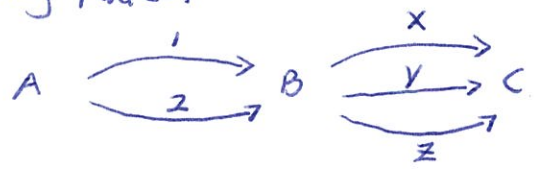
Lecture Notes 4.

Sec 4.4 Counting Rules:

eg:



Q. How many different ways from A to C?

$\Rightarrow \left\{ \begin{matrix} 1X, & 1Y, & 1Z \\ 2X, & 2Y, & 2Z \end{matrix} \right\}$ 6 ways $= 2 \times 3$.

(I): Fundamental Counting Rules. For a sequence of n events.

1st event has k_1 possibilities;
2nd event has k_2 .. ;
...
n^{th} event has k_n .. ;

$\Rightarrow$: Total possibilities of the sequence will be:

$k_1 \cdot k_2 \cdots k_n$

eg: A coin is tossed and a die is rolled,

Q: How many different outcome?

$\Rightarrow$:

Coin	Die
$\left\{ \begin{matrix} H \\ T \end{matrix} \right\}$	$\{1, 2, 4, 5, 3, 6\}$

$2 \times 6 = 12$ outcomes!

eg: Some manager want to make 6-digits IDs for her employee.
How many different IDs

Q1: If she can use digits 1–6 and allow repetition:

$\Rightarrow \underline{6} \times \underline{6} \times \underline{6} \times \underline{6} \times \underline{6} \times \underline{6} = 6^6$.

Q2: What if No repetition?

$\Rightarrow \underline{6} \times \underline{5} \times \underline{4} \times \underline{3} \times \underline{2} \times \underline{1} = 6!$ (6 factorial)

In general, $3! = 3 \times 2 \times 1$

$n! = n \times (n-1) \times \cdots \times 2 \times 1$.

$0! = 1$ $\quad 1! = 1$.

(II). permutation: arrangement of n objects in order!

eg: Some manager want to visit 5 offices in order,
How many possible outcomes?

order:	1	2	3	4	5
case 1:	A	B	C	D	E
case 2:	B	A	C	D	E

→ order difference makes them difference!!!

⇒ total possible outcomes:

$$5 \times 4 \times 3 \times 2 \times 1 = 5!$$

Q: What if she just visit 3 out of 5 places in order?

⇒, $$5 \times 4 \times 3 = \frac{5\times4\times3\times2\times1}{2\times1} = \frac{5!}{(5-3)!}$$

Permutation Rule: Choose r objects out of n objects and put in order.

Total possible outcomes: $$nP_r = \frac{n!}{(n-r)!} \quad (1)$$

eg: Choose 2 musicals out of 9 and play in order,
How many different outcomes?

⇒: $$9P_2 = \frac{9!}{(9-2)!} = \frac{9\times8\times7\times6\times5\times4\times3\times2\times1}{7\times6\times5\times4\times3\times2\times1} = 9\times8 = 72.$$

(III). Combination: select r objects out of n objects. put together!
no order required!

eg: Choose 2 letters from A, B, C.

Q1: How many permutation: AB AC BC
BA CA CB

$$\Rightarrow 6 = {}_3P_2 = \frac{3!}{(3-2)!} = \frac{3\times2\times1}{1} = 6.$$

Q2: How many combinations: 1 + 1 + 1 = 3.
(AB and BA are considered as same in combination).

Combination Rule: $$nC_r = \frac{n!}{(n-r)!\cdot r!} \quad (2)$$

eg: Choose 3 females and 2 males out of
7 females and 5 males

⇒ Q. How many possible outcome?

⇒: $_7C_3 \cdot {}_5C_2$

$= \frac{7!}{(7-3)!\cdot 3!} \cdot \frac{5!}{(5-2)!\,2!} = \dots = 35 \times 10 = 350$

See 4.5: Counting Rule Applications:

eg #1: What's the prob. of getting 4 Aces out of 5 cards that are drawn from a deck?

⇒: $prob = \frac{\#\text{ of outcomes in question/selection}}{\text{total possible outcomes}}$

$= \frac{{}_4C_4 \cdot {}_{48}C_1}{{}_{52}C_5} = \frac{1 \times 48}{\frac{52!}{47!\cdot 5!}} = \dots = \frac{48}{\frac{52\times51\times50\times49\times48\times47!}{47!\cdot 5!}}$

eg #2: Given, there are 4 tools are bad out of 24 total. Randomly select 4 tools out of those 24 tools, find the prob:

Q1: exactly 2 tools are bad.

$prob. = \frac{{}_4C_2 \cdot {}_{20}C_2}{{}_{24}C_4}$

Q2: All are bad. $prob = \frac{{}_4C_4}{{}_{24}C_4}$

Q3: None is bad. ("4 are good") $prob = \frac{{}_{20}C_4}{{}_{24}C_4}$

Q4. At least 1 is bad: consider the "opposite" case: None is bad! $= \bar{E}$

$= E$

$P(E) = 1 - P(\bar{E}) = 1 - Q3 = 1 - \frac{{}_{20}C_4}{{}_{24}C_4}$

eg #3: Select 1 male and 1 female out of 8 couples.
Find the prob. of 2 selected people are couple?

$$\Rightarrow \text{prob} = \frac{\#\text{ of outcome in consideration}}{\text{total possible outcomes}}$$

$$= \frac{8}{{}_8C_1 \cdot {}_8C_1} \neq {}_{16}C_2$$ (It's NOT "select 2 people out 16 people").

eg #4. 2 consumers buy magazine,
out of 6 TV magazines and 8 News magzines.
Find the prob. of both types of magazine are purchased?

$$\Rightarrow \text{prob} = \frac{{}_6C_1 \cdot {}_8C_1}{{}_{14}C_2}$$