1.) (10 pts.) In three (3) years a population of prairie dogs near Hotchkiss, Colorado, grew to 400. In five (5) years there were 1600 prairie dogs. Assuming exponential growth, what was the initial population of prairie dogs ?

assume $N = Ce^{kt}$;

$t = 3$ yrs, $N = 400 \rightarrow 400 = Ce^{3k}$

$t = 5$ yrs, $N = 1600 \rightarrow 1600 = Ce^{5k}$ $\Big\} \rightarrow$

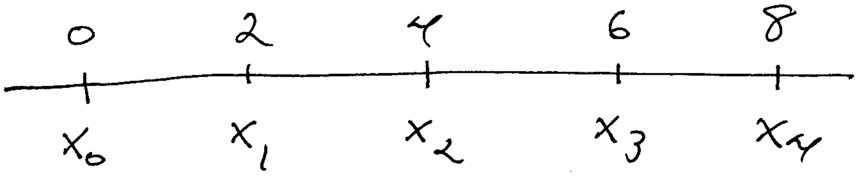
$$C = \frac{400}{e^{3k}} \rightarrow 1600 = \frac{400}{e^{3k}} \cdot e^{5k} \rightarrow$$

$$4 = e^{2k} \rightarrow \ln 4 = 2k \rightarrow k = \frac{1}{2}\ln 4 = \ln 4^{\frac{1}{2}} = \ln 2$$

$\rightarrow$ initial amount

$$C = \frac{400}{e^{3\ln 2}} = \frac{400}{e^{\ln 2^3}} = \frac{400}{8} = \boxed{50 \text{ dogs}}$$

2.) (10 pts.) Compute S_4, the Simpson's Rule estimate with $n = 4$, for the integral $\int_0^8 \sqrt{x^2+1}\,dx$.



$f(x) = \sqrt{x^2+1}$, $n = 4$, $h = \frac{8-0}{4} = 2$ ~~so~~

$$S_4 = \frac{h}{3}\left[f(x_0) + 4f(x_1) + 2f(x_2) + 4f(x_3) + f(x_4)\right]$$

$$= \frac{2}{3}\left[\sqrt{1} + 4\sqrt{5} + 2\sqrt{17} + 4\cdot\sqrt{37} + \sqrt{65}\right]$$

$$\approx 33.72$$

3.) (10 pts. each) Use any method on the following indefinite integrals.

a.) $\int 3^{4x+7}\,dx$ (Let $u = 4x+7 \rightarrow du = 4\,dx \rightarrow \frac{1}{4}du = dx$)

$$= \frac{1}{4}\int 3^u\,du = \frac{1}{4}\cdot\frac{1}{\ln 3}3^u + c$$

$$= \frac{1}{4\ln 3}\cdot 3^{4x+7} + c$$

b.) $\int x^2 e^x\,dx$ (Let $u = x^2$, $dv = e^x dx$, $du = 2x\,dx$, $v = e^x$)

$$= x^2e^x - 2\int xe^x\,dx$$ (Let $u = x$, $dv = e^x dx$, $du = dx$, $v = e^x$)

$$= x^2e^x - 2\left[xe^x - \int e^x dx\right]$$

$$= x^2e^x - 2xe^x + 2e^x + c$$

c.) $\int \frac{x^2}{x^3-8}\,dx$ (Let $u = x^3-8 \rightarrow du = 3x^2dx \rightarrow \frac{1}{3}du = x^2dx$)

$$= \frac{1}{3}\int \frac{1}{u}\,du$$

$$= \frac{1}{3}\cdot \ln|u| + c$$

$$= \frac{1}{3}\ln|x^3-8| + c$$

d.) $\int \sin^2(4x)\,dx = \int \frac{1}{2}\left(1-\cos 2(4x)\right)dx$

$= \frac{1}{2}\int \left(1-\cos 8x\right)dx$

$= \frac{1}{2}\left(x - \frac{1}{8}\sin 8x\right) + c$

e.) $\int \sec^7 x \tan^3 x\,dx = \int \sec^6 x \cdot \tan^2 x \cdot \sec x \tan x\,dx$

$= \int \sec^6 x \cdot \left(\sec^2 x - 1\right) \cdot \sec x \tan x\,dx$

$= \int \left(\sec^8 x - \sec^6 x\right) \cdot \sec x \tan x\,dx$

(Let $u = \sec x \rightarrow du = \sec x \tan x\,dx$)

$= \int \left(u^8 - u^6\right)du = \frac{1}{9}u^9 - \frac{1}{7}u^7 + c$

$= \frac{1}{9}\sec^9 x - \frac{1}{7}\sec^7 x + c$

f.) $\int \frac{x^3}{x^2-4}\,dx$

$$\begin{array}{rl} & x \\ x^2-4 & \overline{)\,x^3} \\ & -(x^3-4x) \\ \hline & 4x \end{array}$$

$= \int \left[x + \frac{4x}{x^2-4}\right]dx$

$= \frac{x^2}{2} + 2\ln|x^2-4| + c$

g.) $\int \frac{x^3}{\sqrt{4-x^2}}\,dx$ (Let $x = 2\sin\theta \rightarrow dx = 2\cos\theta\, d\theta$)

$$= \int \frac{8\sin^3\theta}{\sqrt{4-4\sin^2\theta}} \cdot 2\cos\theta\, d\theta$$

$$= \frac{16}{2}\int \frac{\sin^3\theta\cdot\cos\theta}{\sqrt{\cos^2\theta}}\,d\theta = 8\int \frac{\sin^3\theta\cdot\cancel{\cos\theta}}{\cancel{\cos\theta}}\,d\theta$$

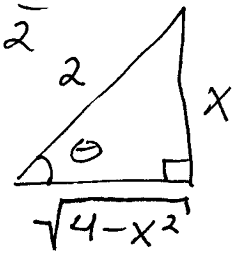
$$= 8\int \sin\theta(\sin^2\theta)\,d\theta = 8\int\sin\theta(1-\cos^2\theta)\,d\theta$$

$$= 8\int(\sin\theta - \cos^2\theta\sin\theta)\,d\theta$$

$$= 8\left(-\cos\theta + \frac{1}{3}\cos^3\theta\right) + C$$

$$= -8\cdot\frac{\sqrt{4-x^2}}{2} + \frac{8}{3}\left(\frac{\sqrt{4-x^2}}{2}\right)^3 + C$$

$\sin\theta = \frac{x}{2}$



h.) $\int \frac{1}{\cos x\cdot(\sin^2 x+1)}\,dx$ (HINT : First multiply and divide by $\cos x$.)

$$= \int\frac{\cos x}{\cos^2 x\cdot(\sin^2 x+1)}\,dx = \int\frac{\cos x}{(1-\sin^2 x)(\sin^2 x+1)}\,dx$$

(Let $u = \sin x \rightarrow du = \cos x\, dx$)

$$= \int\frac{1}{(1-u^2)(u^2+1)}\,du = \int\frac{1}{(1-u)(1+u)(u^2+1)}\,du$$

$$= \int\left[\frac{A}{1-u} + \frac{B}{1+u} + \frac{Cu+D}{u^2+1}\right]du$$

$\{ A(1+u)(u^2+1) + B(1-u)(u^2+1) + (Cu+D)(1-u)(1+u) = 1$

Let $u=1$: $4A = 1 \rightarrow A = 1/4$

Let $u=-1$: $4B = 1 \rightarrow B = 1/4$

Let $u = i$: $(Ci+D)(1-i^2) = 1 \rightarrow$

$(2C)i + (2D) = (0)i + (1) \rightarrow C = 0,\ D = 1/2 \}$

$$= \int\left[\frac{1/4}{1-u} + \frac{1/4}{1+u} + \frac{1/2}{u^2+1}\right]du$$

$$= -\frac{1}{4}\ln|1-u| + \frac{1}{4}\ln|1+u| + \frac{1}{2}\arctan u + C$$

$$= -\frac{1}{4}\ln|1-\sin x| + \frac{1}{4}\ln|1+\sin x| + \frac{1}{2}\arctan(\sin x) + C.$$

The following EXTRA CREDIT problem is OPTIONAL. It is worth 10 points.

1.) Integrate : $\int \frac{x^3 e^{x^2}}{(x^2+1)^2}\,dx$ $= \int \frac{x}{(x^2+1)^2} \cdot x^2 e^{x^2}\,dx$

(Let $u = x^2 e^{x^2}$, $dv = \frac{x}{(x^2+1)^2}\,dx$,

$du = (x^2 \cdot 2x e^{x^2} + 2x e^{x^2})\,dx$

$= 2x(x^2+1)e^{x^2}\,dx$,

$v = \frac{-1}{2} \cdot \frac{1}{x^2+1}$)

$= \frac{-1}{2} \cdot \frac{x^2}{x^2+1} \cdot e^{x^2} + \int x e^{x^2}\,dx$

$= \frac{-1}{2} \cdot \frac{x^2}{x^2+1} \cdot e^{x^2} + \frac{1}{2} e^{x^2} + C$