Problem 1. Part (a) is "washers", part (b) is "cylindrical shells".

$$(a)\ V = \int_{-1}^{1} \pi(1-y^2)^2\, dy = 2\pi(1 - 2/3 + 1/5)$$

$$(b)\ V = \int_{0}^{1} 2\pi\ y(1-y^2)\, dy = \pi/2$$

Problem 3. You need to do two integration by parts for both parts.

$$(a)\ \int e^x\, sin(2x)\, dx = (e^x\, sin(2x) - 2e^x\, cos(2x))/5 + C$$

$$(b)\ \int ln^2(x)\, dx = xln^2(x) - 2\int ln\,(x)\, dx = xln^2(x) - 2xln(x) + 2x + C$$

Problem 4.

$$(a)\ \int (sec^2(x) - 1)\, dx = tan\,(x) - x + C$$

$$(b)\ \frac{1}{4}\int (1 - cos\,(4x))(1 + cos\,(4x))\, dx = \frac{1}{8}\int (1 - cos\,(8x))\, dx = x/8 - sin\,(8x)/64\, + C$$

$$(c)\ \int \sqrt{2sin^2(x)}\, dx = -\sqrt{2}cos\,(x) + C$$

Problem 5.

$$(a)\ (x/2 = tan\,(\theta));\quad \int = \frac{1}{2}tan^{-1}(x/2) + C$$

$$(b)\ (x = \frac{1}{3}tan\,(\theta));\quad \int = \frac{1}{3}lm\,|3x + \sqrt{1+9x^2}| + C$$

$$(c)\ (x/5 = sec\,(\theta));\qquad \int = 5\int tan^2(\theta)\,d\theta = \sqrt{x^2-25} - 5sec^{-1}(x/5) + C$$

Problem 6.

(a) Using the Limit Comparison test:

$$\lim_{x\to\infty}(1/\sqrt{x^4-1})/(1/x^2) = \lim_{x\to\infty}\sqrt{1+1/(x^4-1)} = 1$$

The integral converges as

$$\int_2^\infty \frac{1}{x^2}\,dx$$

converges.

(b) Substitute: $u = \sqrt{x}$, that is $x = u^2$, so $dx = 2udu$. Then

$$\int_1^N e^{-\sqrt{x}}dx = \int_1^M 2ue^{-u}\,du =$$

$$= 2(e^{-1} - Me^{-M}) + 2\int_1^M e^{-u}du \to 4e^{-1}$$

as $M = \sqrt{N} \to \infty$, where the second equality is obtained by a partial integration. The improper integral obtained converges, thus the original integral converges as well.

Problem 7.

(a) Converges; use $ln\,(n) \le n^{1/4}$ for large n and compare the terms to $n^{-5/4}$.

(b) Diverges; as $\sqrt[n]{n} \to 1$, by the n-th term test.

Problem 8.

$$(a)\ |\overrightarrow{AB}| = \sqrt{3},\ \ |\overrightarrow{Ac}| = 2$$

$$(b)\ cos\,(\alpha) = -1/\sqrt{3}$$

$$(c)\ d = |\overrightarrow{AB}|\,sin\,(\alpha) = \sqrt{2}$$

using the fact that: $sin\,(\alpha) = \sqrt{1-cos^2(\alpha)} = \sqrt{2/3}$.