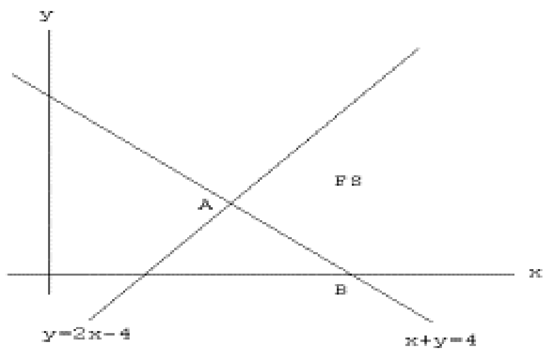
Linear Programming Practice Exam Key

1. Consider the feasible region shown below.
   (a) Determine the coordinates of vertex B. (4, 0)
   (b) Determine the coordinates of vertex A. (8/3, 4/3)
   (c) Write the system of linear inequalities that formed the feasible region.



System of linear inequalities:

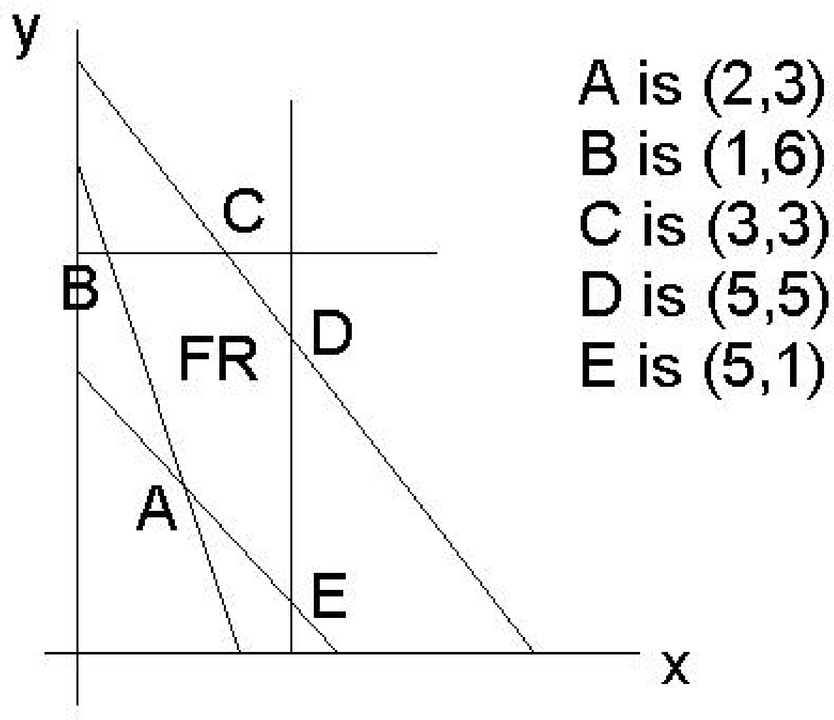
$x + y \geq 4$

$y \leq 2x - 4$

$y \geq 0$

We do not need $x \geq 0$ in this case, for it can be shown that $x \geq 2$ given the other inequalities; however, it doesn't hurt to have $x \geq 0$.

2. The feasible set for a certain linear programming problem is shown below. Determine the values for x and y that minimize 4x+3y. What is the minimum value?



You should find that corner point A is where the minimum occurs and that the minimum value is 17 at that point.

3. Translate the following word problem into a system consisting of an objective function, whether the objective function is to be maximized or minimized, and all constraints. I will need to know what your variables mean, be specific. **DO NOT SOLVE! Simply set up the mathematical model.**

The campaign manager of a candidate for a local political office estimates that each 30-minute speech to a civic group will generate 20 votes, each hour spent on the telephone will generate 10, and each hour spent campaigning in shopping areas will generate 40. The candidate wants to make at least twice as many trips to shopping areas as speeches to civic groups and spend at least 5 hours on the telephone. The campaign manager thinks her candidate can win if he can generate a total of at least 1000 votes by these three methods. How can the candidate meet these goals and keep to a minimum his campaigning time?

I get the following mathematical model:
Minimize $t = (1/2)x + y + z$ where t is in hours,
subject to: $2x \le z$
$y \ge 5$
$20x + 10y + 40z \ge 1000$
with $x \ge 0, y \ge 0, z \ge 0$.

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4. Use the graphing method to solve this linear programming problem. I want to see the feasible set, the coordinates for each of the corner points, and the solution.

   Maximize: $P = 3x + y$
   subject to: $x - y \leq 0$
   $x + 2y \leq 18$
   $y \geq 2$
   $x \geq 0.$

   I found that the maximum occurs at the corner point (6, 6) with a maximum value of 24.

5. Convert the following standard max linear programming problem into its initial simplex tableau.
   Maximize $f = 2x + 3y$,
   subject to: $x + y \leq 2$
   $x \leq 2$
   $x \geq 0, y \geq 0.$

   Initial Simplex Tableau:

$$\begin{array}{ccccc|c} x & y & s_1 & s_2 & f & \\ 1 & 1 & 1 & 0 & 0 & 2 \\ 1 & 0 & 0 & 1 & 0 & 2 \\ \hline -2 & -3 & 0 & 0 & 1 & 0 \end{array}$$

6. Identify the row and column of the next pivot

$$\begin{array}{ccccc|c} x & y & s_1 & s_2 & f & \\ 2 & 0 & 1 & 1 & 0 & 40 \\ 1 & 1 & 0 & 3 & 0 & 60 \\ \hline -1 & 0 & 0 & -5 & 1 & 38 \end{array}$$

   , and then perform the pivot. After pivoting, state the values of all variables including the slack variables. State if this new tableau is the final one, or if there is more pivoting to be done.

   Next pivot element is in row 2, column 4.

   The table looks like this after the above pivot:

$$\begin{array}{ccccc|c} x & y & s_1 & s_2 & f & \\ 5/3 & -1/3 & 1 & 0 & 0 & 20 \\ 1/3 & 1/3 & 0 & 1 & 0 & 20 \\ \hline 2/3 & 5/3 & 0 & 0 & 1 & 138 \end{array}$$

   The values of the variables are: $x = 0$, $y = 0$, $s_1 = 20$, $s_2 = 20$, and $f = 138$.
   This is the final table (no more pivoting is needed).

7. Use the simplex algorithm to solve the following linear programming problem:
   Maximize: $f = 2x + 3y + z$
   subject to: $x + z \leq 8$
   $y - z \leq 10$
   $x - y - z \leq 12$

$x \geq 0$, $y \geq 0$, $z \geq 0$.

I found the following solution: $x = 0$, $y = 18$, $z = 8$, $s_1 = 0$, $s_2 = 0$, $s_3 = 38$, and $f = 62$.