Exam 2

MULTIPLE CHOICE. Choose the one alternative that best completes the statement or answers the question.

Provide an appropriate response.

1) Find dy/dx by implicit differentiation.

$x^3 + y^3 = 5$

A) $\frac{dy}{dx} = -\frac{y^2}{x^2}$ B) $\frac{dy}{dx} = \frac{y^2}{x^2}$ C) $\frac{dy}{dx} = -\frac{x^2}{y^2}$ D) $\frac{dy}{dx} = \frac{x^2}{y^2}$

Evaluate dy/dt for the function at the point.

2) $xy^2 = 4$; $dx/dt = -5$, $x = 4$, $y = 1$

A) $-\frac{5}{8}$ B) $-\frac{8}{5}$ C) $\frac{8}{5}$ D) $\frac{5}{8}$

Solve the problem. Round your answer, if appropriate.

3) A spherical balloon is inflated with helium at a rate of 110π ft^3/min. How fast is the balloon's radius increasing when the radius is 5 ft?

A) 1.10 ft/min B) 2.75 ft/min C) 0.22 ft/min D) 3.30 ft/min

Identify the critical points and find the maximum and minimum value on the given interval I.

4) $f(x) = x^2 + 18x + 81$; $I = [-18, 0]$

A) Critical points: –18, 0, 9; maximum value 81; minimum value 0
B) Critical points: –9; maximum value 18; minimum value 0
C) Critical points: –18, 0, 81; minimum value 0
D) Critical points: –18, –9, 0; maximum value 81; minimum value 0

Use the Concavity Theorem to determine where the given function is concave up and where it is concave down. Also find all inflection points.

5) $q(x) = 3x^3 + 2x + 8$

A) Concave down for all x; no inflection points
B) Concave up on $(-\infty, 0)$, concave down on $(0, \infty)$; inflection point (0, 8)
C) Concave up for all x; no inflection points
D) Concave up on $(0, \infty)$, concave down on $(-\infty, 0)$; inflection point (0, 8)

Solve the problem.

6) Given the distance function, $s(t) = 13t - t^2$, where s is in feet and t is in seconds, find all times when the velocity is 1 ft/sec

A) t = 1 sec B) t = 6 sec C) t = 12 sec D) t = 6.5 sec

7) A car's distance s in miles from its starting point after t hours is given by

$s(t) = 9t^2$

Find the average rate of change of distance with respect to time (average velocity) as t changes from $t_1 = 4$ to $t_2 = 7$.

A) 63 miles/hr B) 49.5 miles/hr C) 42.4 miles/hr D) 99 miles/hr

For the following function, determine the interval(s) of x for which f(x) decreases.

8) $f(x) = 45x^3 - 3x^5$

A) $(-3, 3)$ B) $(-\infty, -3) \cup (3, \infty)$ C) $(-3, 0) \cup (3, \infty)$ D) $(-\infty, -3) \cup (0, 3)$

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Use the Second Derivative Test to find the point(s) of inflection.

9) $f(x) = x^4 - 8x^2 - 4$

A) minimum at (2, –20); maximum at (0, –4)
B) minimum at (–2, –20); maximum at (2, –20)
C) minimum at (2, –20) and (–2, –20); maximum at (0, –4)
D) minimum at (0, –4); maximum at (2, –20) and (–2, –12)

Provide an appropriate response.

10) Let C(x) be the cost function and R(x) the revenue function. Compute the marginal cost, marginal revenue, and the marginal profit functions.

$C(x) = 0.0005x^3 - 0.06x^2 + 200x + 50{,}000$
$R(x) = 400x$

A) $C'(x) = 0.0015x^2 - 0.12x + 200$
$R'(x) = 400$
$P'(x) = -0.0015x^2 + 0.12x + 200$

B) $C'(x) = 0.0015x^2 + 0.12x + 200$
$R'(x) = 400$
$P'(x) = 0.0015x^2 + 0.12x + 200$

C) $C'(x) = 0.0015x^2 - 0.12x + 200$
$R'(x) = 400$
$P'(x) = 0.0015x^2 - 0.12x - 200$

Solve the problem.

11) Find the number of units that must be produced and sold in order to yield the maximum profit, given the following equations for revenue and cost:

$R(x) = 60x - 0.5x^2$
$C(x) = 5x + 4.$

A) 65 units B) 55 units C) 56 units D) 59 units

Using the derivative of f(x) given below, determine the intervals on which f(x) is increasing or decreasing.

12) $f'(x) = (5 - x)(8 - x)$

A) Decreasing on $(-\infty, 5)$; increasing on $(8, \infty)$
B) Decreasing on $(-\infty, -5) \cup (-8, \infty)$; increasing on $(-5, -8)$
C) Decreasing on $(5, 8)$; increasing on $(-\infty, 5) \cup (8, \infty)$
D) Decreasing on $(-\infty, 5) \cup (8, \infty)$; increasing on $(5, 8)$

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SHORT ANSWER. Write the word or phrase that best completes each statement or answers the question. Show all work CLEARLY in the space provided. Give exact answers unless otherwise requested.

Solve the problem. Round your answer, if appropriate.

13) A ladder is slipping down a vertical wall. If the ladder is 20 ft long and the top of it is slipping at the constant rate of 4 ft/s, how fast is the bottom of the ladder moving along the ground when the bottom is16 ft from the wall? (**Hint:** *Draw a diagram. Use the Pythagorean Theorem*)

[10 points]

Rate at which ladder is moving along the ground:

.....

Find an equation in slope–intercept form for the line tangent to the given curve at the indicated point.

14) $y^6 + x^3 = y^2 + 10x$, at (0, 1)

[10 points]

Equation:

.....

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Solve the problem.

15) A carpenter is building a rectangular room with a fixed perimeter of 180 ft. What are the dimensions of the largest room that can be built? What is its area?

[10 points]

Dimensions: ..... Area: .....

Use implicit differentiation to find d^2y/dx^2 in terms of x and y only (that is, dy/dx should not appear in the answer).
(Hint: *Find dy/dx, then differentiate again. Substitute for dy/dx in the final answer.*

16) $y^2 - x^2 = 8$

[12 points]

$$\frac{d^2y}{dx^2} =$$

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For the given function do the following:
(a) Find the coordinates of any local and absolute extreme points
(b) Find the coordinates of all inflection points.
(c) Identify the intervals on which the function is increasing and decreasing.
(d) Identify the intervals on which the function is concave up and concave down.

17) $y = \dfrac{6x}{x^2 + 9}$

Show all work below: **[15 points]**

(a) Minimum/minima point(s): ..... Maximum/maxima point(s): .....

(b) Inflection point(s) .....

(c) Increasing on: ..... Decreasing on: .....

(d) Concave UP on: ..... Concave DOWN on: .....

Answer Key

1) C
2) D
3) A
4) D
5) D
6) B
7) D
8) B
9) C
10) A
11) B
12) C
13) 3.0 ft/s
14) $y = \frac{5}{2}x + 1$
15) 45 ft by 45 ft; 2025 ft^2
16) $\frac{y^2 - x^2}{y^3}$
17) local minimum: (–3, –1)
local maximum: (3, 1)
inflection points: (0, 0), $(-3\sqrt{3}, -\frac{3}{2}\sqrt{3})$,
$(3\sqrt{3}, \frac{3}{2}\sqrt{3})$