1: (a): $\begin{bmatrix} 0 & 1 & 1 \\ 3 & -1 & 1 \\ 1 & 1 & -3 \end{bmatrix} (x, y, z)^T = (6, -7, -14)^T \Rightarrow \begin{bmatrix} 0 & 1 & 1 & 6 \\ 3 & -1 & 1 & -7 \\ 1 & 1 & -3 & -14 \end{bmatrix} \xRightarrow{R_1 \leftrightarrow R_3} \begin{bmatrix} 1 & 1 & -3 & -14 \\ 3 & -1 & 1 & -7 \\ 0 & 1 & 1 & 6 \end{bmatrix}$

$\xRightarrow{R_2 - 3R_1} \begin{bmatrix} 1 & 1 & -3 & -14 \\ 0 & -4 & 10 & 35 \\ 0 & 1 & 1 & 6 \end{bmatrix} \xRightarrow{R_2 \leftrightarrow R_3} \begin{bmatrix} 1 & 1 & -3 & -14 \\ 0 & 1 & 1 & 6 \\ 0 & -4 & 10 & 35 \end{bmatrix} \xrightarrow{R_3 + 4R_2} \begin{bmatrix} 1 & 1 & -3 & -14 \\ 0 & 1 & 1 & 6 \\ 0 & 0 & 14 & 59 \end{bmatrix}$

$\Rightarrow \begin{cases} x + y - 3z = -14 \\ y + z = 6 \\ 14z = 59 \end{cases} \Rightarrow \begin{cases} x = -14 - \frac{25}{14} - 3z \\ y = 6 - \frac{59}{14} = \frac{25}{14} \\ z = 59/14 \end{cases}$

Mathematics 120
Spring 2006

Discussion No. 6

1. Find all solutions of the given linear system, using the Gauss method with back substitution.

(a)

$$\begin{aligned} y + z &= 6 \\ 3x - y + z &= -7 \\ x + y - 3z &= -14 \end{aligned}$$

(b)

$$\begin{aligned} x + 4y - 2z &= 4 \\ 2x + 7y - z &= -2 \\ 2x + 9y - 7z &= 1 \end{aligned}$$

(c)

$$\begin{aligned} w - 3x + 2y - z &= 8 \\ 3w - 7x \qquad + z &= 0 \end{aligned}$$

(d)

$$\begin{aligned} w + x + y &= 3 \\ w + x + y + z &= 4 \\ w - x + y - z &= 0 \\ w - 2x + 2y + z &= 2 \end{aligned}$$

(?) 2. Find all solutions of

$$\begin{bmatrix} 1 & 1 & 0 & 3 & 0 \\ 0 & 0 & 1 & -1 & 0 \\ 0 & 0 & 0 & 0 & 1 \\ 0 & 0 & 0 & 0 & 0 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \\ x_3 \\ x_4 \\ x_5 \end{bmatrix} = \begin{bmatrix} -4 \\ 0 \\ -2 \\ 0 \end{bmatrix} \Rightarrow \begin{cases} x_1 + x_2 + 3x_4 = -4 \\ x_3 - x_4 = 0 \quad \Rightarrow x_3 = x_4 = s \\ x_5 = -2 \\ 0 = 0 \end{cases}$$

Which particular solution satisfies $x_2 = 2, x_3 = 1$?

$\Rightarrow \begin{cases} x_1 = -4 - r - 3s, \\ x_2 = r \\ x_3 = s \\ x_4 = s \\ x_5 = -2 \end{cases} \Rightarrow \begin{pmatrix} x_1 \\ x_2 \\ x_3 \\ x_4 \\ x_5 \end{pmatrix} = \begin{pmatrix} -4 - r - 3s \\ r \\ s \\ s \\ -2 \end{pmatrix}$

When $x_2 = 2 = r$, $x_3 = 1 = s$

$\Rightarrow (x_1, x_2, \ldots x_5) = (-9, 2, 1, 1, -2)$;