Problem 1. TRUE FALSE QUESTIONS. NO PARTIAL CREDIT. CIRCLE TRUE OR FALSE. IF YOU THINK A QUESTION DOESN'T MAKE SENSE, CIRCLE FALSE

(a)

$$\int_0^{\pi}\int_0^1 2e^{-\cos x} y \sin x dy dx = e^{-1} - \frac{1}{e^{-1}} \quad TRUE/FALSE$$

FALSE

(b) If $f(x, y, z)$ is a function, then curl(∇f) is a vector field. TRUE/FALSE
TRUE

(c)

$$\int_1^2 \frac{2\pi}{x + x^5} dx = \int_1^2 \frac{4\pi}{2x} dx + \int_1^2 \frac{2\pi}{x^5} dx = -4\pi \int_2^1 \frac{1}{2x} - 2\pi \int_2^1 x^{-5} dx$$

TRUE/FALSE

FALSE

(d) The directional derivative of a function in a direction tangent to its level curve is always perpendicular to the gradient of the function.

TRUE/FALSE

FALSE

(e) The triple product $\vec{a} \cdot (\vec{b} \times \vec{c})$ is the volume of the parallelepiped with sides $\vec{a}, \vec{b}, \vec{c}$ and hence must always be greater than or equal to zero.

TRUE/FALSE

FALSE

(f) If a surface lies completely below the plane $z = 0$ then it has negative area.

TRUE/FALSE

FALSE

WORKSPACE:

Problem 2. NO PARTIAL CREDIT

(a) Let $f(x,y) = xe^{xy}, x(s,t) = st, y(s,t) = s + t^2, h(s,t) = f(x(s,t), y(s,t))$
Find $\frac{\partial h}{\partial t}(1,1)$

(b) Use differentials to estimate $\sqrt{16.01} \cdot \sqrt{15.98}$ [Hint: Consider the function $f(x,y) = \sqrt{x}\sqrt{y}$]

(c) Find a parametric or vector equation for the line through the point $(1,2,3)$ parallel to the vector $< 2,1,3 >$.

(d) Find an equation for the plane through the points $A(1,1,1), B(1,2,3), C(-1,-1,-1)$.

(e) Find the length of the curve $\vec{r}(t) =< t^3/3, 2t, t^2 >, 0 \le t \le 2$.

(f) $\frac{\partial^{51}[y^3 \tan(\ln x) + e^{x+y}]}{\partial x \partial y^{47} \partial x^3} =$

ANSWERS:
a:
b:
c:
d:
e:
f:

WORKSPACE:

a)

$h_t = f_x x_t + f_y y_t = [e^{xy} + xye^{xy}]s + x^2 e^{xy} 2t = [e^2 + 2e^2]1 + e^2 2 = 5e^2$

b) $f(x,y) = \sqrt{x}\sqrt{y}$ $f(16 + .1, 16 - .2) = f(16,16) + f_x(16,16).01 + f_y(16,16)(-.02) = 16 - \frac{1}{8}.01 = 16 - \frac{1}{800} = 15.99875$

c) $\vec{r}(t) =< 1,2,3 > +t < 2,1,3 >=< 1+2t, 2+t, 3+3t >$,

$$\begin{aligned} x &= 1 + 2t \\ y &= 2 + t \\ z &= 3 + 3t \end{aligned}$$

d) normal vector:
$N = AB \times AC =< 0,1,2 > \times < -2,-2,-2 >=< 2,-4,2 >$
$2(x-1) - 4(y-1) + 2(z-1) = 0.$
$2x - 4y + 2z = 0$
$x - 2y + z = 0.$

WORKSPACE PROBLEM 2

e) $\int_0^2 \sqrt{(t^2)^2 + 2^2 + (2t)^2} dt = \int_0^2 \sqrt{t^4 + 4 + 4t^2} dt = \int_0^2 (t^2 + 2t) dt = 8/3 + 4 = 20/3$

f) e^{x+y}. [Do $y-$ derivatives first]

Problem 3. YOU CAN MAKE 0, 2 OR 4 POINTS ON EACH PART. 2 POINTS WILL ONLY BE AWARDED IN CASE OF A SMALL MISTAKE. THE THREE PARTS OF THE PROBLEM ARE UNRELATED

(a) Find the maximum value of $f(x,y) = xy$ subject to the constraint $x^2 + y^2 = 1$.

(b) Find $\int\int_D e^{x^2+y^2} dA$ where D is the region in the first quadrant between the circles centered at the origin of radius 2 and 3.

(c) Find $\int_C \vec{F} \cdot d\vec{r}$ where C is the curve $\vec{r}(t) =< \sin^3(\pi t/2), t^3, e^t >$, from the point $(0,0,1)$ to the point $(1,1,e)$. $\vec{F} =< x + ye^z, xe^z, xye^z + z^2 >$.

ANSWERS:

a:

b:

c:

WORKSPACE:

a) $g(x,y) = x^2 + y^2$.

$$\begin{aligned} \nabla f &= \nabla g \\ y &= 2\lambda x \\ x &= 2\lambda y \\ x, y &\neq 0 \\ x^2 &= y^2 \\ (x,y) &= (\pm 1/\sqrt{2}, \pm 1/\sqrt{2}) \end{aligned}$$

Max value is 1/2

b)

$$\int_0^{\pi/2} \int_2^3 e^{r^2} r dr d\theta = \pi/4[e^9 - e^4]$$

c) Potential $f = x^2/2 + xye^z + z^3/3$. $f(1,1,e) - f(0,0,1) = [1/2 + e^e + e^3/3] - [1/3] = e^3/3 + e^e + 1/6$

Problem 4. IF YOUR ANSWER IS CORRECT YOU GET FULL SCORE. IF ANSWER IS WRONG YOU CAN GET PARTIAL CREDIT IF YOU EXPLAIN YOURSELF CAREFULLY

(a) Find $\int_C -y^5/5dx + (x^5/5 + 2x^3y^2/3)dy$ where C is the circle $x^2 + y^2 = 1$ counterclockwise. [Hint: You can use Green's Theorem.]

(b) Find the area of the surface parametrized by $\vec{r}(u,v) =< u, uv, v >$, $u^2 + v^2 \leq 1$.

(c) Let S be the surface $z = x, x^2 + y^2 \leq 1$. Find $\int\int_S (x^2 + y^2)dS$

(d) Let S be the surface $z = x + y, 0 \leq x \leq 1, 0 \leq y \leq 1$. Find the upward flux of the vector field $\vec{F} =< z, x, y >$ across S.

ANSWERS:

a:

b:

c:

d:

WORKSPACE:

a)$\int\int x^4 + 2x^2y^2 + y^4 = \int\int (x^2 + y^2)^2 = \int_0^1 \int_0^{2\pi} r^5 d\theta dr = 2\pi/6 = \pi/3.$

b)

$$\begin{aligned}
r_u &= < 1, v, 0 > \\
r_v &= < 0, u, 1 > \\
r_u \times r_v &= < v, -1, u > \\
|r_u \times r_v| &= \sqrt{1 + u^2 + v^2} \\
\int_0^{2\pi} \int_0^1 r\sqrt{1 + r^2} dr d\theta &= 2\pi/3[(1 + 1)^{3/2} - 1] \\
&= 2\pi/3[2\sqrt{2} - 1]
\end{aligned}$$

WORKSPACE PROBLEM 4

c)

$$\begin{aligned}
\vec{r}(x,y) &= <x,y,x> \\
r_x &= <1,0,1> \\
r_y &= <0,1,0> \\
r_x \times r_y &= <-1,0,1> \\
|r_x \times r_y| &= \sqrt{2} \\
\int_0^{2\pi}\int_0^1 r^3\sqrt{2}drd\theta &= \sqrt{2}\pi/2
\end{aligned}$$

d)

$$\begin{aligned}
r &= <x,y,x+y> \\
r_x &= <1,0,1> \\
r_y &= <0,1,1> \\
r_x \times r_y &= <-1,-1,1> \\
F(r(x,y)) &= <x+y,x,y> \\
F \cdot (r_x \times r_y) &= -x-y-x+y = -2x \\
\int\int &= -1
\end{aligned}$$

Problem 5. IF YOUR ANSWER IS CORRECT YOU GET FULL SCORE. IF YOUR ANSWER IS WRONG YOU CAN GET PARTIAL CREDIT IF YOU EXPLAIN YOURSELF CAREFULLY

(a) Find the flux of the vector field $\vec{F} = < y^3 \sin z, 3y, y^5 \ln(x^2+1) >$ out of the unit sphere $x^2 + y^2 + z^2 = 1$.

(b) Let S be the hemisphere $x^2 + y^2 + z^2 = 1, z \geq 0$ oriented upward. Let $\vec{F} = < -ye^z, x \cos z, e^{x^2y^3} >$. Find the integral $\int\int_S$ curl $\vec{F} \cdot d\vec{S}$.

ANSWERS:
a:
b:

WORKSPACE:

a)

$$\begin{aligned} divF &= 1 \\ \int &= 4\pi/3 \end{aligned}$$

b) C is given by $r(t) = < \cos t, \sin t, 0 >$

$$\begin{aligned} r' &= < -\sin t, \cos t, 0 > \\ F(r(t)) &= < -y, x, e^{x^2y^3} > = < -\sin t, \cos t, ... > \\ F \cdot r' &= \sin^2 t + \cos^2 t = 1 \\ \int &= 2\pi \end{aligned}$$