1. **C** As $n \to \infty$, $e^{-n} \to 0$, so $a_n \to \dfrac{1}{5-0} = \dfrac{1}{5}$.

2. **A** $|a_n| = \dfrac{2n^2+2}{3n^2+1} \to \dfrac{2}{3}$, so the terms of the sequence alternate between approaching $\dfrac{2}{3}$ (even-numbered terms) and approaching $-\dfrac{2}{3}$ (odd-numbered terms), which means the sequence diverges.

3. **D** The denominator contains a repeating linear factor and an irreducible quadratic factor, so the form of the partial fraction decomposition is $\dfrac{A}{x+1} + \dfrac{B}{(x+1)^2} + \dfrac{Cx+D}{x^2+2x+3}$.

4. **A** Let s be the sum of the series. Then $s = \lim_{n\to\infty} s_n = 2$, so statement (I) is true. This means the series is convergent, which makes (II) false. Since the series is convergent, $a_n \to 0$ by the Test for Divergence, so (III) is true.

5. **B** If S is the surface area, $S = \int_a^b 2\pi r\, ds$. We choose to integrate with respect to x, so $r = y = e^{2x}$. $\dfrac{dy}{dx} = 2e^{2x}$, so $ds = \sqrt{1+(2e^{2x})^2} = \sqrt{1+4e^{4x}}$. Therefore, $S = \displaystyle\int_0^1 2\pi e^{2x}\sqrt{1+4e^{4x}}\, dx$.

6. **E** Let $x = 2\sin\theta$. Then $dx = 2\cos\theta\, d\theta$. When $x = \sqrt{3}$, $\theta = \sin^{-1}\left(\dfrac{\sqrt{3}}{2}\right) = \dfrac{\pi}{3}$. When $x = 2$, $\theta = \sin^{-1}\left(\dfrac{2}{2}\right) = \dfrac{\pi}{2}$. Then $\displaystyle\int_{\sqrt{3}}^2 \sqrt{4-x^2}\, dx = \int_{\pi/3}^{\pi/2} \sqrt{4-4\sin^2\theta}\,(2\cos\theta\, d\theta)$ $= \displaystyle\int_{\pi/3}^{\pi/2} (2\cos\theta)(2\cos\theta)\, d\theta = 4\int_{\pi/3}^{\pi/2} \cos^2\theta\, d\theta$.

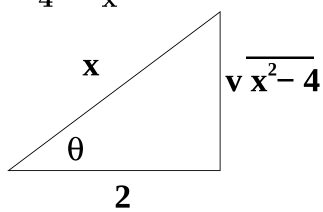
7. **B** The function is unbounded at $x = 0$, so we rewrite the integral as $\displaystyle\int_{-1}^0 \frac{1}{x^2}\, dx + \int_0^3 \frac{1}{x^2}\, dx$. The Type-2 improper integral $\displaystyle\int_0^a \frac{1}{x^p}\, dx$ converges if and only if $p < 1$, so both integrals diverge.

8. **E** Since $\sin^2 x \le 1$, $\dfrac{\sin^2 x}{x^2} \le \dfrac{1}{x^2}$. The Type-1 improper integral $\displaystyle\int_a^\infty \frac{1}{x^p}\, dx$ converges if and only if $p > 1$, so $\displaystyle\int_1^\infty \frac{\sin^2 x}{x^2}\, dx$ converges by comparison with $\displaystyle\int_1^\infty \frac{1}{x^2}\, dx$.

9. **E** The Test for Divergence fails since $\dfrac{n}{n^3-5} \to 0$. Compare with $\displaystyle\sum_{n=2}^\infty \frac{1}{n^2}$, which is convergent by the P-test. Since $\dfrac{n}{n^3-5} > \dfrac{1}{n^2}$, the Comparison Test fails. Using the Limit Comparison Test, we see that $\displaystyle\lim_{n\to\infty} \frac{\frac{n}{n^3-5}}{\frac{1}{n^2}} = \lim_{n\to\infty} \frac{n^3}{n^3-5} = 1$, so the given series is convergent.

10. **B** (a), (b), and (d) are P-series with $p = 1, \dfrac{3}{2}, \dfrac{1}{3}$ respectively, so only (b) is convergent. Since $\displaystyle\int_2^\infty \frac{1}{x\ln x}\, dx = \lim_{a\to\infty} \ln(\ln x)\Big|_2^a = \infty$, (c) is divergent by the Integral Test.

11. Let $x = 2\sec\theta$. Then $dx = 2\sec\theta\tan\theta\, d\theta$. Substituting into the integral yields $\displaystyle\int \frac{2\sec\theta\tan\theta}{4\sec^2\theta\sqrt{4\sec^2\theta - 4}}\, d\theta = \int \frac{2\sec\theta\tan\theta}{4\sec^2\theta(2\tan\theta)}\, d\theta$ $\displaystyle = \int \frac{1}{4\sec\theta}\, d\theta = \frac{1}{4}\int \cos\theta\, d\theta = \frac{1}{4}\sin\theta + C$. Using the reference triangle below, $\sin\theta = \dfrac{\sqrt{x^2-4}}{x}$, so the integral $= \mathbf{\dfrac{1}{4}\dfrac{\sqrt{x^2-4}}{x} + C}$.



12. .

(a) $\displaystyle\sum_{n=0}^{\infty} \frac{2+2^n}{10^n} = \sum_{n=0}^{\infty} \frac{2}{10^n} + \sum_{n=0}^{\infty} \frac{2^n}{10^n}$, assuming both series are convergent. The first is a geometric series with $a = 2, r = \dfrac{1}{10}$, and the second is a geometric sereis with $a = 1, r = \dfrac{2}{10}$, so both series are convergent and the total sum is $\dfrac{2}{1-\frac{1}{10}} + \dfrac{1}{1-\frac{2}{10}} = \dfrac{20}{9} + \dfrac{5}{4} = \mathbf{\dfrac{125}{36}}$.

(b) Using partial fractions, we find that $\dfrac{2}{(n+1)(n+3)} = \dfrac{1}{n+1} - \dfrac{1}{n+3}$. Therefore, the Nth partial sum of the series is given by $s_N = \left(1 - \dfrac{1}{3}\right) + \left(\dfrac{1}{2} - \dfrac{1}{4}\right) + \left(\dfrac{1}{3} - \dfrac{1}{5}\right) + \cdots + \left(\dfrac{1}{N} - \dfrac{1}{N+2}\right) + \left(\dfrac{1}{N+1} - \dfrac{1}{N+3}\right) = 1 + \dfrac{1}{2} - \dfrac{1}{N+2} - \dfrac{1}{N+3}$ as a telescoping series. Since $s = \displaystyle\lim_{N\to\infty} s_N = 1 + \frac{1}{2}$, the series converges to $\mathbf{\dfrac{3}{2}}$.

13.

(a) $\dfrac{dx}{dt} = -\sin t + \sin t + t\cos t = t\cos t$. $\dfrac{dy}{dt} = \cos t - \cos t + t\sin t = t\sin t$. Therefore, the length of the curve is given by $\displaystyle\int_0^{\pi/2} \sqrt{(t\cos t)^2 + (t\sin t)^2}\, dt = \int_0^{\pi/2} t\, dt$ $\displaystyle = \frac{1}{2}t^2\Big|_0^{\pi/2} = \frac{\pi^2}{8}$.

(b) $S = \displaystyle\int_a^b 2\pi r\, ds$. Since we are rotating about the y-axis, $r = x = \cos t + t\sin t$ and, from the previous problem, $ds = t$. Therefore, the surface area is given by $\mathbf{2\pi \displaystyle\int_0^{\pi/2} (\cos t + t\sin t)t\, dt}$

14. $\dfrac{3x^2 - 4x + 11}{(x-1)(x^2+4)} = \dfrac{A}{x-1} + \dfrac{Bx+C}{x^2+4}$. Eliminating the fractions yields $3x^2 - 4x + 11 = A(x^2+4) + (Bx+C)(x-1)$. If $x = 1$, $10 = 5A$, so $A = 2$. Expanding the right-hand side yields $3x^2 - 4x + 11 = 2x^2 + 8 + Bx^2 - Bx + Cx - C$. From the x^2 coefficients, we must have $B = 1$, and from the constants, we must have $C = -3$. Therefore, the given integral is equivalent to $\displaystyle\int \left(\frac{2}{x-1} + \frac{x-3}{x^2+4}\right) dx$ $\displaystyle = \int \left(\frac{2}{x-1} + \frac{x}{x^2+4} - \frac{3}{x^2+4}\right) dx =$ $\mathbf{2\ln|x-1| + \dfrac{1}{2}\ln|x^2+4| - \dfrac{3}{2}\tan^{-1}\left(\dfrac{x}{2}\right) + C}$.

15.

(a) Let $f(x) = 3x^2e^{-x^3}$. f is continuous, positive, and decreasing $(f'(x) = 3xe^{-x^3}(2 - 3x^3) < 0)$, so we can apply the Integral Test. $\displaystyle\int_1^{\infty} 3x^2e^{-x^3}\, dx = \lim_{a\to\infty} -e^{-x^3}\Big|_1^a = e^{-1}$, so the integral converges and therefore the given series is convergent by the Integral Test.

(b) By the remainder theorem, $s - s_3 \le \displaystyle\int_3^{\infty} 3x^2e^{-x^3}\, dx$ $s - s_3 \le$ $\displaystyle\lim_{a\to\infty} -e^{-x^3}\Big|_3^a = \mathbf{e^{-27}}$.