Systems of Linear Equations and Matrix Algebra Practice Exam

Answers to the problems are at the end of the page.

1. State the next elementary row operation which must be performed to put the matrix in diagonal form, then perform the operation.

$$\left[\begin{array}{ccc|c} 1 & 0 & 3 & 9 \\ 4 & 1 & -3 & 2 \\ 0 & -5 & 4 & 3 \end{array}\right]$$

2. Solve the following systems using the Gauss-Jordan elimination method.

a. $$\begin{aligned} 2x + 3y &= 4 \\ y &= 5x - 1 \end{aligned}$$

b. $$\begin{aligned} x + y &= 140 \\ 4x + 3y + 5z &= 520 \\ 3x + 5y + z &= 588 \end{aligned}$$

3. A small manufacturing plant makes three types of inflatable boats: one-person, two-person, and four-person models. Each boat requires the services of three departments: cutting, assembly, and packaging. For the one-person model it takes 0.5 hour in cutting, 0.6 hour in assembly, and 0.2 hour in packaging to produce the boat. For the two-person model it takes 1 hour, 0.9 hour, and 0.3 hour respectively to produce the boat. And for the four-person model it takes 1.5 hours, 1.2 hours, and 0.5 hour to produce. The cutting, assembly, and packaging departments have available a maximum of 380, 330, and 120 labor-hours per week, respectively. How many boats of each type must be produced each week for the plant to operate at full capacity? Express the problem in terms of a system of linear equations. **Do not solve! Set up the system only.**

4. A nursery stocks three brands of grass seed whose ingredients are summarized in the following table (each entry represents kilograms per bag):

	Ryegrass	Fescue	Bluegrass
Brand P	2	2	6
Brand Q	4	2	4
Brand R	0	6	4

Determine how many bags of each brand are needed to satisfy a customer who wants a mixture containing 30 kg of ryegrass seed, 30 kg of fescue seed, and 50 kg of bluegrass seed. Express the problem in terms of a system of linear equations. **Do not solve! Set up the system only.**

5. Each of the following matrices was the result of applying the Gauss-Jordan elimination method to the matrix of a system of equations. State the general solution of each system of equations.

a. $\left[\begin{array}{ccc|c} 1 & 0 & 3 & 4 \\ 0 & 1 & 5 & 2 \\ 0 & 0 & 0 & 2 \end{array}\right]$ b. $\left[\begin{array}{ccc|c} 1 & 0 & 0 & 4 \\ 0 & 1 & 0 & 8 \\ 0 & 0 & 1 & 12 \end{array}\right]$ c. $\left[\begin{array}{ccc|c} 1 & 0 & 4 & 5 \\ 0 & 1 & 8 & 2 \\ 0 & 0 & 0 & 0 \end{array}\right]$

6. Perform the indicated operations on the given matrices.

$A = \begin{bmatrix} 2 & 3 \\ 5 & 6 \end{bmatrix}$ $B = \begin{bmatrix} 3 & 1 & 5 \\ 6 & -4 & 7 \end{bmatrix}$ $C = \begin{bmatrix} 1 & 3 & 6 \\ 4 & 2 & 7 \\ 2 & 4 & 5 \end{bmatrix}$

$D = \begin{bmatrix} 2 & 6 \\ 3 & 9 \end{bmatrix}$ $E = \begin{bmatrix} 1 & 3 \\ 2 & 5 \\ 4 & 6 \end{bmatrix}$ $F = \begin{bmatrix} 3 & 1 & -4 \\ -2 & 5 & 6 \end{bmatrix}$

a. $B + F$
b. BE
c. $C + E$
d. $2A - D$
e. F^{-1}
f. D^{-1}
g. DE
h. E^T

7. Find the inverse of the matrix:

$$\begin{bmatrix} 1 & 0 & 0 \\ 0 & 2 & -2 \\ -2 & 0 & 1 \end{bmatrix}$$

8. Set up and use a matrix equation to solve the following problem: A graduate student in paleontology is analyzing prehistoric populations of three species of dinosaur. She was part of an expedition that discovered a mound of bones belonging to a total of 100 animals. There is some variation in all species, and she knows, for example, that long thigh bones are found in one-half of the animals in the first species, one-quarter of the animals in the second species, and all of the animals in the third species. Short forearm bones are found in three-quarters of the specimens of the first species, all of the second species, and one-half of the third species. In this mound, there are 60 long thighbones and 75 short forearms. How many animals of each species are in this mound?

9. Refer to problem 8 above to answer this problem: A more careful count reveals that the mound is the grave of only 90 extinct beasts. Furthermore, only 50 of the thighbones are long, and only 70 forearms are short. What now is the species count in this mound?

10. Find the decoding matrix A^{-1}, and use it to decode the encoded message. Use the symbol-number correlation given in the text.

Encoded Message (other than Lial): 48, 21, 33, -3, 27, -6, 66, -24, 32, 5, 38, 11, 40, 25, 27, -12, 48, -12, 130, -20.

Lial Text Encoded Message: -9, 36, 15, 21, 17, 16, 48, 33, 4, 23, -2, 29, -17, 32, 25, 14, 32, 28

$$A = \begin{bmatrix} 2 & -1 \\ 1 & 1 \end{bmatrix}$$

11. During her first two years at college, Sara took course hours, plane trips home, and beach weekends, as indicated in this matrix:

$A = \begin{bmatrix} 12 & 16 & 15 & 17 \\ 2 & 3 & 1 & 2 \\ 0 & 2 & 1 & 2 \end{bmatrix}$ where row one contains the hours taken, row two the number of trips home, row three the number of beach trips, column one is those quantities for first semester, column two for second semester, column three for third semester, and column four for fourth semester. Given that tuition costs \$80 per hour, each round-trip plane fare home costs \$260, and each beach weekend costs \$250, find a matrix B so that a multiplication with the matrix A will give the total cost for each semester for the three activities listed. Then do the matrix multiplication.

Solutions

1. $-4R_1 + R_2 \to R_2$

$$\left[\begin{array}{ccc|c} 1 & 0 & 3 & 9 \\ 0 & 1 & -15 & -34 \\ 0 & -5 & 4 & 3 \end{array}\right]$$

2. a. $\left\{\left(\frac{7}{17}, \frac{18}{17}\right)\right\}$ b. $\{(60,80,8)\}$

3. $\begin{aligned} 0.5x + y + 1.5z &= 380 \\ 0.6x + 0.9y + 1.2z &= 330 \\ 0.2x + 0.3y + 0.5z &= 120 \end{aligned}$ where $\begin{aligned} x &= \#\text{ of one-person boats} \\ y &= \#\text{ of two-person boats} \\ z &= \#\text{ of four-person boats} \end{aligned}$

4. $\begin{aligned} 2x + 4y &= 30 \\ 2x + 2y + 6z &= 30 \\ 6x + 4y + 4z &= 50 \end{aligned}$ where $\begin{aligned} x &= \#\text{ of bags of Brand P} \\ y &= \#\text{ of bags of Brand Q} \\ z &= \#\text{ of bags of Brand R} \end{aligned}$

5. a. no solution (an inconsistent system) b. {(4,8,12)} c. x = 5-4z, y = 2-8z, z is any real number.

6. a. $\begin{bmatrix} 6 & 2 & 1 \\ 4 & 1 & 13 \end{bmatrix}$ b. $\begin{bmatrix} 25 & 44 \\ 26 & 40 \end{bmatrix}$ c. Not of same dimension

d. $\begin{bmatrix} 2 & 0 \\ 7 & 3 \end{bmatrix}$ e. No inverse, for matrix is not square.

f. No inverse, for the determinant is zero.

g. Cannot multiply because the columns of left matrix do not match the number of rows of the right matrix.

h. $\begin{bmatrix} 1 & 2 & 4 \\ 3 & 5 & 6 \end{bmatrix}$

7. $\begin{bmatrix} 1 & 0 & 0 \\ 2 & \frac{1}{2} & 1 \\ 2 & 0 & 1 \end{bmatrix}$

8. 20 of first species, 40 of the second, and 40 of the third

9. 20 of first species, 40 of the second, and 30 of the third
Notice that if you used the matrix approach to solve the system in problem 7, which means using the inverse of the coefficient matrix, then you only had to re-multiply in problem 8 to find your answer. That is multiply the inverse matrix with the constant matrix where the constant matrix is different from problem 7. The inverse matrix is the same however, for the coefficients did not change; only the constants changed from problem 7 to problem 8. If you used the Gauss-Jordan elimination approach, you found that you had to re-set up the matrix and do all that pivoting again. Problems 7 and 8 show one advantage of the matrix equation approach for solving systems of linear equations. This same advantage can be seen when using the input-output model developed by Wassily Leontif in economics, and also in cryptography.

10. *I like finite math!*

11. $B = \begin{bmatrix} 80 & 260 & 250 \end{bmatrix}$. The product matrix that will show the total cost for each semester is: BA = $\begin{bmatrix} 1480 & 2560 & 1710 & 2380 \end{bmatrix}$.