Practice Probability Exam Answers

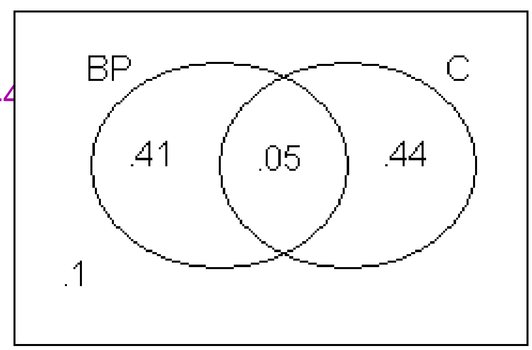
1. An experiment consists of arranging a white ball (W), a black ball (B), and a red ball (R) in a row.
   (a) Write a sample space S for this experiment?
   {WBR, RWB, BRW, BWR, RBW, WRB}
   (b) Write an event space for the event E = "the white ball is in the middle," as a subset of the sample space that you wrote in part (a).
   { RWB, BWR}
   (c) Find P(E).
   2/6 = 1/3

2. Let E and F be events such that P(E) = 0.6, P(F) = 0.3, $P(E \cap F) = 0.2$, find $P(E \cup F)$.
   0.6 + 0.3 – 0.2 = 0.7

3. Events E and F are mutually exclusive and $P(F) \neq 0$. Select the following statement that must be true.
   (a) E and F are independent
   (b) P(E | F) = P(E)
   (c) $P(E \cap F) = P(E) \cdot P(F)$
   (d) P(E | F) = 0
   (e) None of the above.

4. Events E and F are independent and $P(F) \neq 0$. Select the following statement that must be true.
   (a) P(E | F) = 0
   (b) E and F are mutually exclusive
   (c) P(E | F) = P(F)
   (d) $P(E \cup F) = P(E) + P(F)$
   (e) None of the above.

5. A survey of senior citizens at a doctor's office shows that 46% take blood pressure lowering medication, 49% take cholesterol-lowering medication and 5% take both medications. What is the probability that a senior takes:
   (a) at least one type of medication?
   0.46 + 0.49 - .05 = 0.9
   or you could add 0.41 + 0.05 + 0.44
   (b) neither type of medication?
   0.1
   (c) exactly one type of medication?
   0.41 + 0.44 = 0.85



6. The table shows the number of college students who prefer a given pizza topping.

toppings	freshman	sophomore	junior	senior	total
cheese	14	11	28	19	72
meat	22	19	11	14	66
veggie	11	14	22	19	66
total	47	44	61	52	204

One student is chosen at random.

(a) What is the probability that the student prefers cheese toppings?
72/204

(b) What is the probability that the student is a junior and prefers veggie toppings?
22/204

(c) What is the probability that the student is a junior or prefers veggie toppings?
61/204 + 66/204 – 22/204 = 105/204
Notice that in this situation the events are not mutually exclusive, so we need to remove the double counting by subtracting out the intersection of the events.

(d) What is the probability that the student is a freshman or a junior?
47/204 + 61/204 = 108/204
Notice that in this situation the events are mutually exclusive, so no need to subtract out the intersection of the events.

(e) What is the probability that the student prefers meat toppings given that the student is a senior?
14/52

(f) What is the probability that the student is a sophomore given that the student prefers veggie toppings?
14/66

(g) Are the events "prefers meat toppings" and "is a sophomore" independent? Justify your answer using the formulas.

There are a couple of ways to determine independence. One way is to see if $P(A \cap B) = P(A)P(B)$. The probability of the intersection of these two events is 19/204 and the product probability is (66/204)(44/204). You can check with you calculator, and you should find that these are not equal; therefore, the events are not independent. Another way to determine independence is to check and see if $P(A \mid B) = P(A)$. In our case the probability of "prefers meat toppings" given "is a sophomore" is 19/44, and the probability of "prefers meat toppings" is 66/204. Again you can check it out with a calculator and you find that the two probabilities are not equal; therefore, the two events are not independent.

7. Let $S = \{s_1, s_2, s_3, s_4, s_5, s_6\}$ be the sample space associated with an experiment having the following probability distribution.

Outcome	s_1	s_2	s_3	s_4	s_5	s_6
Probability	1/9	2/9	2/9	1/9	2/9	1/9

Find the probability of the event

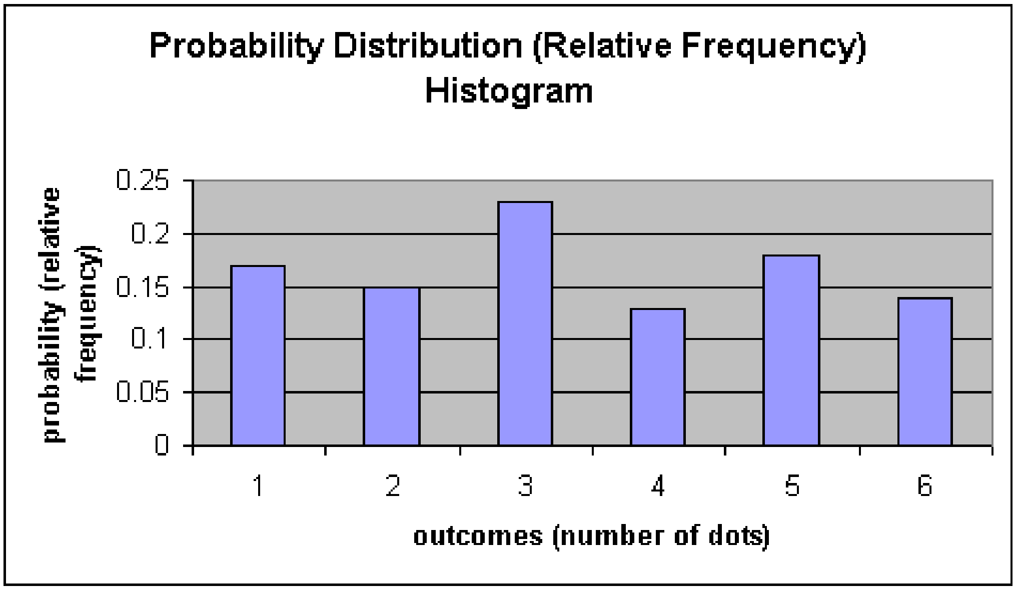
(a) $A = \{s_1, s_2\}$
1/9 + 2/9 = 3/9 = 1/3
(b) $B = \{s_2, s_4, s_6\}$
2/9 + 1/9 + 1/9 = 4/9
(c) $A \cap B = \{s_2\}$
2/9
(d) $A \cup B = \{s_1, s_2, s_4, s_6\}$
1/9 + 2/9 + 1/9 + 1/9 = 5/9
(e) Find $P(A \mid B) = [P(A \cap B) / P(B)] = (2/9) / (4/9) = 2/4 = 1/2$

8. A die was repeatedly tossed and the number of dots on the top face recorded. The results were as follows:

Number of Dots	Frequency	Probability
1	100	100/600 ≈ .17
2	90	90/600 ≈ .15
3	140	140/600 ≈ .23
4	80	80/600 ≈ .13
5	105	105/600 ≈ .18
6	85	85/600 ≈ .14
total	600	1.0

(a) Make an empirical probability distribution (that is a relative frequency table) for this information (simply finish filling in the above table).
(b) If this die is rolled, find the probability that an odd number is rolled.
17 + .23 + .18 = .58
(c) Find the expected number of dots per roll.
1(.17) + 2(.15) + 3(.23) + 4(.13) + 5(.18) + 6(.14) = 3.42
I get 3.425 when I use fractions instead of the decimal approximations.

(d) Construct the probability distribution (relative frequency) histogram.



I used Excel to make the above histogram.

9. The letters of the word "clamber" are scrambled and arranged in a random order. What is the probability of an arrangement starting with a vowel?
   $(2\times6!)/7! = 2/7$

10. An urn contains six red balls and eight green balls. A sample of seven balls is selected at random. Find the probability that
    (a) five red and two green balls are selected.
    $[C(6,5)C(8,2)] / C(14,7) \approx .04895$
    (b) all the balls selected are red.
    This is an impossible event, so the probability is 0.
    (c) at least five red balls are selected.
    exactly 5 or exactly 6 = $[C(6,5)C(8,2)] / C(14,7) + [C(6,6)C(8,1)] / C(14,7) \approx .04895 + .00233 = .05128$

11. In the movie "Cheaper by the Dozen" the featured family included 12 children. What is the probability of such a family having 4 girls and 8 boys? Assume a probability of 0.5 that a birth will be a girl.
    $C(12,4) / 2^{12} = 495/4096 \approx .121$

12. Six conventions are to be scheduled at J and J Resort during the twelve week winter session.
    (a) What is the probability that all six organizations request the same week for their conventions?
    $12 / 12^6 = 1/12^5 \approx .000004$
    (b) What is the probability all six organizations request different weeks for their conventions?
    $P(12,6) / 12^6 \approx .22$
    (c) What is the probability that two or more organizations request the same week for their

conventions?

1 – P(none the same week) = 1 - .22 = .78

13. Two cards are drawn at random from a standard 52-card deck without replacement.

(a) Find the probability that the first card drawn is the king of clubs.

1/52

(b) Find the probability that the second card drawn is red, given that the first card drawn is the king of clubs.

26/51

(c) Find the probability that the first card drawn is the king of clubs, given that the second card drawn is red.

1/51

(d) Find the probability that the second card drawn is a club, given that the first card drawn is black.

$$P(C_2|B_1) = \frac{P(B_1 \cap C_2)}{P(B_1)} = \frac{P(C_2) \cdot P(B_1|C_2)}{\frac{26}{52}} = \frac{P(C_2) \cdot \frac{25}{51}}{\frac{1}{2}} =$$

$$\frac{\frac{1}{4} \cdot \frac{25}{51}}{\frac{1}{2}} = \frac{25}{204} \cdot \frac{2}{1} = \frac{25}{102} \approx 0.2451$$

$$P(C_2) = P(C_1 \cap C_2) + P(C_1' \cap C_2) = P(C_1) \cdot P(C_2|C_1) + P(C_1') \cdot P(C_2|C_1') =$$

$$\frac{13}{52} \cdot \frac{12}{51} + \frac{39}{52} \cdot \frac{13}{51} = \frac{1}{4}$$

This one is pretty wild; so do not get too hung up about it. At best this would be an extra credit type question. I used the idea of creating a partitioning of the sample space on the first draw with drawing a club on the first draw and not drawing a club on the first draw. This way I can determine the conditional probabilities for drawing a club on the second draw. This concept is called total probability and is not something shown in our text. The power of this idea is that I can arbitrarily make up such a partitioning in a manner that makes my computation easier. It also forms the basis for Bayes' Theorem. Another way to justify that the probability of getting a club on the second draw is ¼, is to realize that we have no prior knowledge, so drawing a club on the second draw is the same as drawing a club on the first draw. We do not know what happened on the first draw, so our perception of the second card remains unchanged. Now if we speculate about the first card, then our perception of the likelihood of getting a club on the second draw will change, but then that is a conditional probability and not what we have.

14. In the current first-year class of a community college, all the students come from three local high

schools. Schools I, II, III supply respectively 30%, 50%, 20% of the students. The failure rate of students is 4%, 2%, 6% respectively. Hint: a tree may be helpful for this problem.

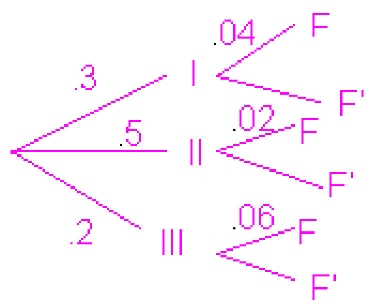
(a) What is the probability that a student will fail given that they came from school I?
0.04
Notice that this probability is a conditional probability so it is located in one of the secondary braches of the tree below.

(b) What is the probability that a student chosen at random will have come from school II and passed?
(.5)(.98) = .49
This probability is for an intersection of two events, so it is found at one of the leaves of the tree, and is computed by multiplying all of the probabilities along the way (following the braches) to the leaf.

(c) What is the probability that a student chosen at random will fail?
(.3)(.04)+(.5)(.02)+(.2)(.06) = .034
This probability is found by using the tree like a probability distribution table, simply identify the leaves that have this event (F), and then sum their probabilities.

(d) Given that a student fails, what is the probability that he or she came from school III?
[(.2)(.06)] / .034 ≈ .353
This conditional probability is a Bayes type. The events are reversed from the events of conditional probabilities in the secondary braches of the tree. To compute these, I like to use the formula for computing conditional probabilities as a guide.

Here is a tree diagram to help out with the computations for problem 14.



15. Angie has a six-sided red die numbered 1 through 6 and a six-sided green die numbered 4 through 9. She rolls the dice.

(a) What is the probability that the dice show the same number on the uppermost faces?
3/36 = 1/12

(b) What is the probability that the sum of the numbers on the uppermost faces is 7 or 8?
3/36 + 4/36 = 7/36
These are mutually exclusive events, so we do not need to subtract out the probability of the intersection of the two events.

(c) What is the probability that the red die shows an odd number and the green die shows a multiple of 3?
6/36 = 1/6
This is an intersection of two events. Above I simply found the probability by counting the number of outcomes in the intersection. I like to think of this as a global approach. Another way to compute the probability of the intersection is to use the formula,

$P(A \cap B) = P(A) \cdot P(B \mid A)$, which yields $\frac{3}{6} \cdot \frac{2}{6} = \frac{6}{36} = \frac{1}{6}$. Notice that since these are independent events I could have used the simpler formula, which states that the probability of the intersection equals the product of their probabilities. We get the same result either way, simply different ways to conceptualize the problem.

(d) What is the probability that the red die shows an odd number or the green die shows a multiple of 3?
18/36 + 12/36 – 6/36 = 24/36 = 2/3
These are not mutually exclusive events, so we must subtract out the probability of the intersection of the two events.

16. Jane has two friends who do not know each other. Each of them has heard the same rumor. The probability that each will tell Jane is 60%. What is the probability that Jane does not hear the rumor from either of these friends?
(.4)(.4) = .16, this assumes independence of course.

17. A factory produces screws, which are packaged in boxes of 30. Four screws are selected from each box for inspection. The box fails inspection if two or more of these four screws are defective. What is the probability that a box containing 2 defective screws will pass inspection?
To pass inspection means that the sample must contain either no defects or exactly 1 defect. Any more and the box would be rejected, so to compute we need to determine the probability of no defectives and add it to the probability of exactly 1 defective:

$$\frac{C(28,4) \cdot C(2,0)}{C(30,4)} + \frac{C(28,3) \cdot C(2,1)}{C(30,4)} \approx .98621$$

18. Box *A* has 2 blue marbles and 3 yellow marbles. Box *B* has 1 blue marble and 2 yellow marbles. A box is chosen, and a marble is selected from it. The probability of selecting box *A* is ¾. If the selected marble is yellow, find the probability that it came from box *B*.

$$P(B \mid Y) = \frac{P(B \cap Y)}{P(Y)} = \frac{\frac{1}{4} \cdot \frac{2}{3}}{\frac{3}{4} \cdot \frac{3}{5} + \frac{1}{4} \cdot \frac{2}{3}} \approx 0.27$$

You may find a tree helpful in organizing this one. This is another Bayes type conditional probability.

19. Nuclear power plants have a threefold security system, each of which is 98% reliable, and independent of the others, to prevent unauthorized persons from entering the premises. What is the probability that an unauthorized person will:

(a) Get through all three security systems?
(.02)(.02)(.02) = 0.000008

(b) Get through the first two systems, but not the third?
(.02)(.02)(.98) = 0.000392

20. Two cards are to be drawn, without replacement, from a deck of 52 cards and the number of kings recorded.

(a) Find the sample space for this experiment and the probability of each outcome in that sample space.

S = {0, 1, 2}

Outcome	Probability
0	$\frac{C(48,2)}{C(52,2)} \approx 0.8507$
1	$\frac{C(4,1)\cdot C(48,1)}{C(52,2)} \approx 0.1448$
2	$\frac{C(4,2)}{C(52,2)} \approx 0.0045$
	1.0

(b) Find the probability that at least one king will be selected.

1 – P(no kings) = 1 - .8507 = . 1493

(c) Find the odds that exactly one king will be selected.

P(one king) = 192/1326 = 32/221 which translates to 32 to 189 as odds in favor of getting exactly one king.

21. It is said that the odds of seeing the ghost of Windsor Buckingham in Maplehurst castle are 2 to 63. What is the probability of seeing the ghost?

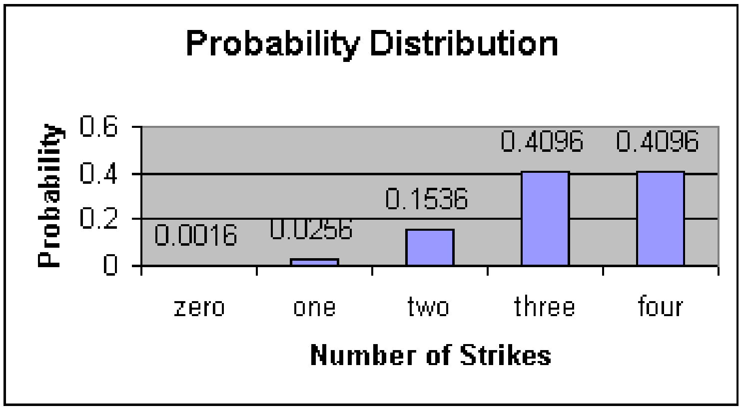
2/65

22. Lefty "Can't Miss" LeMarc can throw a strike with a probability of 0.8. Assuming that each throw is independent of previous throws, find the probability distribution for the number of strikes that Lefty could throw out of 4 pitches. Also graph the probability histogram for this distribution. What is the probability that Lefty could walk a batter on four consecutive pitches (i.e. throw 4 consecutive balls, or non-strikes for non-baseball folks)?

This experiment has all of the characteristics of a Binomial Experiment, so we can use the related formula to compute the probabilities.

Outcome (# of strikes out of four throws)	Probability of outcome
0	$C(4,0)(.8)^0(.2)^4 = .0016$
1	$C(4,1)(.8)^1(.2)^3 = .0256$
2	$C(4,2)(.8)^2(.2)^2 = .1536$
3	$C(4,3)(.8)^3(.2)^1 = .4096$
4	$C(4,4)(.8)^4(.2)^0 = .4096$
	Sum of all probabilities = 1.0

Below is the probability distribution histogram. I used Microsoft Excel to produce the histogram. Actually I call these bar charts, to be a genuine histogram I like to see the rectangles (bars) touching.



The probability that Lefty could throw 4 consecutive balls is 0.0016.

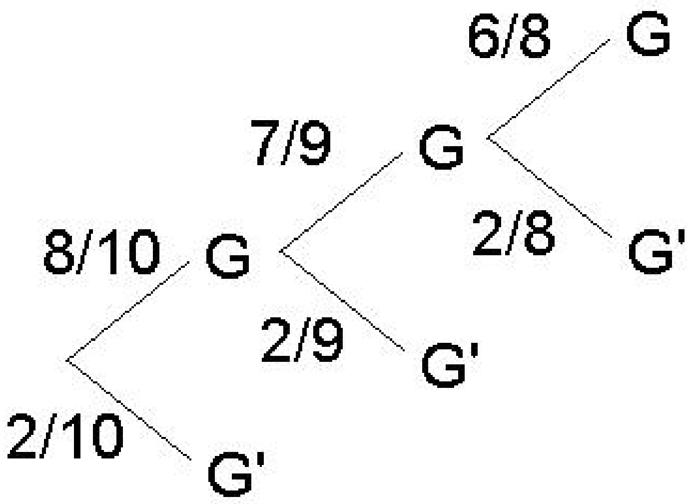
23. Find the expected number of strikes out of 4 pitches for the probability distribution in problem 22 above.
We could multiply each outcome with its probability in the table above and sum these products to find the expected value of this random variable, but for binomial experiments we have a nice short formula that we can use instead: $E(X) = np = 4(0.8) = 3.2$. Recall that n is the number of trials in the experiment (here it is 4 for the four pitches), and p is the probability of a success, which in this case is 0.8 – the probability of a strike being thrown, so we defined a success as a strike.

24. In a game three coins are tossed. If you bet \$1 you win \$1 for every heads that comes up. Find your expected winnings for a \$1 bet in this game. State why this game is either a fair game or is not a fair game.

Net winnings	Probability
-1	1/8
0	3/8
1	3/8
2	1/8

E(X) = (-1)(1/8)+0(3/8)+1(3/8)+2(1/8) = ½ = \$0.50, and since this expected value is not zero, the game is not fair.

25. James has a box that contains 8 good apples and 2 rotten apples. Jamie draws apples from the box until she draws a rotten apple or until she has three apples. If this game is to be fair, how much should Jamie pay to play the game if James pays out \$1 for each draw?
A tree diagram is helpful to set up this problem.



Let p = the amount to pay.

X (net winnings)	P(X)
1 – p	2/10 = 1/5
2 – p	(8/10)(2/9) = 8/45
3 – p	(4/5)(7/9)(3/4)+(4/5)(7/9)(1/4) = 28/45

Since the game is to be fair, the expected value must be zero; therefore:
$(1/5)(1 - p)+(8/45)(2 - p)+(28/45)(3 - p) = 0$. Solve this equation for p and you get \$2.42 should be paid to play the game.