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1. A survey of a group's viewing habits over the last year revealed the following information:

(i) 28% watched gymnastics
(ii) 29% watched baseball
(iii) 19% watched soccer
(iv) 14% watched gymnastics and baseball
(v) 12% watched baseball and soccer
(vi) 10% watched gymnastics and soccer
(vii) 8% watched all three sports.

Calculate the percentage of the group that watched none of the three sports during the last year.

(A) 24

(B) 36

(C) 41

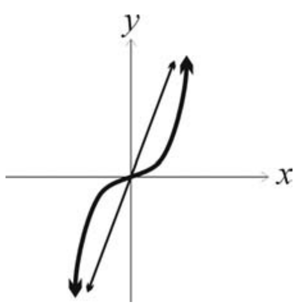
(D) 52

(E) 60

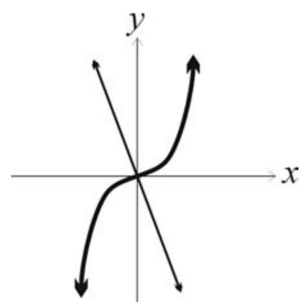
2. Each of the graphs below contains two curves.

Identify the graph containing a curve representing a function $y = f(x)$ and a curve representing its second derivative $y = f''(x)$.

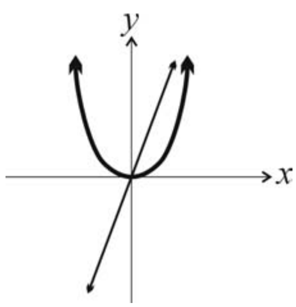
(A)



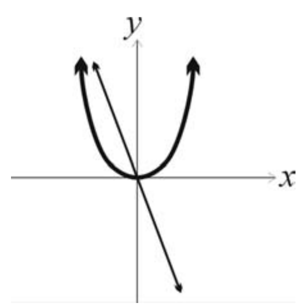
(B)



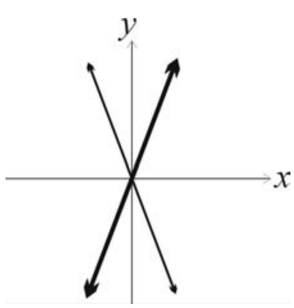
(C)



(D)



(E)



3. Let f and g be differentiable functions such that

$$\lim_{x \to 0} f(x) = c$$
$$\lim_{x \to 0} g(x) = d$$

where $c \neq d$.

Determine $\lim_{x \to 0} \dfrac{cf(x) - dg(x)}{f(x) - g(x)}$.

(A) 0

(B) $\dfrac{cf'(0) - dg'(0)}{f'(0) - g'(0)}$

(C) $f'(0) - g'(0)$

(D) $c - d$

(E) $c + d$

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4. The time to failure of a component in an electronic device has an exponential distribution with a median of four hours.

Calculate the probability that the component will work without failing for at least five hours.

(A) 0.07

(B) 0.29

(C) 0.38

(D) 0.42

(E) 0.57

5. An insurance company examines its pool of auto insurance customers and gathers the following information:

(i) All customers insure at least one car.

(ii) 70% of the customers insure more than one car.

(iii) 20% of the customers insure a sports car.

(iv) Of those customers who insure more than one car, 15% insure a sports car.

Calculate the probability that a randomly selected customer insures exactly one car and that car is not a sports car.

(A) 0.13

(B) 0.21

(C) 0.24

(D) 0.25

(E) 0.30

6. Let X and Y be continuous random variables with joint density function

$$f(x, y) = \begin{cases} \frac{8}{3}xy & \text{for } 0 \le x \le 1,\ x \le y \le 2x \\ 0 & \text{otherwise.} \end{cases}$$

Calculate the covariance of X and Y.

(A) 0.04

(B) 0.25

(C) 0.67

(D) 0.80

(E) 1.24

7. Given $\int_0^2 f(x)\,dx = 3$ and $\int_2^4 f(x)\,dx = 5$,

calculate $\int_0^2 f(2x)\,dx$.

(A) $3/2$

(B) 3

(C) 4

(D) 6

(E) 8

8. An auto insurance company insures drivers of all ages. An actuary compiled the following statistics on the company's insured drivers:

Age of Driver	Probability of Accident	Portion of Company's Insured Drivers
16-20	0.06	0.08
21-30	0.03	0.15
31-65	0.02	0.49
66-99	0.04	0.28

A randomly selected driver that the company insures has an accident.

Calculate the probability that the driver was age 16-20.

(A) 0.13

(B) 0.16

(C) 0.19

(D) 0.23

(E) 0.40

9. An insurance company determines it cannot write medical malpractice insurance profitably and stops selling the coverage. In spite of this action, the company will have to pay claims for many years on existing medical malpractice policies.

The company pays 60 for medical malpractice claims the year after it stops selling the coverage. Each subsequent year's payments are 20% less than those of the previous year.

Calculate the total medical malpractice payments that the company pays in all years after it stops selling the coverage.

(A) 75

(B) 150

(C) 240

(D) 300

(E) 360

10. Let X and Y be continuous random variables with joint density function

$$f(x,y) = \begin{cases} 15y & \text{for } x^2 \le y \le x \\ 0 & \text{otherwise.} \end{cases}$$

Let g be the marginal density function of Y.

Which of the following represents g?

(A) $g(y) = \begin{cases} 15y & \text{for } 0 < y < 1 \\ 0 & \text{otherwise} \end{cases}$

(B) $g(y) = \begin{cases} \dfrac{15y^2}{2} & \text{for } x^2 < y < x \\ 0 & \text{otherwise} \end{cases}$

(C) $g(y) = \begin{cases} \dfrac{15y^2}{2} & \text{for } 0 < y < 1 \\ 0 & \text{otherwise} \end{cases}$

(D) $g(y) = \begin{cases} 15y^{3/2}\left(1 - y^{1/2}\right) & \text{for } x^2 < y < x \\ 0 & \text{otherwise} \end{cases}$

(E) $g(y) = \begin{cases} 15y^{3/2}\left(1 - y^{1/2}\right) & \text{for } 0 < y < 1 \\ 0 & \text{otherwise} \end{cases}$

11. The value of a particular investment changes over time according to the function

$$S(t) = 5000e^{0.1\left(e^{0.25t}\right)},$$

where $S(t)$ is the value after t years.

Calculate the rate at which the value of the investment is changing after 8 years.

(A) 618

(B) 1,934

(C) 2,011

(D) 7,735

(E) 10,468

12. Let X be a continuous random variable with density function

$$f(x) = \begin{cases} \frac{|x|}{10} & \text{for } -2 \le x \le 4 \\ 0 & \text{otherwise.} \end{cases}$$

Calculate the expected value of X.

(A) $\frac{1}{5}$

(B) $\frac{3}{5}$

(C) 1

(D) $\frac{28}{15}$

(E) $\frac{12}{5}$

13. A charity receives 2025 contributions. Contributions are assumed to be independent and identically distributed with mean 3125 and standard deviation 250.

Calculate the approximate 90^{th} percentile for the distribution of the total contributions received.

(A) 6,328,000

(B) 6,338,000

(C) 6,343,000

(D) 6,784,000

(E) 6,977,000

14. Let f be a differentiable function such that:

$$f(x+h)-f(x)=3x^2h+3xh^2+h^3+2h \text{ for all } x \text{ and } h$$

$$f(0)=1$$

Let $g(x)=e^{-x}f(x)$.

Calculate $g'(3)$.

(A) $-34e^{-3}$

(B) $-29e^{-3}$

(C) $-5e^{-3}$

(D) $-4e^{-3}$

(E) $63e^{-3}$

15. An insurance policy pays a total medical benefit consisting of two parts for each claim. Let X represent the part of the benefit that is paid to the surgeon, and let Y represent the part that is paid to the hospital. The variance of X is 5000, the variance of Y is 10,000, and the variance of the total benefit, $X+Y$, is 17,000.

Due to increasing medical costs, the company that issues the policy decides to increase X by a flat amount of 100 per claim and to increase Y by 10% per claim.

Calculate the variance of the total benefit after these revisions have been made.

(A) 18,200

(B) 18,800

(C) 19,300

(D) 19,520

(E) 20,670

16. A device runs until either of two components fails, at which point the device stops running. The joint density function of the lifetimes of the two components, both measured in hours, is

$$f(x,y) = \frac{x+y}{27} \text{ for } 0 < x < 3 \text{ and } 0 < y < 3 .$$

Calculate the probability that the device fails during its first hour of operation.

(A) 0.04

(B) 0.41

(C) 0.44

(D) 0.59

(E) 0.96

17. Determine $\lim_{n\to\infty}\frac{1}{n}\left(e^{1/n}+e^{2/n}+\cdots+e^{n/n}\right)$.

(A) 0

(B) 1

(C) $e-1$

(D) e

(E) ∞

18. An insurance company estimates that 40% of policyholders who have only an auto policy will renew next year and 60% of policyholders who have only a homeowners policy will renew next year. The company estimates that 80% of policyholders who have both an auto and a homeowners policy will renew at least one of those policies next year.

Company records show that 65% of policyholders have an auto policy, 50% of policyholders have a homeowners policy, and 15% of policyholders have both an auto and a homeowners policy.

Using the company's estimates, calculate the percentage of policyholders that will renew at least one policy next year.

(A) 20

(B) 29

(C) 41

(D) 53

(E) 70

19. Let $f(x)=e^{-2x}$. For $x>0$, let $P(x)$ be the perimeter of the rectangle with vertices

$(0,0),(x,0),(x,f(x))$ and $(0,f(x))$.

Which of the following statements is true?

(A) The function P has an absolute minimum but not an absolute maximum on the interval $(0,\infty)$.

(B) The function P has an absolute maximum but not an absolute minimum on the interval $(0,\infty)$.

(C) The function P has both an absolute minimum and an absolute maximum on the interval $(0,\infty)$.

(D) The function P has neither an absolute maximum nor an absolute minimum on the interval $(0,\infty)$, but the graph of the function P does have a point of inflection with positive x-coordinate.

(E) The function P has neither an absolute maximum nor an absolute minimum on the interval $(0,\infty)$, and the graph of the function P does not have a point of inflection with positive x-coordinate.

20. A family buys two policies from the same insurance company. Losses under the two policies are independent and have continuous uniform distributions on the interval from 0 to 10. One policy has a deductible of 1 and the other has a deductible of 2. The family experiences exactly one loss under each policy.

Calculate the probability that the total benefit paid to the family does not exceed 5.

(A) 0.13

(B) 0.25

(C) 0.30

(D) 0.32

(E) 0.42

21. The profitability, P, of a new product is related to the amount of labor, L, and capital, C, invested in it according to the formula

$$P = 3.5\, L^{6/5}\, C^{1/2}.$$

At a time when $L = 12$ and $C = 4$, the rate of change in labor is 2.5 and the rate of change in capital is 0.5 .

Calculate how fast profit is increasing at that time.

(A) 1.7

(B) 2.2

(C) 31.1

(D) 43.1

(E) 50.1

22. An insurer's annual weather-related loss, X, is a random variable with density function

$$f(x) = \begin{cases} \dfrac{2.5(200)^{2.5}}{x^{3.5}} & \text{for } x > 200 \\ 0 & \text{otherwise.} \end{cases}$$

Calculate the difference between the 30th and 70th percentiles of X.

(A) 35

(B) 93

(C) 124

(D) 231

(E) 298

23. The time, T, that a manufacturing system is out of operation has cumulative distribution function

$$F(t) = \begin{cases} 1-\left(\dfrac{2}{t}\right)^2 & \text{for } t > 2 \\ 0 & \text{otherwise.} \end{cases}$$

The resulting cost to the company is $Y = T^2$.

Determine the density function of Y, for $y > 4$.

(A) $\dfrac{4}{y^2}$

(B) $\dfrac{8}{y^{3/2}}$

(C) $\dfrac{8}{y^3}$

(D) $\dfrac{16}{y}$

(E) $\dfrac{1024}{y^5}$

24. Let X represent the age of an insured automobile involved in an accident. Let Y represent the length of time the owner has insured the automobile at the time of the accident.

X and Y have joint probability density function

$$f(x,y) = \begin{cases} \frac{1}{64}\left(10 - xy^2\right) & \text{for } 2 \le x \le 10 \text{ and } 0 \le y \le 1 \\ 0 & \text{otherwise.} \end{cases}$$

Calculate the expected age of an insured automobile involved in an accident.

(A) 4.9

(B) 5.2

(C) 5.8

(D) 6.0

(E) 6.4

25. An insurance policy pays for a random loss *X* subject to a deductible of *C*, where $0 < C < 1$. The loss amount is modeled as a continuous random variable with density function

$$f(x) = \begin{cases} 2x & \text{for } 0 < x < 1 \\ 0 & \text{otherwise.} \end{cases}$$

Given a random loss *X*, the probability that the insurance payment is less than 0.5 is equal to 0.64 .

Calculate *C*.

(A) 0.1

(B) 0.3

(C) 0.4

(D) 0.6

(E) 0.8

26. Let $g(x) = \dfrac{x+4}{x^2+2x-8}$.

Determine all values of x at which g is discontinuous, and for each of these values of x, define g in such a manner so as to remove the discontinuity, if possible.

(A) g is discontinuous only at -4 and 2.
Define $g(-4) = -\frac{1}{6}$ to make g continuous at -4.
$g(2)$ cannot be defined to make g continuous at 2.

(B) g is discontinuous only at -4 and 2.
Define $g(-4) = -\frac{1}{6}$ to make g continuous at -4.
Define $g(2) = 6$ to make g continuous at 2.

(C) g is discontinuous only at -4 and 2.
$g(-4)$ cannot be defined to make g continuous at -4.
$g(-2)$ cannot be defined to make g continuous at 2.

(D) g is discontinuous only at 2.
Define $g(2) = 6$ to make g continuous at 2.

(E) g is discontinuous only at 2.
$g(2)$ cannot be defined to make g continuous at 2.

27. A life insurance company invests 5000 in a bank account in order to fund a death benefit of 20,000. Growth in the investment over time can be modeled by the differential equation

$$\frac{dA}{dt} = Ai$$

where i is the interest rate and $A(t)$ is the amount invested at time t (in years).

Calculate the interest rate that the investment must earn in order for the company to fund the death benefit in 24 years.

(A) $\dfrac{-\ln 2}{12}$

(B) $\dfrac{-\ln 2}{24}$

(C) $\dfrac{\ln 2}{24}$

(D) $\dfrac{\ln 2}{12}$

(E) $\dfrac{\ln 2}{6}$

28. Let X and Y be continuous random variables with joint density function

$$f(x,y)=\begin{cases}24xy & \text{for } 0<x<1 \text{ and } 0<y<1-x\\ 0 & \text{otherwise.}\end{cases}$$

Calculate $P\left[Y<X \middle| X=\frac{1}{3}\right]$.

(A) $\frac{1}{27}$

(B) $\frac{2}{27}$

(C) $\frac{1}{4}$

(D) $\frac{1}{3}$

(E) $\frac{4}{9}$

29. An investment account earns an annual interest rate R that follows a uniform distribution on the interval $(0.04, 0.08)$. The value of a 10,000 initial investment in this account after one year is given by $V = 10,000e^R$.

Determine the cumulative distribution function, $F(v)$, of V for values of v that satisfy $0 < F(v) < 1$.

(A) $\dfrac{10,000e^{v/10,000} - 10,408}{425}$

(B) $25e^{v/10,000} - 0.04$

(C) $\dfrac{v - 10,408}{10,833 - 10,408}$

(D) $\dfrac{25}{v}$

(E) $25\left[\ln\left(\dfrac{v}{10,000}\right) - 0.04\right]$

30. Let $f(x)=\sum_{k=0}^{\infty}(-1)^{k(k+1)/2}x^k$, for $-1<x<1$.

Which of the following is an equivalent expression for $f(x)$, for $-1<x<1$?

(A) $\dfrac{1}{1+x}$

(B) $\dfrac{1}{1-x}$

(C) $\dfrac{1-2x}{1-x}$

(D) $\dfrac{x^2+x}{1+x^2}$

(E) $\dfrac{1-x}{1+x^2}$

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31. A health study tracked a group of persons for five years. At the beginning of the study, 20% were classified as heavy smokers, 30% as light smokers, and 50% as nonsmokers.

Results of the study showed that light smokers were twice as likely as nonsmokers to die during the five-year study, but only half as likely as heavy smokers.

A randomly selected participant from the study died over the five-year period.

Calculate the probability that the participant was a heavy smoker.

(A) 0.20

(B) 0.25

(C) 0.35

(D) 0.42

(E) 0.57

32. A random variable X has the cumulative distribution function

$$F(x) = \begin{cases} 0 & \text{for } x < 1 \\ \dfrac{x^2 - 2x + 2}{2} & \text{for } 1 \le x < 2 \\ 1 & \text{for } x \ge 2. \end{cases}$$

Calculate the variance of X.

(A) $\dfrac{7}{72}$

(B) $\dfrac{1}{8}$

(C) $\dfrac{5}{36}$

(D) $\dfrac{4}{3}$

(E) $\dfrac{23}{12}$

33. Let $f(x) = \dfrac{2x}{x+1}$.

Define: $f^2(x) = f(f(x))$
$f^3(x) = f(f^2(x))$
$\vdots$
$f^n(x) = f(f^{n-1}(x))$

Determine $\lim_{n\to\infty} f^n(x)$ for $x > 0$.

(A) 0

(B) 1

(C) 2

(D) x

(E) $\dfrac{1}{x}$

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34. The lifetime of a machine part has a continuous distribution on the interval (0, 40) with probability density function *f*, where $f(x)$ is proportional to $(10 + x)^{-2}$.

Calculate the probability that the lifetime of the machine part is less than 6.

(A) 0.04

(B) 0.15

(C) 0.47

(D) 0.53

(E) 0.94

35. Let $f(x, y) = y^2 - 2x^2 y + 4x^3 + 20x^2$. The only critical points are (–2, 4), (0, 0), and (5, 25).

Which of the following correctly describes the behavior of *f* at these points?

(A) $(-2, 4)$: local minimum
$(0, 0)$: local minimum
$(5, 25)$: local maximum

(B) $(-2, 4)$: local minimum
$(0, 0)$: local maximum
$(5, 25)$: local maximum

(C) $(-2, 4)$: neither a local minimum nor a local maximum
$(0, 0)$: local maximum
$(5, 25)$: local minimum

(D) $(-2, 4)$: local maximum
$(0, 0)$: neither a local minimum nor a local maximum
$(5, 25)$: local minimum

(E) $(-2, 4)$: neither a local minimum nor a local maximum
$(0, 0)$: local minimum
$(5, 25)$: neither a local minimum nor a local maximum

36. An insurance policy pays an individual 100 per day for up to 3 days of hospitalization and 25 per day for each day of hospitalization thereafter.

The number of days of hospitalization, X, is a discrete random variable with probability function

$$P(X = k) = \begin{cases} \dfrac{6-k}{15} & \text{for } k = 1,2,3,4,5 \\ 0 & \text{otherwise.} \end{cases}$$

Calculate the expected payment for hospitalization under this policy.

(A) 85

(B) 163

(C) 168

(D) 213

(E) 255

37. An insurance company pays hospital claims. The number of claims that include emergency room or operating room charges is 85% of the total number of claims. The number of claims that do not include emergency room charges is 25% of the total number of claims. The occurrence of emergency room charges is independent of the occurrence of operating room charges on hospital claims.

Calculate the probability that a claim submitted to the insurance company includes operating room charges.

(A) 0.10

(B) 0.20

(C) 0.25

(D) 0.40

(E) 0.80

38. At time $t = 0$, a store has 19 units of a product in inventory. The cumulative number of units sold is given by $S(t) = e^{3t} - 1$ where t is measured in weeks. The inventory will be replenished when it drops to 1 unit. The cost of carrying inventory until then is 15 per unit per week (pro-rated for a portion of a week).

Calculate the inventory carrying cost that will be incurred before the inventory is replenished.

(A) 90

(B) 199

(C) 204

(D) 210

(E) 294

39. X and Y are independent random variables with common moment generating function $M(t) = e^{t^2/2}$.

Let $W = X + Y$ and $Z = Y - X$.

Determine the joint moment generating function, $M(t_1, t_2)$, of W and Z.

(A) $e^{2t_1^2+2t_2^2}$

(B) $e^{(t_1-t_2)^2}$

(C) $e^{(t_1+t_2)^2}$

(D) $e^{2t_1t_2}$

(E) $e^{t_1^2+t_2^2}$

40. A particle travels along the curve defined by $x = t^2 - 7t + 2$ and $y = \frac{t^2}{4} - 6t$ for $t \geq 0$.

Determine the time t at which the minimum speed occurs.

(A) $\frac{7}{2}$

(B) 4

(C) $\frac{21}{2}$

(D) 12

(E) 24