UNIT REVIEW QUESTIONS

The following questions are taken from SOA exams given prior to the year 2000 syllabus. Questions under the new syllabus may be quite different. Nevertheless, these old questions remain an excellent source for mastering the basic theory.

1. You are given the survival function $s_X(x) = \frac{\sqrt{100 - x}}{10}$ for $0 \le x \le 100$. Calculate $F_X(75)$, $f_X(75)$, and $\mu(75)$.

	$F_X(75)$	$f_X(75)$	$\mu(75)$
(A)	0.2	0.02	0.04
(B)	0.5	0.01	0.02
(C)	0.5	0.01	0.04
(D)	0.2	0.02	0.02
(E)	0.5	0.02	0.04

2. You are given: ${}_1|q_{x+1} = 0.095$ ${}_2|q_{x+1} = 0.171$ $q_{x+3} = 0.200$

Calculate $q_{x+1} + q_{x+2}$.

(A) 0.15 (B) 0.20 (C) 0.25 (D) 0.27 (E) 0.30

3. You are given the following information:

(i) K = probability that (x) dies in the first $\frac{1}{3}$ of the year, under the Balducci assumption.

(ii) L = probability that (x) dies in the last $\frac{2}{3}$ of the year, under the uniform distribution of deaths assumption.

(iii) $\ell_x = 9$

(iv) $\ell_{x+1} = 6$

Calculate $K + L$.

(A) $\frac{19}{63}$ (B) $\frac{1}{3}$ (C) $\frac{23}{63}$ (D) $\frac{11}{28}$ (E) $\frac{23}{56}$

4. If K is the curtate future lifetime of (96), calculate $Var(K)$, given the following life table:

x	ℓ_x
96	180
97	130
98	73
99	31
100	0

(A) .39 (B) .53 (C) .91 (D) 1.11 (E) 1.50

5. Mortality follows de Moivre's Law and $\mathring{e}_{16} = 42$. Calculate $Var\left(T(16)\right)$.
(Answer to nearest integer)

(A) 595 (B) 588 (C) 505 (D) 472 (E) 300

6. You are given the survival function $s_X(x) = \frac{9000 - 10x - x^2}{9000}$ for $0 \le x \le 90$.
Calculate the exact value of $q_{50} - \mu(50)$.

(A) $\frac{-1}{600}$ (B) $\frac{-1}{6000}$ (C) 0 (D) $\frac{1}{6000}$ (E) $\frac{1}{600}$

7. If $\ell_x = 10(100 - x)^2$, calculate $Var(T(x))$.

(A) $(100 - x)^2/18$ (B) $(100 - x)/3$ (C) $(100 - x)^3/6$ (D) $(100 - x)^2/6$ (E) $(100 - x)^2/3$

8. You are given: $\mu(x) = \frac{x}{100}$. Calculate ${}_{20|10}q_5$.

(A) $\frac{e^5 - 1}{e^6}$ (B) $\frac{e^4 - 1}{e^6}$ (C) $\frac{e^3 - 1}{e^6}$ (D) $\frac{e^2 - 1}{e^6}$ (E) $\frac{e - 1}{e^6}$

9. You are given the following extract from a 2-year select-and-ultimate mortality table:

$[x]$	$\ell_{[x]}$	$\ell_{[x]+1}$	ℓ_{x+2}	$x+2$
92	...	...	6300	94
93	...	...	5040	95
94	...	...	3024	96

The following relationships hold for all x:

(i) $2 \cdot q_{[x]+1} = 3 \cdot q_{[x+1]}$ (ii) $3 \cdot q_{x+2} = 4 \cdot q_{[x+1]+1}$

Calculate $\ell_{[94]}$.

(A) 4800 (B) 4851 (C) 4985 (D) 5103 (E) 5166

10. Calculate ${}_{20}p_{40}$, given that $\mu(x) = kx$ for all $x > 0$, and ${}_{10}p_{35} = .81$.

(A) .36 (B) .41 (C) .45 (D) .59 (E) .66

11. You are given ${}_t|q_x = 0.10$, for $t = 0, 1, \ldots 9$. Calculate ${}_2p_{x+5}$.

(A) 0.40 (B) 0.60 (C) 0.72 (D) 0.80 (E) 0.81

12. You are given:

(i) Deaths are uniformly distributed over each year of age.

(ii)

x	ℓ_x
35	100
36	99
37	96
38	92
39	87

Which of the following are true?

I. ${}_{1|2}q_{36} = .091$
II. $m_{37} = .043$
III. ${}_{.33}q_{38.5} = .021$

(A) I and II only (B) I and III only (C) II and III only (D) I, II and III
(E) The correct answer is not given by (A), (B), (C) or (D)

13. You are given:

(i) Deaths are uniformly distributed over each year of age.
(ii) $\mu(45.5) = .50$

Calculate $\mathring{e}_{45:\overline{1}|}$.

(A) .40 (B) .50 (C) .60 (D) .70 (E) .80

14. You are given:

(i) $q_{70} = .040$
(ii) $q_{71} = .044$
(iii) Deaths are uniformly distributed over each year of age.

Calculate $\mathring{e}_{70:\overline{1.5}|}$.

(A) 1.435 (B) 1.445 (C) 1.455 (D) 1.465 (E) 1.475

15. You are given that $q_x = .25$.

Based on the constant force of mortality assumption, the force of mortality is $\mu_x^A(s)$, $0 < s < 1$. Based on the uniform distribution of deaths assumption, the force of mortality is $\mu_x^B(s)$, $0 < s < 1$. Calculate the smallest s such that $\mu_x^B(s) \geq \mu_x^A(s)$.

(A) .4523 (B) .4758 (C) .5001 (D) .5242 (E) .5477

16. For a 2-year select and ultimate mortality table, you are given:

(i) $q_{96} = .350, q_{97} = .475, q_{98} = .675$
(ii) $q_{[x]} = .5q_x$ for all x
(iii) $q_{[x]+1} = .5q_{x+1}$ for all x
(iv) $\ell_{[96]} = 10{,}000$

Calculate $\ell_{[97]}$.

(A) 4047 (B) 4076 (C) 4094 (D) 4136 (E) 4158

17. For a two-year select-and-ultimate mortality table, you are given:

(i) $q_{[x]} = (1 - 2k)q_x$
(ii) $q_{[x]+1} = (1 - k)q_{x+1}$
(iii) $\ell_{[32]} = 90$
(iv) $\ell_{32} = 100$
(v) $\ell_{33} = 90$
(vi) $\ell_{34} = 63$

Calculate $\ell_{[32]+1}$.

(A) 82 (B) 83 (C) 84 (D) 85 (E) 86

18. You are given:

(i) $\mathring{e}_{x:\overline{1}|} = F$ under the uniform distribution of deaths assumption
(ii) $\mathring{e}_{x:\overline{1}|} = G$ under the Balducci assumption
(iii) $q_x = .1$

Calculate $1000(F - G)$.

(A) .00 (B) .24 (C) 1.00 (D) 1.76 (E) 2.52

19. A survival function, $s_X(x)$, is defined as follows:

$$s_X(x) = \left(1 - \frac{x}{\omega}\right)^r \quad 0 \le x < \omega, \quad r > 0$$

For age y, $0 \le y < \omega$, you are given:

(i) $\mu_y = .1$
(ii) $\mathring{e}_y = 8.75$

Calculate r.

(A) 1 (B) 3 (C) 5 (D) 7 (E) 9

20. You are given that $q_x = .1200$.

Which of the following are true?

I. ${}_{1/3}q_{x+1/2} = .0426$ under the uniform distribution of deaths assumption.
II. ${}_{1/3}q_x = .0435$ under the Balducci assumption.
III. ${}_{1/2}q_x = .0619$ under the constant force of mortality assumption.

(A) I and II only (B) I and III only (C) II and III only (D) I, II and III
(E) The correct answer is not given by (A), (B), (C) or (D)

21. You are given:

(i) $q_x = .04$
(ii) $\mu_x(t) = .04 + .001644t, \ 0 \le t \le 1$
(iii) $\mu_y(t) = .08 + .003288t, \ 0 \le t \le 1$

Calculate q_y.

(A) .0784 (B) .0792 (C) .0800 (D) .0808 (E) .0816

22. For a two-year select-and-ultimate mortality table, you are given:

$[x]$	$\ell_{[x]}$	$\ell_{[x]+1}$	ℓ_{x+2}	$x+2$
30	1000	998	995	32
31	996	994	988	33
32	994	990	982	34
33	987	983	970	35

Which of the following are true?

I. ${}_2p_{[31]} > {}_2p_{[30]+1}$ II. ${}_{1|}q_{[31]} > {}_{1|}q_{[30]+1}$ III. ${}_2q_{[33]} > {}_2q_{[31]+2}$

(A) None (B) I only (C) II only (D) III only
(E) The correct answer is not given by (A), (B), (C), or (D)

23. Assume mortality follows de Moivre's law, for $0 \le x \le \omega$.

Which of the following expressions equal $\mu(x)$?

I. $\dfrac{1}{2\mathring{e}_x}$

II. ${}_n|q_x, \ 0 \le n \le \omega - x - 1$

III. $\dfrac{m_x}{1 + .5m_x}, \ x \le \omega - 1$

(A) I and II only (B) I and III only (C) II and III only (D) I, II and III
(E) The correct answer is not given by (A), (B), (C) or (D)

24. Which of the following can serve as survival functions for $x \geq 0$?

I. $s_X(x) = exp[x - .7(2^x - 1)]$ II. $s_X(x) = \frac{1}{(1+x)^2}$ III. $s_X(x) = exp(-x^2)$

(A) I and II only (B) I and III only (C) II and III only (D) I, II and III
(E) The correct answer is not given by (A), (B), (C) or (D)

25. Deaths are assumed to be uniformly distributed over each year of age. The distribution function of the curtate future lifetime of (x) is

k	0	1	2	3	4	5	6
$F_k(k)$	.05	.15	.35	.60	.75	.85	1.00

Calculate $\mathring{e}_{x:\overline{5}|}$.

(A) 3.100 (B) 3.475 (C) 3.625 (D) 3.850 (E) 4.275

26. You are given:

$s_X(x) = \left(1 - \frac{x}{\omega}\right)^{\alpha}$, $0 \leq x < \omega$, where α is a positive constant.

Calculate $\mu_x \cdot \mathring{e}_x$.

(A) $\frac{\alpha}{\alpha+1}$ (B) $\frac{\alpha\omega}{\alpha+1}$ (C) $\frac{\alpha^2}{\alpha+1}$ (D) $\frac{\alpha^2}{\omega-x}$ (E) $\frac{\alpha(\omega-x)}{(\alpha+1)\omega}$

27. You are given:
(i) $q_{60} = .30$
(ii) $q_{61} = .40$
(iii) f is the probability that (60) will die between ages 60.5 and 61.5 under the uniform distribution of deaths assumption.
(iv) g is the probability that (60) will die between ages 60.5 and 61.5 under the Balducci assumption.

Calculate $10{,}000(g - f)$.

(A) 0 (B) 85 (C) 94 (D) 178 (E) 213

28. You are given $\mu(x) = \sqrt{\frac{1}{80 - x}}$, $0 \leq x < 80$.

Calculate the median future lifetime of (20).

(A) 5.25 (B) 6.08 (C) 8.52 (D) 26.08 (E) 30.00

29. You are given:

(i) ${}_tp_x = (0.8)^t, t \geq 0$

(ii) $\mathring{e}_{x:2} = 6.4$

Calculate T_{x+1}.

(A) 4.5 (B) 7.2 (C) 28.7 (D) 35.9 (E) 44.8

30. For a ten-year select-and ultimate table, you are given:

(i) $t_{[30]+t} = \frac{\sqrt{60}}{9}\left(1 - \frac{t}{100}\right), 0 \leq t < 10$

(ii) $t_{30+t} = \frac{\sqrt{70 - t}}{10}, 10 \leq t \leq 70$

Calculate $\mathring{e}^{\circ}_{[30]}$.

(A) 21.0 (B) 39.0 (C) 42.0 (D) 45.5 (E) 48.5

31. You are given:

(i) $\mu_{35+t} = \mu, 0 \leq t \leq 1$

(ii) $p_{35} = .985$

(iii) μ'_{35+t} is the force of mortality for (35) subject to an additional hazard, $0 \leq t \leq 1$.

(iv) $\mu'_{35+t} = \mu + c, 0 \leq t \leq .5$

(v) The additional force of mortality decreases uniformly from c to 0 between age 35.5 and age 36.

Determine the probability that (35) subject to the additional hazard will not survive to age 36.

(A) $0.015e^{-.25c}$ (B) $0.015e^{.25c}$ (C) $1 - 0.985e^{-.5c}$ (D) $1 - 0.985e^{-.25c}$ (E) $1 - 0.985e^{-.75c}$

32. You are given:

(i) $s_Y(x) = \frac{\sqrt{k^2 - x}}{k}, 0 \leq x \leq k^2, k > 0$

(ii) $\mathring{e}_{40} = 2\mathring{e}_{80}$

Calculate $\mathring{e}_0$.

(A) 10 (B) 20 (C) 30 (D) 40 (E) 50

Use the following information for Questions 33-36.

For a select-and-ultimate mortality table with a one-year select period, you are given:

x	$\ell_{[x]}$	$d_{[x]} = q_{[x]}\ell_{[x]}$	$\mathring{e}_{[x]}$
85	1000	100	5.225
86	850	100	

Assume deaths are uniformly distributed over each year of age.

33. Which of the following is correct?

	$p_{[85]}$	$p_{[86]}$	p_{86}
(A)	10/100	100/100	100/850
(B)	90/100	75/90	75/85
(C)	90/100	75/85	75/90
(D)	90/100	75/85	100/100
(E)	90/100	75/100	75/90

34. Calculate $\int_2^\infty {}_tp_{[85]}\,dt$.

(A) 1.775 (B) 3.245 (C) 3.450 (D) 3.754 (E) 3.950

35. Calculate $\mathring{e}_{[87]}$.

(A) 3.45 (B) 3.75 (C) 4.15 (D) 4.40 (E) 4.60

36. Calculate $\mathring{e}_{[86]}$.

(A) 3.45 (B) 3.75 (C) 4.15 (D) 4.40 (E) 5.00

Use the following information for Questions 37-40.

You are given:

$$s_X(x) = \frac{(10-x)^2}{100}, \quad 0 \le x \le 10$$

37. Calculate the average number of years of future lifetime of the ℓ_1 survivors of the group at age 1.

(A) 2.37 (B) 2.43 (C) 2.70 (D) 2.92 (E) 3.00

38. Calculate the difference between the force of mortality at age 1, and the probability that (1) dies before age 2.

(A) .007 (B) .010 (C) .012 (D) .016 (E) .024

39. Calculate the average number of years lived between ages 1 and 2 by those of the survivorship group who die between those ages.

(A) .461 (B) .473 (C) .484 (D) .490 (E) .500

40. Instead of using $s_X(x)$ within each year of age, use the constant force of mortality assumption to calculate the average number of years lived between ages 1 and 2 by those of the survivorship group who die between those ages.

(A) .461 (B) .480 (C) .490 (D) .500 (E) .508

Use the following information for Questions 41-44.

You are given:

(i) T is the random variable for the future lifetime of (x).
(ii) The p.d.f. of T is $f_T(t) = 2e^{-2t}, t \geq 0$.

41. Calculate $\mathring{e}_x$.

(A) 0.5 (B) 2.0 (C) 10.0 (D) 20.0 (E) 30.0

42. Calculate $Var[T]$.

(A) 0.25 (B) 0.50 (C) 1.00 (D) 2.00 (E) 4.00

43. Calculate $m(x)$, the median future lifetime of (x).

(A) $\frac{e^{-4}}{2}$ (B) $\frac{e^{-2}}{2}$ (C) $\frac{ln\,2}{2}$ (D) $\frac{ln\,4}{2}$ (E) 1

44. Calculate m_x, the central-death-rate at age x.

(A) $\frac{e^{-2}}{2}$ (B) e^{-2} (C) $2e^{-2}$ (D) 1 (E) 2