Mathematics 120
Spring 2006

# Homework Assignment No. 6

1. Use Lagrange multipliers to find the maximum of $f(x, y) = 3x+4y$ subject to the constraint $x^2 + 4y^2 = 1$

2. Find the maximum and the minimum of

$$f(x, y, z) = x - y - z$$

subject to the constraints

$$\begin{aligned} x^2 + y^2 + z^2 &= 6 \\ x + y + z &= 0 \end{aligned}$$

3. The planes $x + y - z - 2w = 1$ and $x - y + z + \overset{w}{2} = 2$ intersect in a set $F$ in $\Re^4$. Find the point in $F$ that is nearest to the origin.

4. Let $a_1, \ldots, a_n$ be $n$ postive real numbers. Use the method of Lagrange multipliers to show that

$$(a_1 \cdot a_2 \cdots a_n)^{1/n} \leq \frac{a_1 + a_2 + \cdots + a_n}{n}$$

where equality holds if and only if $a_1 = \cdots = a_n$. (Find the local extreme values of $(a_1 \cdot a_2 \cdots a_n)^{1/n}$ subject to $a_1 + \cdots + a_n = const.$)

H. W. #6



#1:

$$F(x,y) = f(x,y) + \lambda g(x,y)$$
$$= 3x + 4y + \lambda(x^2 + 4y^2 - 1)$$

$$\left.\begin{aligned} \frac{\partial F}{\partial x} &= 3 + 2\lambda x = 0 \\ \frac{\partial F}{\partial y} &= 4 + 8\lambda y = 0 \\ x^2 + 4y^2 &= 1 \end{aligned}\right\} \Rightarrow \begin{cases} x = \frac{3}{13}\sqrt{13} \\ y = \frac{1}{13}\sqrt{13} \end{cases} \text{ or } \begin{cases} x = -\frac{3}{13}\sqrt{13} \\ y = \frac{1}{13}\sqrt{13} \end{cases}$$

$\downarrow$ $f = \sqrt{13}$, $\updownarrow$ maximum

$\downarrow$ $f = -\sqrt{13}$; $\uparrow$ minimum

#2:

$$F(x,y,z) = f(x,y,z) + \lambda_1 g_1(x,y,z) + \lambda_2 g_2(x,y,z)$$
$$= (x - y - z) + \lambda_1(x^2 + y^2 + z^2 - 6) + \lambda_2(x + y + z - 0)$$

$$\begin{cases} \frac{\partial F}{\partial x} = 0 \\ \frac{\partial F}{\partial y} = 0 \\ \frac{\partial F}{\partial z} = 0 \end{cases} \Rightarrow \cdots\cdots$$

#3: Equivalent prob: minimize $x^2 + y^2 + z^2$

s.t. $x + y - z - 2W = 1$

$x - y + z + 2W = 2$

$\Rightarrow F(x,y,z) = \cdots\cdots$

#4: Equivalent to: Maximize/minimize: $(x_1 x_2 \cdots x_n)^{\frac{1}{n}}$

s.t.: $x_1 + x_2 + \cdots + x_n = nC$

C: some constant;

(see the attached paper for H.W. #4.)