Often Tested Mortality Models

DeMoivre's Law (DML): $X \sim U(0,\omega) \Rightarrow T = T(x) \sim U(0,\omega - x)$

Usually characterized in problems by

$$\mu(x) = \frac{1}{\omega - x} \quad \text{or} \quad l_x = l_0 \cdot \left(1 - \frac{x}{\omega}\right) \quad \text{or} \quad s(x) = {}_x p_0 = \frac{\omega - x}{\omega} \quad \text{or} \quad F(x) = {}_x q_0 = \frac{x}{\omega}$$

Other Important Formulas

$${}_t p_x = \frac{\omega - x - t}{\omega - x} \qquad {}_t q_x = \frac{t}{\omega - x} = t \cdot q_x$$

$$f_X(x) = \frac{1}{\omega} \qquad f_T(t) = {}_t p_x \cdot \mu(x+t) = \frac{1}{\omega - x} = q_x$$

$${}_{k|t} q_x = {}_t q_x = \frac{t}{\omega - x}$$

$$\overset{0}{e}_x = \frac{\omega - x}{2} = e_x + \frac{1}{2} \qquad Var(T(x)) = \frac{(\omega - x)^2}{12}$$

$$\overset{0}{e}_{x:\overline{n}|} = n - \frac{n^2}{2(\omega - x)} \qquad \overset{0}{e}_{xx} = \frac{\omega - x}{3}$$ (Independent lives (x) and (x))

$$A^1_{x:\overline{n}|} = \frac{1}{\omega - x} \cdot a_{\overline{n}|} \qquad \bar{A}^1_{x:\overline{n}|} = \frac{1}{\omega - x} \cdot \bar{a}_{\overline{n}|}$$

$${}^2A^1_{x:\overline{n}|} = \frac{1}{\omega - x} \cdot \frac{a_{\overline{2n}|}}{s_{\overline{2}|}} \qquad {}^2\bar{A}^1_{x:\overline{n}|} = \frac{1}{\omega - x} \cdot \bar{a}_{\overline{2n}|}$$

$$A_x = \frac{1}{\omega - x} \cdot a_{\overline{\omega - x}|} \qquad \bar{A}_x = \frac{1}{\omega - x} \cdot \bar{a}_{\overline{\omega - x}|}$$

$${}^2A_x = \frac{1}{\omega - x} \cdot \frac{a_{\overline{2(\omega - x)}|}}{s_{\overline{2}|}} \qquad {}^2\bar{A}_x = \frac{1}{\omega - x} \cdot \bar{a}_{\overline{2(\omega - x)}|}$$

Constant Force (CF): $X \sim EX(mean = \frac{1}{\mu}) \Rightarrow T = T(x) \sim EX(mean = \frac{1}{\mu})$

Usually characterized in problems by

$\mu(x) = \mu$ or ${}_x p_0 = e^{-\mu \cdot x}$

Other Important Formulas

$f_X(x) = \mu \cdot e^{-\mu \cdot x}$ $\quad f_T(t) = {}_t p_x \cdot \mu(x+t) = \mu \cdot e^{-\mu \cdot t}$

${}_t p_x = e^{-\mu \cdot t}$ $\quad {}_t q_x = 1 - e^{-\mu \cdot t}$ (Neither of these depends on *x*)

$\therefore p = p_x = p_{x+1} = \cdots = e^{-\mu \cdot}$ and $q = q_x = q_{x+1} = \cdots = 1 - e^{-\mu \cdot} = 1 - p$

${}_{k|t} q_x = p^k - p^{k+t}$ $\quad \overset{0}{e}_x = \frac{1}{\mu}$ $\quad e_x = \frac{p}{q}$ $\quad Var(T(x)) = \frac{1}{\mu^2}$ $\quad Var(K(x)) = \frac{p}{q^2}$

$\overset{0}{e}_{x:\overline{n}|} = \overset{0}{e}_x \cdot {}_n q_x = \frac{1}{u}(1 - p^n)$ $\quad e_{x:\overline{n}|} = e_x \cdot {}_n q_x = \frac{p}{q}(1 - p^n)$

$A^{1}_{x:\overline{n}|} = \frac{q}{q+i} \cdot (1 - {}_n E_x) = \frac{q}{q+i} \cdot (1 - v^n \cdot p^n)$ $\quad {}^2A^{1}_{x:\overline{n}|} = \frac{q}{q+2i+2i^2} \cdot (1 - {}_n^2 E_x) = \frac{q(1 - v^{2n} \cdot p^n)}{q+2i+i^2}$

$\overline{A}^{1}_{x:\overline{n}|} = \frac{\mu}{\mu+\delta} \cdot (1 - {}_n E_x) = \frac{\mu}{\mu+\delta} \cdot (1 - v^n \cdot p^n)$ $\quad {}^2\overline{A}^{1}_{x:\overline{n}|} = \frac{\mu}{\mu+2\delta} \cdot (1 - {}_n^2 E_x) = \frac{\mu(1 - v^{2n} \cdot p^n)}{\mu+2\delta}$

$A_x = \frac{q}{q+i}$ $\quad {}^2A_x = \frac{q}{q+2i+i^2}$

$\overline{A}_x = \frac{\mu}{\mu+\delta}$ $\quad {}^2\overline{A}_x = \frac{\mu}{\mu+2\delta}$

If the life of (*x*) is CF(μ_1) and the life of (*y*) is CF(μ_2) and is independent of the life of (*x*), then replace μ by $\mu_1 + \mu_2$ in the respective formulas to get formulas for $\overset{0}{e}_{xy}$, e_{xy}, $\overset{0}{e}_{xy:\overline{n}|}$, $e_{xy:\overline{n}|}$, $\overline{A}^{1}_{xy:\overline{n}|}$, ${}^2\overline{A}^{1}_{xy:\overline{n}|}$, $\overline{A}_{xy}$, and ${}^2\overline{A}_{xy}$.

Generalized DeMoivre's Law (GDML):

Usually characterized in problems by

$$\mu(x) = \frac{\alpha}{\omega - x} \quad \text{or} \quad l_x = l_0 \cdot \left(1 - \frac{x}{\omega}\right)^{\alpha} \quad \text{or} \quad s(x) = {}_x p_0 = \left(\frac{\omega - x}{\omega}\right)^{\alpha}$$

Other Important Formulas

$$ {}_t p_x = \left(\frac{\omega - x - t}{\omega - x}\right)^{\alpha} \qquad \overset{0}{e}_x = \frac{\omega - x}{\alpha + 1} \qquad Var(T(x)) = \frac{\alpha \cdot (\omega - x)^2}{(\alpha + 1)^2 (\alpha + 2)}$$

UDD (Uniform Distribution of Deaths: $_t d_x = t \cdot d_x$ **where** $0 \le t \le 1$**)**
(Usually told to use this assumption, or told to linearly interpolate between integer ages.)

Important Formulas

$$\mu(x+t) = \frac{q_x}{1 - t \cdot q_x}$$

$$l_{x+t} = l_x - t \cdot d_x$$

$$_t p_x = 1 - t \cdot q_x \qquad\qquad _t q_x = t \cdot q_x$$

$$f_T(t) = {_t p_x} \cdot \mu(x+t) = \frac{1}{\omega - x} = q_x$$

$$\overset{0}{e}_x = e_x + 0.5 \qquad \overset{0}{e}_{x:\overline{n}|} = e_{x:\overline{n}|} + 0.5({_n q_x}) \qquad \overset{0}{e}_{x:\overline{1}|} = 1 - 0.5 \cdot q_x$$

$$Var(T(x)) = Var(K(x)) + \frac{1}{12}$$

For $0 \le s + t \le 1$, $_t q_{x+s} = \dfrac{t \cdot q_x}{1 - s \cdot q_x}$

$m_x = \dfrac{q_x}{1 - 0.5 \cdot q_x}$ (central death rate at age x)

$$\bar{A}_x = \frac{i}{\delta} A_x \qquad \bar{A}^1_{x:\overline{n}|} = \frac{i}{\delta} A^1_{x:\overline{n}|}$$

So, $\bar{A}_{x:\overline{n}|} = \bar{A}^1_{x:\overline{n}|} + A_{x:\overset{1}{\overline{n}|}} = \dfrac{i}{\delta} A^1_{x:\overline{n}|} + A_{x:\overset{1}{\overline{n}|}}$

$$_k V(\bar{A}_x) = \frac{i}{\delta} \cdot {_k V_x} \qquad _k V(\bar{A}^1_{x:\overline{n}|}) = \frac{i}{\delta} \cdot {_k V^1_{x:\overline{n}|}}$$

So, $_k V(\bar{A}_{x:\overline{n}|}) = \dfrac{i}{\delta} \cdot {_k V^1_{x:\overline{n}|}} + {_k V_{x:\overset{1}{\overline{n}|}}}$