

## Analysis of Covariance (ANCOVA)

ANOVA+Regression

$$y_{ij} = \mu + \tau_i + \beta_1 x_{ij} + e_{ij}$$

### Uses of ANCOVA

- Account (adjust) for “pre-existing” condition
  - initial weight
  - soil moisture/fertility at planting
  - baseline value (y at start of experiment)
- Convenient alternative to regression contrasts (orthogonal polynomials) in treatment design with quantitative treatment levels

### What does ANCOVA Accomplish?

- Adjusts the treatment mean to a common X when the randomization falls short (ONLY when the treatment and X are independent!!!)
- Reduces the magnitude of the error variance ( $\sigma_e^2$ ) – like blocking

### Glossary of ANCOVA

-General (separate slopes) ANCOVA model

$$y_{ij} = \mu + \tau_i + \beta_i x_{ij} + e_{ij}$$

```
PROC MIXED;  
CLASS TRT;  
MODEL Y=TRT X(TRT); OR MODEL Y=TRT X X*TRT;
```

- Common slopes ANCOVA model

$$y_{ij} = \mu + \tau_i + \beta x_{ij} + e_{ij}$$

```
PROC MIXED;  
CLASS TRT;  
MODEL Y=TRT X;
```

### Example 1: Completely Randomized Design(CRD)

## 4 treatments, 8 e.u. per treatment

Obs	trt	x	y
1	c	385	1.91
2	c	413	2.55
3	c	435	2.03
4	c	403	1.88
5	c	432	2.33
6	c	439	2.11
7	c	407	2.54
8	c	399	1.86
9	1	330	1.55
10	1	340	1.92
11	1	353	2.31
12	1	384	2.26
13	1	325	1.48
14	1	374	2.54
15	1	323	1.30
16	1	345	2.21
17	2	421	2.64
18	2	379	1.81
19	2	391	2.09
20	2	371	2.36
21	2	355	1.62
22	2	364	2.08
23	2	389	2.44
24	2	373	1.69
25	3	345	1.46
26	3	380	1.26
27	3	337	1.40
28	3	414	2.31
29	3	368	2.08
30	3	345	1.56
31	3	372	1.77
32	3	410	1.77

Run 1: ANOVA without Covariate

$$y_{ij} = \mu + \tau_i + e_{ij}$$

```
PROC MIXED;
  CLASS TRT;
  MODEL Y=TRT;
  LSMEANS TRT/DIFF;
RUN;
```

Cov Parm	Estimate
Residual	0.1380

Type 3 Tests of Fixed Effects

Effect	Num DF	Den DF	F Value	Pr > F
trt	3	28	2.32	<b>0.0964</b>

Least Squares Means  
Standard

Effect	trt	Estimate	Error	DF	t Value	Pr >  t
trt	1	1.9463	0.1314	28	14.82	<.0001
trt	2	2.0913	0.1314	28	15.92	<.0001
trt	3	1.7013	0.1314	28	12.95	<.0001
trt	c	2.1513	0.1314	28	16.38	<.0001

Differences of Least Squares Means  
Standard

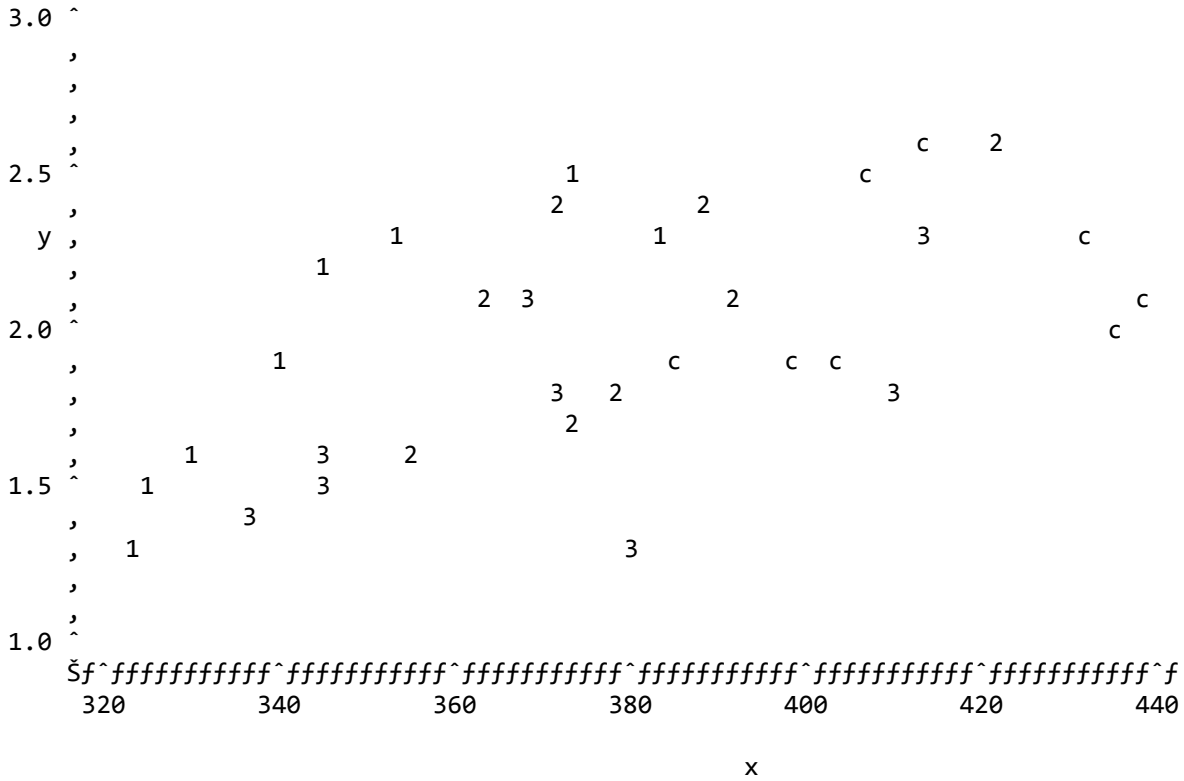
Effect	trt	_trt	Estimate	Error	DF	t Value	Pr >  t
trt	1	2	-0.1450	0.1858	28	-0.78	0.4416
trt	1	3	0.2450	0.1858	28	1.32	0.1979
trt	1	c	-0.2050	0.1858	28	-1.10	0.2792
trt	2	3	0.3900	0.1858	28	2.10	<b>0.0449</b>
trt	2	c	-0.06000	0.1858	28	-0.32	0.7491
trt	3	c	-0.4500	0.1858	28	-2.42	<b>0.0221</b>

```
PROC MEANS DATA=ANCOV2;
BY TRT;
VAR X;
RUN;
```

T	N	Mean	Std Dev	Minimum	Maximum
1	8	346.7500000	22.4737943	323.0000000	384.0000000
2	8	380.3750000	20.3325039	355.0000000	421.0000000
3	8	371.3750000	29.1985200	337.0000000	414.0000000
C	8	414.1250000	19.3718315	385.0000000	439.0000000

```
PROC PLOT;
PLOT Y*X=TRT;
RUN;
```

Plot of y\*x. Symbol is value of trt.



**Run 2: ANOVA with Covariate, separate slopes :**

$$y_{ij} = \mu + \tau_i + \beta x_{ij} + e_{ij}$$

**Test for equal slopes:  $H_0$ : all  $\beta_i$  equal ( $\beta$ )**

```
PROC MIXED;  
CLASS TRT;  
MODEL Y=TRT X(TRT)/NOINT SOLUTION;  
CONTRAST 'EQUAL SLOPES' X(TRT) 1 0 0 -1,  
X(TRT) 0 1 0 -1,  
X(TRT) 0 0 1 -1;  
RUN;
```

Label	Num DF	Den DF	F Value	Pr > F
equal slopes	3	24	1.51	0.2361

**Fail to reject  $H_0$ :**

**Did not present the Type III sums of squares because they provide no useful information.**

**Run 3: equal slopes model:**

$$y_{ij} = \mu + \tau_i + \beta x_{ij} + e_{ij}$$

```
PROC MIXED;  
CLASS TRT;  
MODEL Y=TRT X/SOLUTION;  
LSMEANS TRT/DIFF;  
LSMEANS TRT/AT X=0;  
ESTIMATE 'INTCPT T=1' INTERCEPT 1 TRT 1 0 0 0;  
ESTIMATE 'INTCPT T=2' INTERCEPT 1 TRT 0 1 0 0;  
ESTIMATE 'INTCPT T=3' INTERCEPT 1 TRT 0 0 1 0;  
ESTIMATE 'INTCPT T=C' INTERCEPT 1 TRT 0 0 0 1;  
ESTIMATE 'MEAN AT T=1' INTERCEPT 1 TRT 1 0 0 0 X 346.75;  
ESTIMATE 'MEAN AT T=2' INTERCEPT 1 TRT 0 1 0 0 X 371.375;  
ESTIMATE 'MEAN AT T=3' INTERCEPT 1 TRT 0 0 1 0 X 380.375;  
ESTIMATE 'MEAN AT T=C' INTERCEPT 1 TRT 0 0 0 1 X 414.125;  
RUN;
```

COV PARM ESTIMATE  
 RESIDUAL 0.08384

SOLUTION FOR FIXED EFFECTS

EFFECT	TRT	ESTIMATE	STANDARD ERROR	DF	T VALUE	PR >  T
INTERCEPT		-2.1243	0.9836	27	-2.16	0.0398
TRT	1	0.4906	0.2152	27	2.28	0.0307
TRT	2	0.2884	0.1653	27	1.75	0.0923
TRT	3	-0.00864	0.1765	27	-0.05	0.9613
TRT	C	0	.	.	.	.
X		0.01032	0.002362	27	4.37	0.0002

TYPE 3 TESTS OF FIXED EFFECTS

EFFECT	NUM DF	DEN DF	F VALUE	PR > F
TRT	3	27	3.95	0.0186
X	1	27	19.10	0.0002

ESTIMATES

LABEL	ESTIMATE	STANDARD ERROR	DF	T VALUE	PR >  T
INTCPT T=1	-1.6337	0.8255	27	-1.98	0.0581
INTCPT T=2	-1.8359	0.9043	27	-2.03	0.0523
INTCPT T=3	-2.1329	0.8832	27	-2.41	0.0228
INTCPT T=C	-2.1243	0.9836	27	-2.16	0.0398
MEAN AT T=1	1.9463	0.1024	27	19.01	<.0001
MEAN AT T=2	1.9983	0.1046	27	19.11	<.0001
MEAN AT T=3	1.7942	0.1046	27	17.16	<.0001
MEAN AT T=C	2.1513	0.1024	27	21.01	<.0001

LEAST SQUARES MEANS

EFFECT	TRT	X	ESTIMATE	STANDARD ERROR	DF	T VALUE	PR >  T
TRT	1	378.16	2.2705	0.1264	27	17.96	<.0001
TRT	2	378.16	2.0683	0.1025	27	20.18	<.0001

TRT	3	378.16	1.7713	0.1036	27	17.09	<.0001
TRT	C	378.16	1.7799	0.1330	27	13.38	<.0001
TRT	1	0.00	-1.6337	0.8255	27	-1.98	0.0581
TRT	2	0.00	-1.8359	0.9043	27	-2.03	0.0523
TRT	3	0.00	-2.1329	0.8832	27	-2.41	0.0228
TRT	C	0.00	-2.1243	0.9836	27	-2.16	0.0398

DIFFERENCES OF LEAST SQUARES MEANS

EFFECT	TRT	_TRT	X	ESTIMATE	STANDARD ERROR	DF	T VALUE
TRT	1	2	378.16	0.2022	0.1651	27	1.22
TRT	1	3	378.16	0.4992	0.1560	27	3.20
TRT	1	C	378.16	0.4906	0.2152	27	2.28
TRT	2	3	378.16	0.2971	0.1463	27	2.03
TRT	2	C	378.16	0.2884	0.1653	27	1.75
TRT	3	C	378.16	-0.00864	0.1765	27	-0.05

DIFFERENCES OF LEAST SQUARES MEANS

EFFECT	TRT	_TRT	PR >  T
TRT	1	2	0.2315
TRT	1	3	0.0035
TRT	1	C	0.0307
TRT	2	3	0.0523
TRT	2	C	0.0923
TRT	3	C	0.9613