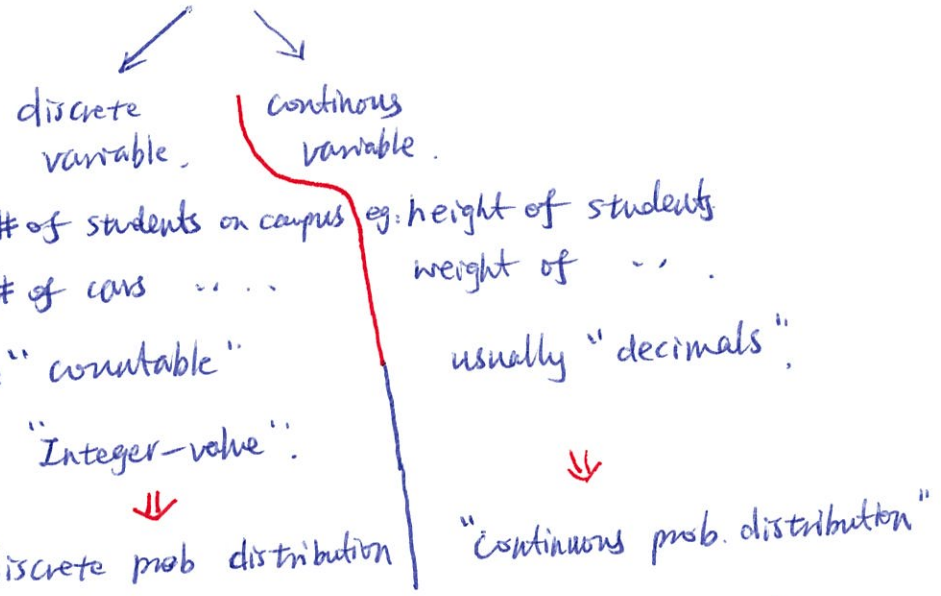
Sec 5.1: Prob. distributions:

I: Recall: Random Variable (variable whose value determined by chance)



eg #1: Construct prob. distribution for rolling a die:

outcome (X):	1,	2,	3,	4,	5,	6
p(X):	$\frac{1}{6}$	$\frac{1}{6}$	$\frac{1}{6}$	$\frac{1}{6}$	$\frac{1}{6}$	$\frac{1}{6}$

eg #2 construct prob. distribution for rolling 2 coins:

outcome: {(H,H), (H,T), (T,H), (T,T)}.

X: # of heads: 2, 1, 1, 0

$\Rightarrow$

X	0	1	2
p(X):	$\frac{1}{4}$	$\frac{2}{4}$	$\frac{1}{4}$

eg #3: What if 3 coins?

outcome: (H,H,H), (H,T,H), (H,H,T), (H,T,T)
(T,H,H), (T,T,H), (T,H,T), (T,T,T)
3, 2, 2, 1, 2, 1, 1, 0

X: # of heads:	0	1	2	3
p(X):	$\frac{1}{8}$	$\frac{3}{8}$	$\frac{3}{8}$	$\frac{1}{8}$

Note: prob. distribution need to satisfy:

(1): for all $p(x)$: $0 \le p(x) \le 1$.

(2): sum up all $p(x)$: $\sum p(x) = 1$.

(Verify 3 examples in previous page.)

eg: Check whether the following is a prob. distribution?

a):

x	4	6	8	10
$p(x)$	-0.6	0.2	0.7	0.5

$\Rightarrow$ $-0.6 < 0$, condition (1) breaks!

$\Rightarrow$ NOT a prob. distribution!

b):

x	1	3	5	7	9
$p(x)$	0.3	0.1	0.2	0.4	0.1

$\Rightarrow$ condition (1): ✓

condition (2): $0.3+0.1+0.2+0.4+0.1 = 1.1 > 1$ ✗

$\Rightarrow$ NOT a prob. distribution.

c):

x	8	9	10
$p(x)$	$\frac{2}{3}$	$\frac{1}{6}$	$\frac{1}{6}$

$\Rightarrow$ condition (1): ✓

condition (2): ✓

$\Rightarrow$ Yes, it's a prob. distribution!

Introduction to Stats.

Sec 5.2 : Mean, Variance, Expectation.

(I): Recall prob. distribution for tossing 2 coins:

$\{(H,H), (H,T), (T,H), (T,T)\}$

# of heads: 2 1 1 0

Average # of heads (out of this sample)

$$\bar{x} = \frac{2+1+1+0}{4} = 1.$$

Variance out of this sample space:

$$s^2 = \frac{\Sigma (x_i - \bar{x})^2}{n-1}$$

x	$p(x)$	$x \cdot p(x)$	$x^2 \cdot p(x)$
0	$\frac{1}{4}$	0	0
1	$\frac{2}{4}$	$\frac{2}{4}$	$\frac{2}{4}$
2	$\frac{1}{4}$	$\frac{2}{4}$	$4 \cdot \frac{1}{4} = 1$

Now, out of this prob. distribution:

Define: $= E(x)$

Mean $= \mu = \Sigma x p(x) =$ Expectation.

$\Rightarrow E(x) = \Sigma x p(x)$

$= 0 + \frac{2}{4} + \frac{2}{4} = 1$

Q: What does expectation tell us?

A: In the long run, if we toss 2 coins many times, then we expected to get avg. # of heads $= E(x) = 1$.

Variance out of this prob. distri:

$$\sigma^2 = \Sigma x^2 p(x) - \mu^2$$

$= 0 + \frac{2}{4} + \frac{1}{4} \times 4 - 1^2 = \frac{1}{2}$

$\Rightarrow$ s.d. $= \sigma = \sqrt{\frac{1}{2}}$

What if we toss 3 coins many times, (each time we toss 3 coins).

How many # of heads we expected to get?

$\Rightarrow$: Intuitively, it should be 1.5 heads. (because every time we toss a coin on average, we can get 0.5 heads. 3 coins $\Rightarrow 3 \times 0.5 = 1.5$ heads.

$\Rightarrow$ prob. distribution for x: # of heads.

x	$p(x)$	$x \cdot p(x)$	$x^2 \cdot p(x)$
0	$\frac{1}{8}$	0	0
1	$\frac{3}{8}$	$\frac{3}{8}$	$\frac{3}{8}$
2	$\frac{3}{8}$	$\frac{6}{8}$	$\frac{12}{8}$
3	$\frac{1}{8}$	$\frac{3}{8}$	$\frac{9}{8}$

$\Rightarrow E(x) = \Sigma x p(x)$

$= 0 + \frac{3}{8} + \frac{6}{8} + \frac{3}{8} = \frac{12}{8}$

$= 1.5$

Variance $\sigma^2 = \Sigma x^2 p(x) - \mu^2$

$= \ldots$

Eg #1: Lottery Ticket: 1000 tickets sold at $3 each.

offer 1st prize: $1000. ⇒ Q: What's your expected return

2nd prize: $500 if you buy a ticket?

3rd prize: $100

⇒ X: return of a ticket:

	Win 1st prize	2nd	3rd	Nothing
X:	$1000-3=997$	$500-3=497$	97	-3
$p(X)$:	$\frac{1}{1000}$	$\frac{1}{1000}$	$\frac{1}{1000}$	$\frac{997}{1000}$

$$\Rightarrow E(X) = \sum x p(x) = \frac{997}{1000} + \frac{497}{1000} + \frac{97}{1000} - \frac{3\times 997}{1000}$$

$$= -1.4 < 0$$ (should be <0, otherwise they can't make money)

Eg #2: Someone bought $360 for $100,000 term life insurance.

Given the prob. of death rate: 0.000943.

Q: Find your expected return if you bought that insurance:

	survive	die.
(Return) X:	-360	$100,000-360$
$p(X)$:	$1-0.000943$	0.000943

$$\Rightarrow E(X) = \sum x p(x) = -360\cdot(1-0.000943) + (100,000-360)\cdot 0.000943$$

$$= -265.7 < 0$$ (should be <0).

otherwise, the insurance company will lose money).