1.) (8 pts. each) Integrate each of the following. DO NOT SIMPLIFY answers.

a.) $\int \sqrt{x}(1+x)\ dx = \int \left(x^{1/2} + x^{3/2}\right) dx$

$= \dfrac{x^{3/2}}{3/2} + \dfrac{x^{5/2}}{5/2} + c$

b.) $\int \dfrac{x^2+3}{x+2}\ dx = \int \left(x - 2 + \dfrac{7}{x+2}\right) dx$

$$\begin{array}{r|l} & x-2 \\ x+2 & x^2+3 \\ & -(x^2+2x) \\ \hline & -2x+3 \\ & -(-2x-4) \\ \hline & 7 \end{array}$$

$= \dfrac{x^2}{2} - 2x + 7\ln|x+2| + c$

c.) $\int \dfrac{e^x}{e^x+5}\ dx$ $\quad$ (Let $u = e^x + 5 \rightarrow du = e^x dx$)

$= \int \dfrac{1}{u}\, du = \ln|u| + c = \ln|e^x + 5| + c$

d.) $\int \dfrac{x}{(3x^2+1)^4}\ dx$ $\quad$ (Let $u = 3x^2 + 1 \rightarrow du = 6x\,dx \rightarrow \frac{1}{6}du = dx$)

$= \dfrac{1}{6}\int \dfrac{1}{u^4}\, du = \dfrac{1}{6}\int u^{-4} du = \dfrac{1}{6}\cdot\dfrac{u^{-3}}{-3} + c$

$= \dfrac{-1}{18}\left(3x^2+1\right)^{-3} + c$

e.) $\int \sin^2 x\ \cos x\ dx$ $\quad$ (Let $u = \sin x \rightarrow du = \cos x\, dx$)

$= \int u^2 du = \dfrac{1}{3}u^3 + c = \dfrac{1}{3}\sin^3 x + c$

2.) Consider the function $f(x) = xe^{x^4}$.

a.) (6 pts.) Verify that f is an odd function. Show $f(-x) = -f(x)$:

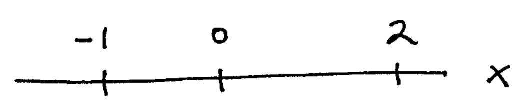
$$f(-x) = (-x)\cdot e^{(-x)^4}$$
$$= -xe^{x^4}$$
$$= -f(x)$$

b.) (4 pts.) Evaluate $\int_{-3}^{3} xe^{x^4}\,dx$. Since $f(x) = xe^{x^4}$ is odd,

$$\int_{-3}^{3} xe^{x^4}\,dx = 0$$

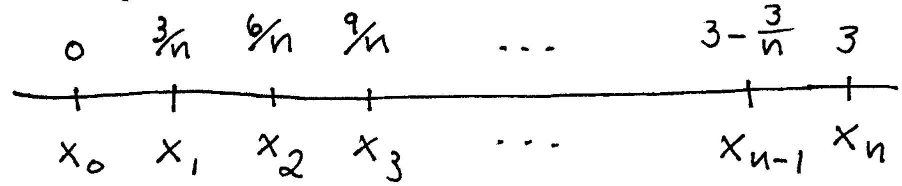
3.) (6 pts.) If $\int_{-1}^{2} f(x)\,dx = 3$ and $\int_{0}^{2} f(x)\,dx = -4$, what is $\int_{0}^{-1} f(x)\,dx$?

By properties of a definite integral:



$$\int_{-1}^{2} f(x)\,dx = \int_{-1}^{0} f(x)\,dx + \int_{0}^{2} f(x)\,dx \rightarrow$$
$$3 = \int_{-1}^{0} f(x)\,dx + -4 \rightarrow$$
$$\int_{-1}^{0} f(x)\,dx = 7 \rightarrow$$
$$\int_{0}^{-1} f(x)\,dx = -7.$$

4.) (12 pts.) Use the limit definition of the definite integral (for convenience, you may choose equal subdivisions and right-hand endpoints) to evaluate $\int_0^3 (x^2 + 2x)\, dx$.



$f(x) = x^2 + 2x$

Divide $[0, 3]$ into n equal parts each of length $3/n$. Then right-hand endpoints are

$$x_i = 0 + \frac{3}{n} i = \frac{3}{n} i \text{ with } \Delta x_i = \frac{3}{n} \text{ for } i = 1, 2, 3, \ldots, n.$$

$$\int_0^3 (x^2 + 2x)\, dx = \lim_{n\to\infty} \sum_{i=1}^{n} f(x_i) \cdot \Delta x_i$$

$$= \lim_{n\to\infty} \sum_{i=1}^{n} \left[x_i^2 + 2x_i\right] \cdot \Delta x_i$$

$$= \lim_{n\to\infty} \sum_{i=1}^{n} \left[\frac{9}{n^2} i^2 + \frac{6}{n} i\right] \cdot \frac{3}{n}$$

$$= \lim_{n\to\infty} \sum_{i=1}^{n} \left(\frac{27}{n^3} i^2 + \frac{18}{n^2} i\right)$$

$$= \lim_{n\to\infty} \left\{ \frac{27}{n^3} \cdot \left(\sum_{i=1}^{n} i^2\right) + \frac{18}{n^2} \cdot \left(\sum_{i=1}^{n} i\right) \right\}$$

$$= \lim_{n\to\infty} \left\{ \frac{27}{n^3} \cdot \frac{n(n+1)(2n+1)}{6} + \frac{18}{n^2} \cdot \frac{n(n+1)}{2} \right\}$$

$$= \lim_{n\to\infty} \left\{ \frac{9}{2} \cdot \cancel{\frac{n}{n}} \cdot \frac{n+1}{n} \cdot \frac{2n+1}{n} + 9 \cdot \cancel{\frac{n}{n}} \cdot \frac{n+1}{n} \right\}$$

$$= \lim_{n\to\infty} \left\{ \frac{9}{2} \cdot \left(1 + \frac{1}{n}\right)\left(2 + \frac{1}{n}\right) + 9 \cdot \left(1 + \frac{1}{n}\right) \right\}$$

$$= \frac{9}{2} \cdot (1)(2) + 9(1) = 18$$

5.) (8 pts.) Compute the area of the region bounded by the graphs of $y = x^2$, $y = 2 - x$, and $y = 0$.

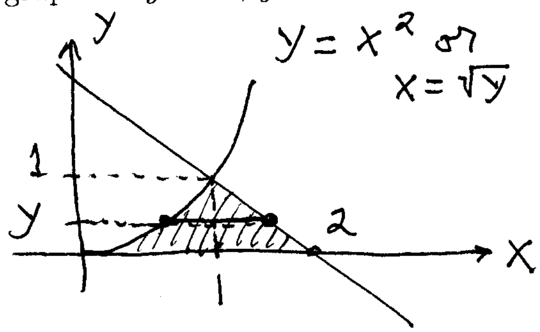
$$\text{Area} = \int_0^1 \left[(2-y) - \sqrt{y}\,\right] dy$$

$$= \left(2y - \frac{y^2}{2} - \frac{2}{3}y^{3/2}\right)\Big|_0^1$$

$$= 2 - \frac{1}{2} - \frac{2}{3}$$

$$= \frac{12}{6} - \frac{3}{6} - \frac{4}{6}$$

$$= \frac{5}{6}$$



$y = x^2$ or $x = \sqrt{y}$

$Y = 2 - X$ or $X = 2 - Y$

6.) (8 pts.) The temperature T of a room at time t minutes is $T(t) = \sqrt{16+t}\ \ {}^oF$. Find the *average* temperature of the room from $t = 0$ to $t = 20$ minutes.

$$\text{AVE} = \frac{1}{20-0}\int_0^{20} \sqrt{16+t}\ dt$$

$$= \frac{1}{20}\cdot\frac{2}{3}(16+t)^{3/2}\Big|_0^{20}$$

$$= \frac{1}{30}\left(36^{3/2} - 16^{3/2}\right)$$

$$= \frac{1}{30}(216 - 64)$$

$$= \frac{152}{30} \approx 5.07\ {}^\circ F$$

7.) (5 pts. each) Use FTC1 to differentiate each function.

a.) $F(x) = \int_x^4 \cos\sqrt{t}\,dt \quad = -\int_4^x \cos\sqrt{t}\,dt \quad \xrightarrow{D}$

$F'(x) = -\cos\sqrt{x}$

b.) $F(x) = \int_{x^2}^{3x} e^{t^2}\,dt. \quad = \int_{x^2}^{0} e^{t^2}\,dt + \int_0^{3x} e^{t^2}\,dt$

$= -\int_0^{x^2} e^{t^2}\,dt + \int_0^{3x} e^{t^2}\,dt \quad \xrightarrow{D}$

$F'(x) = -e^{(x^2)^2} \cdot (2x) + e^{(3x)^2} \cdot (3)$

$= -2x e^{x^4} + 3e^{9x^2}$

8.) (6 pts.) Write the following limit as a definite integral, then evaluate the integral. HINT : First identify x_i.

$$\lim_{n\to\infty} \sum_{i=1}^{n} \left\{3 + \frac{2i}{n}\right\} \frac{4}{n}$$

Let $x_i = 3 + \frac{2i}{n}$ for $i = 1, 2, 3, \ldots, n$;

$3 \quad 3+\frac{2}{n} \quad 3+\frac{4}{n} \quad 3+\frac{6}{n} \quad \ldots \quad 3+\frac{2\cancel{n}}{\cancel{n}} = 5$

$x_1 \quad x_2 \quad x_3 \quad \ldots \quad x_n$

$\Delta x_i = \frac{2}{n}$ for $i = 1, 2, 3, \ldots, n$; then

$\lim_{n\to\infty} \sum_{i=1}^{n} \left(3 + \frac{2i}{n}\right) \cdot \frac{4}{n} = \lim_{n\to\infty} \sum_{i=1}^{n} 2\left(3 + \frac{2i}{n}\right) \cdot \frac{2}{n}$

$= \lim_{n\to\infty} \sum_{i=1}^{n} 2x_i \cdot \Delta x_i = \int_3^5 2x\,dx = x^2 \Big|_3^5 = 5^2 - 3^2 = 16$

The following EXTRA CREDIT problem is OPTIONAL. It is worth 10 points.

1.) Use the limit definition of the definite integral, $\lim_{mesh \to 0} \sum_{i=1}^{n} f(c_i)(x_i - x_{i-1})$, to evaluate $\int_a^b \frac{1}{x^2}\,dx$. Use an arbitrary partition $a = x_0 < x_1 < x_2 < \ldots < x_{n-1} < x_n = b$ for the interval $[a, b]$ and sampling numbers $c_i = \sqrt{x_{i-1} x_i}$ for $i = 1, 2, 3, \ldots, n$.

$$\int_a^b \frac{1}{x^2}\,dx = \lim_{mesh \to 0} \sum_{i=1}^{n} f\left(\sqrt{x_{i-1}x_i}\right) \cdot (x_i - x_{i-1})$$

$$= \lim_{mesh \to 0} \sum_{i=1}^{n} \frac{1}{x_{i-1}\,x_i} (x_i - x_{i-1})$$

$$= \lim_{mesh \to 0} \sum_{i=1}^{n} \left(\frac{x_i}{x_{i-1}x_i} - \frac{x_{i-1}}{x_{i-1}x_i} \right)$$

$$= \lim_{mesh \to 0} \sum_{i=1}^{n} \left(\frac{1}{x_{i-1}} - \frac{1}{x_i} \right)$$

$$= \lim_{mesh \to 0} \left\{ \left(\frac{1}{x_0} - \cancel{\frac{1}{x_1}}\right) + \left(\cancel{\frac{1}{x_1}} - \cancel{\frac{1}{x_2}}\right) + \left(\cancel{\frac{1}{x_2}} - \cancel{\frac{1}{x_3}}\right) + \cdots + \left(\cancel{\frac{1}{x_{n-2}}} - \cancel{\frac{1}{x_{n-1}}}\right) + \left(\cancel{\frac{1}{x_{n-1}}} - \frac{1}{x_n}\right) \right\}$$

$$= \lim_{mesh \to 0} \left\{ \frac{1}{a} - \frac{1}{b} \right\}$$

$$= \frac{1}{a} - \frac{1}{b}$$