5.7 EXERCISES

5.1 Discrete Stochastic Models

5-1 Derive Equation (5.19a).

5-2 Show that the right hand sides of Equations (5.19a) and (5.19b) are equal.

✓ 5-3 For the model of Example 5.1, calculate $Cov(Z^1_{x:\overline{3}|}, {}_3|Z_x)$.

✓ 5-4 A 2-year discrete term insurance is issued to (x) at interest rate $i=0$. Given $q_x = .50$ and $Var(Z^1_{x:\overline{2}|}) = .1771$, calculate q_{x+1}.

✓ 5-5 A one-year endowment insurance issued to (x) pays b at the end of the year if (x) fails in $(x, x+1]$ and pays e at the end of the year if (x) survives to $x+1$. Let $Z^*_{x:\overline{1}|}$ denote the present value of benefit for this insurance. Show that $Var(Z^*_{x:\overline{1}|}) = v^2(b-e)^2 \cdot p_x \cdot q_x$.

5-6 Show that $A_x = v \cdot q_x + v \cdot p_x \cdot A_{x+1}$.

5-7 Calculate the value of $Var(Z_{51})$, given the following values:

$$A_{51} - A_{50} = .004 \qquad i = .02$$

$$^2A_{51} - {}^2A_{50} = .005 \qquad p_{50} = .98$$

✓ 5-8 Let Z_1 denote the present value random variable for a 25-year term insurance of amount 7, and let Z_2 denote the present value random variable for a 10-year term insurance of amount 4 that is deferred for 25 years. Calculate the value of $Var(Z_1 + Z_2)$, given the following values:

$$E[Z_1] = 2.80 \qquad Var(Z_1) = 5.76$$

$$E[Z_2] = 0.12 \qquad Var(Z_2) = 0.10$$

✓ 5-9 Let $Z = 1000Z_{x:\overline{n}|}$ denote the present value random variable for an n-year endowment insurance of amount 1000. Calculate the value of $Var(Z)$, given the following values:

$$^2A_x = .2196 \qquad A_{x:\overline{n}|} = .7896$$

$$^2A_{x+n} = .2836 \qquad ^2A^{\;\;1}_{x:\overline{n}|} = .5649$$

✓ 5-10 A special ten-year endowment insurance pays 1000 for survival to time 10, or a benefit at the end of the year of failure, whichever occurs first. Let Z_1 denote the present value random variable for this insurance if the failure benefit is $1000 \cdot {}_{10}E_x$, let Z_2 denote the present value random variable if the failure benefit is $750 \cdot {}_{10}E_x$, and let Z_3 denote the present value random variable if the failure benefit is $500 \cdot {}_{10}E_x$. Given also that $A_{x:\overline{10}|} = .57$ and $\frac{E[Z_1]}{E[Z_2]} = 1.005$, calculate the value of $E[Z_3]$.

5.2 Group Deterministic Approach

5-11 Show that ${}_n|A_x = {}_nE_x \cdot A_{x+n}$.

5-12 Use the group deterministic approach to interpret each of the following expected present value functions as net single premiums.

(a) $A^1_{x:\overline{n}|}$ (b) ${}_n|A_x$ (c) $A_{x:\overline{n}|}$

✓ 5-13 Calculate A_{77}, given that $A_{76} = .800$, $v \cdot p_{76} = .90$, and $i = .03$.

✓ 5-14 A special n-year endowment contract, with net single premium of 600, pays 1000 for survival to time n but pays only the net single premium for failure before time n. Given that $A_{x:\overline{n}|} = .8($ find the value of ${}_nE_x$.

5.3 Continuous Stochastic Models

5-15 A benefit of 50 is paid at the precise time of failure t. The PDF of T, the random variable for time of failure, is given by

$$f_T(t) = \begin{cases} \frac{t}{5000} & \text{for } 0 < t \leq 100 \\ 0 & \text{otherwise} \end{cases}$$

Find the APV of the benefit using force of interest $\delta = .10$.

5-16 (a) Define random variable $\bar{Z}^1_{x:\overline{n}|}$ by reference to Equation (5.7).

(b) Give expressions for the first and second moments of $\bar{Z}^1_{x:\overline{n}|}$.

5-17 (a) Define random variable $_{n|}\bar{Z}_x$ by reference to Equation (5.11).

(b) Give expressions for the first and second moments of $_{n|}\bar{Z}_x$.

(c) Show that $_{n|}\bar{A}_x + \bar{A}^1_{x:\overline{n}|} = \bar{A}_x$.

5-18 The random variable $_{n|}\bar{Z}_x$ has a mixed distribution. If the future lifetime random variable T_x has PDF given by $(110-x)^{-1}$, describe the discrete part of the mixed distribution of $_{20|}\bar{Z}_{40}$.

5-19 Calculate the value of $Var(\bar{Z}_{x:\overline{n}|})$, given the following values:

$$E[\bar{Z}^1_{x:\overline{n}|}] = .23 \qquad v^n = .20 \qquad Var(\bar{Z}^1_{x:\overline{n}|}) = .08 \qquad {}_np_x = .50$$

5-20 Find an expression for $Var(\bar{Z}_x)$ when the age-at-failure random variable X has an exponential distribution with parameter λ.

5-21 Find an expression for $_{n|}\bar{A}_x$ when the age-at-failure random variable X has an exponential distribution with parameter λ.

5-22 Let the age-at-failure random variable X have a uniform distribution with $\omega = 110$. Let $f_Z(z)$ denote the PDF of the random variable $\bar{Z}_{40}$. Calculate the value of $f_Z(.80)$, given also that $\delta = .05$.

5-23 Let $M_{T_x}(r)$ denote the moment generating function of the random variable T_x. Show that $\bar{A}_x = M_{T_x}(-\delta)$.

5.4 Contingent Payment Models with Varying Payments

5-24 Let $\bar{B}_x$ denote the present value random variable for a continuously increasing contingent payment contract with benefit $b_t = 1+.10t$ for failure at time t. Given also that $v^t = (1+.10t)^{-2}$ and the PDF of T_x, the random variable for time of failure, is given by $f(t) = .02$ for $0 \leq t \leq 50$, find the variance of $\bar{B}_x$.

5-25 (a) Show that $(IA)_x = A_x + {}_1E_x \cdot (IA)_{x+1}$.

(b) Calculate the value of $(IA)_{36}$, given the following values:

$(IA)_{35} = 3.711$	$A_{35:\overline{1}|} = .9434$
$A_{35} = .1300$	$p_{35} = .9964$

5-26 A three-year warranty on a new computer will pay $b_k = 500(4-k)$ for failure in the k^{th} year, for $k = 1,2,3$. The benefit is paid at the end of the year of failure. The age-at-failure random variable follows a Pareto distribution (see Section 12.1.2) with mean 4 and variance 48. At annual interest rate $i = .05$, find the APV of the warranty payment.

5-27 A student loan of amount 10,000 is amortized over 20 years by continuous payment at $\delta = .08$. The loan is subject to default at constant force of default $\lambda = .01$. A government agency guarantees the outstanding balance of the loan in case of default. Using force of interest .05, calculate the APV of the guarantee.