Elementary Loss Models

In this section, X denotes a non-negative random variable representing loss amount.

Conditional vs Unconditional pdfs

$$f_{X|X>d}(x \mid x > d) = \begin{cases} 0 \cdots (X \le d) \\ \dfrac{f_X(x)}{\Pr(X > d)} \cdots (X > d) \end{cases}$$

Summing n independent and identically distributed random variables ($X_i \sim X$)

$$S = \sum_{i=1}^{n} X_i$$
$$E[S] = n \cdot E[X]$$
$$Var(S) = n \cdot Var(X)$$

Deductibles – Y_L denotes the random variable representing the amount paid per *loss* by an insurer after a deductible of d is applied.

$$Y_L = (X - d)_+ = \begin{cases} 0 \cdots (X \le d) \\ X - d \cdots (X > d) \end{cases}$$
$$E[(Y_L)^n] = E[(X-d)^n \mid X > d] \cdot \Pr(X > d) = \int_d^{\infty} (x-d)^n \cdot f_X(x)dx$$

Deductibles – Y_P denotes the random variable representing the amount paid per *payment* by an insurer after a deductible of d is applied.

$$Y_P = Y_L \mid Y_L > 0 = X - d \mid X > d$$
$$E[(Y_P)^n] = E[(Y_L)^n \mid Y_L > 0] = \frac{E[(Y_L)^n]}{\Pr(X > d)} = E[(X-d)^n \mid X > d] = \frac{\int_d^{\infty} (x-d)^n \cdot f_X(x)dx}{\Pr(X > d)}$$

Policy Limits – Y denotes the random variable representing the amount paid per loss by an insurer after a policy limit of L is applied.

$$Y = X \wedge L = \begin{Bmatrix} X \cdots (X \le L) \\ L \cdots (X > L) \end{Bmatrix}$$

$$E[Y^n] = E[X^n \mid X \le L] \cdot \Pr(X \le L) + L^n \cdot \Pr(X > L) = \int_0^L x^n \cdot f_X(x)dx + L^n \cdot \int_L^\infty f_X(x)dx$$

Proportional Insurance – Y denotes the random variable representing the amount paid per loss by an insurer that insures a proportion α of the loss ($0 < \alpha < 1$).

$$Y = \alpha \cdot X$$

$$E[Y^n] = \alpha^n \cdot E[X^n]$$

$$Var(Y) = \alpha^2 \cdot Var(X)$$

Often Tested Facts

1. $X \sim U[0,c] \Rightarrow X - d \mid X > d \sim U[0, c-d]$

2. $X \sim EX(mean = \mu) \Rightarrow X - d \mid X > d \sim EX(mean = \mu)$
   (memoryless property of exponential distribution)

3. $X = (X \wedge c) + (X - c)_+$