x) > 0$  when  $x > \frac{5}{3}$  and  $f''(x) < 0$  when  $x < \frac{5}{3}$ . Therefore,  $f$  is **concave upward when  $\mathbf{x > \frac{5}{3}}$  and concave downward when  $\mathbf{x < \frac{5}{3}}$** .

15. Our goal is to minimize  $C = 4lw + 2lh + 2wh$  under the restrictions that  $l = w$  and  $lwh = 8$ , where  $l$ ,  $w$ , and  $h$  are the dimensions of the box. Substituting  $l = w$  into the other equations yields  $C = 4w^2 + 4wh$  and  $w^2h = 8$ , or  $h = \frac{8}{w^2}$ . Thus, our goal is to minimize  $C(w) = 4w^2 + 4w\left(\frac{8}{w^2}\right) = 4w^2 + \frac{32}{w}$  ( $w > 0$ ).  $C'(w) = 8w - \frac{32}{w^2} = 0$  when  $8w^3 - 32 = 0$ , or  $w = \sqrt[3]{4}$ . By testing the signs of  $C'(w)$  on either side or by showing  $C''(\sqrt[3]{4}) > 0$ , we see that this critical value is a local, and hence absolute minimum. Thus,  $l = \sqrt[3]{4}$  and  $h = \frac{8}{\sqrt[3]{4^2}} = 2\sqrt[3]{4}$ . Therefore, the dimensions required are  $\sqrt[3]{4} \times \sqrt[3]{4} \times 2\sqrt[3]{4}$  ft.

again or the fact that  $\frac{\tan x}{x} \rightarrow 1$  when  $x \rightarrow 0$ , the limit =  $e^{-1/2}$ .

16. .

- (a) Area  $\approx \sum_{i=1}^n f(x_i^*)\Delta x_i$ . For the given partition,  $n = 2$ , so Area  $\approx \sum_{i=1}^2 \cos^2(x_i^*)\Delta x_i$ .
- (b) Expanding and using the given partition yields Area  $\approx \cos^2\left(\frac{\pi}{4}\right)\left(\frac{\pi}{4}\right) + \cos^2\left(\frac{\pi}{3}\right)\left(\frac{\pi}{12}\right) = \frac{1}{2} \cdot \frac{\pi}{4} + \frac{1}{4} \cdot \frac{\pi}{12} = \frac{7\pi}{48}$



17. Since the base approaches 1 and the exponent approaches  $\infty$ , rewrite using exponential and logarithms:

$$\begin{aligned} \lim_{x \rightarrow 0} (\cos x)^{1/x^2} &= e^{\lim_{x \rightarrow 0} \ln((\cos x)^{1/x^2})} \\ &= e^{\lim_{x \rightarrow 0} \frac{\ln(\cos x)}{x^2}} = e^{\lim_{x \rightarrow 0} \frac{-\frac{\sin x}{\cos x}}{2x}} \\ &= e^{\lim_{x \rightarrow 0} \frac{-\tan x}{2x}}. \quad \text{Using L'Hospital's Rule} \end{aligned}$$