#2: $J(f) = \left[\frac{\partial f_1}{\partial t}, \frac{\partial f_2}{\partial t}\right]^T = [-\sin t, \cos t]$

Mathematics 120
Spring 2006

Homework Assignment No. 2

1. Find the Jacobian matrix of $F(x,y,z) = (\overset{F_1}{\sin(xy)}, \overset{F_2}{\cos(xy)})$. $\begin{bmatrix} \frac{\partial F_1}{\partial x} & \frac{\partial F_1}{\partial y} \\ \frac{\partial F_2}{\partial x} & \frac{\partial F_2}{\partial y} \end{bmatrix} = \begin{bmatrix} y\cos(xy) & x\cos(xy) \\ -y\sin(xy) & -x\sin(xy) \end{bmatrix}$
2. Find the derivative matrix of $f(t) = (\overset{f_1}{\cos t}, \overset{f_2}{\sin t})$.
3. Find the gradient of $f(s,t) = \ln(st)$. $\left(\frac{\partial f}{\partial s}, \frac{\partial f}{\partial t}\right) = \nabla f$.
4. Find the tangent plane to $f(x,y) = x^4 - 3y^2 + 3$ at $(x,y) = (1,1)$.
5. Find a linear approximation to $f(x,y,z) = (x^2 - y^2, yz)$ at $(x,y,z) = (2,2,1)$. $f(x,y,z) \doteq f(2,2,1) + \nabla f(2,2,1) \cdot (x-2, y-2, z-1)$
6. Find the directions of maximum descent and maximum ascent of $f(x,y) = \arctan(x^2+y^2)$ at $(x,y) = (3,4)$. $\nabla f(3,4)$, $-\nabla f(3,4)$
7. Sketch the level curves $f(x,y) = c$, if $f(x,y) = xy$ and $c = \pm 1, \pm 2$
8. Find the Taylor approximation of degree 3 of $f(x) = x \ln x$ at $x = 1$.
9. Find the Taylor approximation of degree 2 of $f(x,y) = y \sin x$ at $(x,y) = \left(\frac{\pi}{2}, 1\right)$.
10. Find the Taylor approximation of degree 2 of $f(x,y,z) = x + y^2 + z$ at $(x,y,z) = (0,0,0)$.

Follow Example #17 on Page 19.

or use Taylor's thm for multivariables case:

$$f(x_0+h) = f(x_0) + \sum_{i=1}^{n} h_i \frac{\partial f}{\partial x_i}(x_0) + \sum_{i,j=1}^{n} h_i h_j \frac{\partial f}{\partial x_i \partial x_j}(x_0) + \cdots$$

$x = (x_1, x_2, \cdots x_n)$, $h = (h_1, h_2, \cdots h_n)$,