3. (15 points) The lines given paramerically by

$$(x, y, z) = (7 + 2t, -1 - t, -2t), \quad -\infty < t < \infty$$

and

$$(x, y, z) = (4 - s, -1 + 2s, 2 + 2s), \quad -\infty < s < \infty$$

intersect at the point $\langle x, y, z\rangle = \langle 3, 1, 4\rangle$. Find an equation for the **plane** which contains both lines.

SOLUTION: A normal vector $\vec{v}$ to the plane is the cross product of the vector multiplied by t in the first line and the vector multiplied by s in the other line:

$$\vec{v} = \begin{vmatrix} \vec{i} & \vec{j} & \vec{k} \\ 2 & -1 & -2 \\ -1 & 2 & 2 \end{vmatrix} = -6\vec{i} - 6\vec{j} + 3\vec{k}.$$

Divide $\vec{v}$ by 3. So the plane is given by the equation $-2(x-3) - 2(y-1) + (z-4) = 0$, or

$$-2x - 2y + z = -4.$$

4. (15 points) For the function $f(x, y) = e^{-2y}\sin 2x$, find the **second partial derivatives**

$$f_{xx} = \frac{\partial^2 f}{\partial x^2}$$

and

$$f_{yy} = \frac{\partial^2 f}{\partial y^2}$$

.

SOLUTION: $f_x = 2e^{-2y}\cos 2x$, so $f_{xx} = -4e^{-2y}\sin 2x$. For the y ipartial derivatives, $f_y = -2e^{-2y}\sin 2x$ and $f_{yy} = +4e^{-2y}\sin 2x$.

5. (10 points) Suppose $z = f(x, y)$ is a function with partial derivatives $f_x(3, 1) = 5$ and $f_y(3, 1) = 2$. If x and y are both functions of t: $x = 5 - 2t$ and $y = 2 + t - 2t^2$, find

$$\frac{dz}{dt}$$

at $t = 1$.

SOLUTION: $x = g(t) = 5 - 2t$ so $x = g(1) = 3$ at $t = 1$, and $fracdxdt = g'(t) = -2$. Meanwhile, $y = h(t) = 2 + t - 2t^2$, so $y = h(1) = 1$ at $t = 1$. $\frac{dy}{dt} = h'(t) = 1 - 2t$, so $h'(1) = -1$. The **chain rule** says that

$$\frac{dz}{dt} = f_x(3, 1)g'(1) + f_y(3, 1)h'(1) = (5)(-2) + (2)(-3) = -16.$$

6. (15 points) The point $\langle x, y, z \rangle = \langle 2, 1, 0 \rangle$ lies on the surface S:

$$x^2 - y^2 + xz + xy - 4z^2 = 5.$$

Find the equation of the **tangent plane** to the surface S at $\langle 2, 1, 0 \rangle$, in the form $ax + by + cz = d$.

SOLUTION: The normal vector to the tangent plane to the surface $g(x, y, z) = 0$ is the gradient $\vec{\nabla} g$. But $g_x = 2x + z + y = 2 + 0 + 1 = 3$; $g_y = -2y + x = -2 + 2 = 0$; and $g_z = x - 8z = 2 - 0 = 2$. So $\vec{\nabla} g(2, 1, 0) = \langle 3, 0, 2 \rangle$. The equatopn of the tangent plane is $3(x - 2) + 0(y - 1) + 2(z - 0) - 0$ or equivalently:

$$3x + 2z = 6.$$

7. **(15 points) (a)** Find the **gradient** of the function $f(x, y, z) = e^z \ln(x + 2y)$ at the point $\langle x, y, z \rangle = \langle e, 0, 1 \rangle$. **(b)** Find the **directional derivative** of f at the point $\langle e, 0, 1 \rangle$ in the direction

$$\vec{u} = \frac{1}{3}\left(\vec{i} - 2\vec{j} + 2\vec{k}\right).$$

(15 points) **SOLUTION:** $f_x = \frac{e^z}{x+2y} = 1$; $f_y = 2 frac e^z x + 2y = 2$; and $f_z = e^z \ln(x + 2y) = e$. So the gradient is

$$vec \nabla f(e, 0, 1) = \vec{i} + 2\vec{j} + e\vec{k}.$$