Mathematics 120
Fall Spring 2006

Homework Assignment No. 1

1. Show that the vectors

$$\vec{v}_1 = \begin{bmatrix} 1 \\ 2 \\ 3 \end{bmatrix}, \vec{v}_2 = \begin{bmatrix} 4 \\ 5 \\ 6 \end{bmatrix}, \vec{v}_3 = \begin{bmatrix} 7 \\ 8 \\ 9 \end{bmatrix}$$

are linearly independent. $a\vec{v}_1 + b\vec{v}_2 + c\vec{v}_3 = 0 \Rightarrow a = b = c = 0$

2. Show that the vectors

$$\vec{v}_1 = \frac{1}{2}\begin{bmatrix} 1 \\ 1 \\ 1 \\ 1 \end{bmatrix}, \vec{v}_2 = \frac{1}{2}\begin{bmatrix} 1 \\ 1 \\ -1 \\ -1 \end{bmatrix}, \vec{v}_3 = \frac{1}{2}\begin{bmatrix} 1 \\ -1 \\ 1 \\ -1 \end{bmatrix}, \vec{v}_4 = \frac{1}{2}\begin{bmatrix} 1 \\ -1 \\ -1 \\ 1 \end{bmatrix}$$

form an orthonormal basis of $\Re^4$ and find the coordinates of an arbitrary vector

$$\vec{b} = \begin{bmatrix} b_1 \\ b_2 \\ b_3 \\ b_4 \end{bmatrix}$$

show: $V_i \cdot V_j = \begin{cases} 1 & i = j \\ 0 & i \neq j. \end{cases}$

with respect to this basis. $\Rightarrow$ the coefficients of $\vec{V_i}$ is $\vec{b} \cdot \vec{V_i}$

3. Find all (real) eigenvectors of the following matrices:

(a)

$$A = \begin{bmatrix} 3 & 4 \\ 4 & 3 \end{bmatrix}$$

① find eigenvalues from $|A - \lambda I| = 0$

(b)

$$B = \begin{bmatrix} 1 & 1 \\ 0 & 1 \end{bmatrix}$$

② then find eigenvectors for each eigenvalue.

(c)

$$C = \begin{bmatrix} 3 & 4 & 0 & 0 \\ 4 & 3 & 0 & 0 \\ 0 & 0 & 1 & 1 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

4. For the matrix

$$A = \begin{bmatrix} \frac{41}{25} & 0 & \frac{12}{25} \\ 0 & 1 & 0 \\ \frac{12}{25} & 0 & \frac{34}{25} \end{bmatrix}$$

find an orthonormal basis of eigenvectors and a matrix S so that $S^T AS$ is a diagonal matrix.

$\Rightarrow$. Follow Example #5 in page 6 in notes.

(skip) 5. Sketch the solution sets of the following quadratic form:

(a) $x^2 - y^2 = 1$

(b) $x^2 - 2xy + y^2 = 1$

(c) $x^2 - 3xy + y^2 = 1$

(d) $x^2 - xy + y^2 = 1$

6. Find the maximum and the minimum of $f(x, y) = x^2 + y^2$ with respect to the following constraints, or state that no maximum or minimum exists:

(a) $x^2 - y^2 = 1$ minimum: $f(x_0, y_0) = 1$. at $(x_0, y_0) = (1, 0)$ or $(-1, 0)$

(b) $x^2 - 2xy + y^2 = 1$ $\Rightarrow (x-y)^2 = 1$. minimum: $\frac{1}{2} = f(x_0, y_0)$. $(x_0, y_0) = \pm\left(\frac{1}{2}, \frac{1}{2}\right)$

(c) $x^2 - 3xy + y^2 = 1$

(d) $x^2 - xy + y^2 = 1$

$\Rightarrow$ follow Example #6 on Page 8.