Introduction to mathematical Statistics Lecture 1

1. Review of Probability, the Monty Hall Problem

$$P(Win\_By\_Stay)$$
$$= P(WBST \mid First\_Door\_Chosen\_Has\_Prize) \cdot P(F.D.C\_Has\_Prize)$$
$$+ P(WBST \mid First\_Door\_Chosen\_Has\_No\_Prize) \cdot P(F.D.C\_Has\_No\_Prize)$$
$$= 1 \cdot \frac{1}{3} + 0 \cdot \frac{2}{3} = \frac{1}{3}$$

$$P(Win\_By\_Switch)$$
$$= P(WBSW \mid First\_Door\_Chosen\_Has\_Prize) \cdot P(F.D.C\_Has\_Prize)$$
$$+ P(WBSW \mid First\_Door\_Chosen\_Has\_No\_Prize) \cdot P(F.D.C\_Has\_No\_Prize)$$
$$= 0 \cdot \frac{1}{3} + 1 \cdot \frac{2}{3} = \frac{2}{3}$$

- Please be aware that the pre-requisite for AMS312 is AMS311.
- All exams including quizzes are close book exams.
- Please review Chapters 1, 2, 3 of our text book.
- Please preview Chapter 4 of our text book.

2. Review of Normal Distribution

$$X \sim N(\mu, \sigma^2)$$

1) c.d.f. (Cumulative Distribution Function): $F(x) = P(X \le x) = \int_{-\infty}^{x} f(u)du$

2) p.d.f. (Probability Distribution Function): $f(x) = \frac{1}{\sqrt{2\pi}\sigma} e^{\frac{-(x-\mu)^2}{2\sigma^2}}$

$$x \in (-\infty, +\infty), \mu \in (-\infty, +\infty), \sigma \in (0, +\infty)$$

3) Mathematical expectations:

The **mathematical expectation** of any function g(X) of a random variable X is defined as: $E[g(X)] = \int_{-\infty}^{+\infty} g(x) * f(x)dx$

Special cases of mathematical expectations:

a. **Mean**:

$$\mu = E(X) = \int_{-\infty}^{+\infty} xf(x)dx$$

b. **Variance**:

$$\sigma^2 = Var(X) = E(X-\mu)^2 = \int_{-\infty}^{+\infty} (x-\mu)^2 f(x)dx$$

c. The **Moment Generation Function (m.g.f.)** is defined as a special mathematical expectation with g(X) = $E(e^{tX})$

$$M_X(t) = E(e^{tX}) = \int_{-\infty}^{\infty} e^{tx} f(x)dx$$

Homework Assignment #1 (Due before class on Friday, February 4)

Q1.

Derive the moment generating function (mgf) for the Normal Distribution $X \sim N(\mu, \sigma^2)$

Q2.

$X_1 \sim N(\mu_1, \sigma_1^2)$ & $X_2 \sim N(\mu_2, \sigma_2^2)$, *furthermore* X_1 & X_2 *are independent. Please derive the distribution of:*

(1) $X_1 + X_2$

(2) $X_1 - X_2$

(3) 3 $X_1 - 2X_2 + 5$