Derivatives Markets (Part 1)

A **derivative** is a financial instrument (an agreement between two people) that has a value determined by the price of something else.

Motivation to use of derivatives:
- Risk Management (Hedging)
- Speculation
- Reduce Transaction Costs (Lower Costs)
- Regulatory Arbitrage (Circumvent Regulatory Restrictions and Taxes)

Perspectives on Derivatives
- End-User
- Market-Maker
- Economic Observer

Financial Markets Role in Risk-Sharing

Diversifiable Risk is risk that is unrelated to other risks. For example, the risk that your house will burn down is diversifiable risk. If many investors share a small piece of this risk, by taking out insurance, then it has no significant effect on anyone.

Non-diversifiable Risk is risk that does not vanish when spread across many investors. For example, the risk of a stock market crash is non-diversifiable.

Long and Short Positions

Generally, the term **long** is used to describe the buyer and the term **short** is used to describe the seller.

Buying and Selling Financial Assets (Consider stocks)

Main Idea: You pay a market-maker to buy or sell a stock. If the price of the stock is quoted at $50, you may have to pay the **ask price** (or **offer price**) of $50.50 to purchase the stock, or you may only be given the **bid price** of $49.50 to sell the shares you own.

The **bid-ask spread** in this case is $49.50 – $50.50. This terminology is from the market-maker's perspective.

Forwards

A **forward contract** is an agreement made today to buy or sell an item, the underlying asset, at a specified date in the future (at time *T*), the expiration date, for a specified price, the forward price, $F_{0,T}$. The buyer of the forward contract has a long position and is obligated to buy the underlying asset for $F_{0,T}$ on the expiration date and the seller of the forward contract has a short position and is obligated to sell the underlying asset for $F_{0,T}$ on the expiration date, regardless of the spot price, S_T, on the expiration date. The forward premium is defined to be the ratio of the forward price and the current asset price, $\frac{F_{0,T}}{S_0}$. The annualized forward premium, α, is found by solving $F_{0,T} = S_0 \cdot e^{\alpha \cdot T}$. Therefore $\alpha = \frac{1}{T} \cdot \ln\left(\frac{F_{0,T}}{S_0}\right)$.

The payoff to a forward contract is:

Payoff to long forward = $S_T - F_{0,T}$

Payoff to short forward = $F_{0,T} - S_T$

A **prepaid forward contract** is a forward contract in which the underlying asset is exchanged at time *T*, but the contract is paid for at time 0. The price of a prepaid forward contract is $F_{0,T}^P = S_0 - PV(dividends)$. Note that an assumption of no arbitrage implies that $F_{0,T} = F_{0,T}^P \cdot e^{r \cdot T}$, where *r* is the continuously compounded risk-free interest rate.

Special Cases of annualized forward premiums:

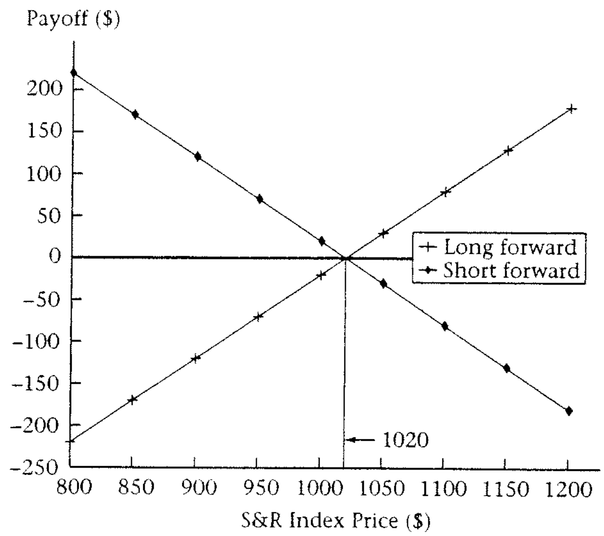
1. If the asset pays no dividends, then $F_{0,T}^P = S_0$, and so $\alpha = r$
2. If the asset pays continuous dividends at rate δ, then $PV(dividends) = S_0(1 - e^{-\delta \cdot T})$. Therefore $F_{0,T}^P = S_0 \cdot e^{-\delta \cdot T}$, and so $F_{0,T} = S_0 \cdot e^{\alpha \cdot T} \Rightarrow S_0 \cdot e^{-\delta \cdot T} \cdot e^{r \cdot T} = S_0 \cdot e^{\alpha \cdot T} \Rightarrow \alpha = r - \delta$

Payoff Diagrams for Forward Contracts:

Long and Short 6-month Forward Positions with Current Price, $S_0 = \$1000$, and Forward Price, $F_{0,0.5} = 1020$

FIGURE 2.2

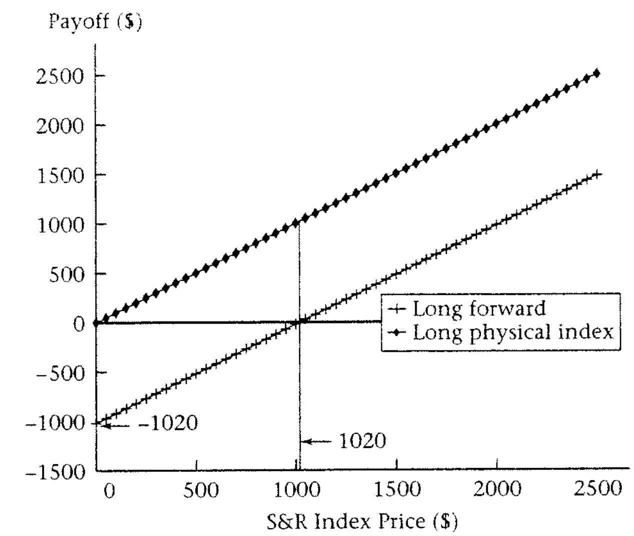
Long and short forward positions on the S&R 500 index.



Comparison of Long Forward verses Long Position in Underlying Asset

FIGURE 2.3

Comparison of payoff after 6 months of a long position in the S&R index versus a forward contract in the S&R index.

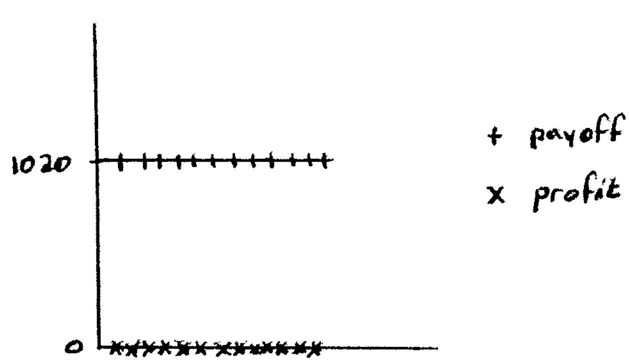


A **Profit Diagram** subtracts the future value of the investment in the position, at the risk-free interest rate, from the payoff. Since there is no initial investment for a forward contract, its profit diagram is the same as the payoff diagram for a forward contract.

Payoff and Profit at redemption (expiration) for a Zero-Coupon Bond Redeemable at 1020 in 6-months and bought to yield the risk-free 2% seir

Payoff = 1020

Profit = 0

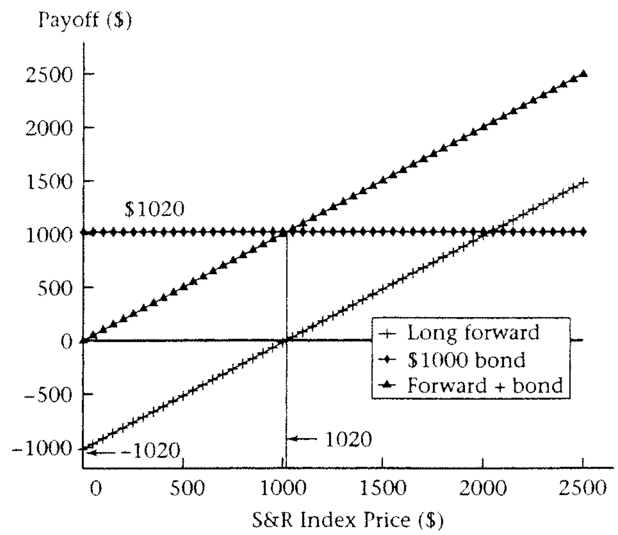


Payoff Diagram for Long Forward ($F_{0,0.5} = 1020$) + above Zero-Coupon Bond

Note: The profit diagram looks just like profit diagram for the long forward since the profit diagram for a zero-coupon bond is a horizontal line at 0; i.e. ignore the bond to get the profit diagram for a long forward + bond

FIGURE 2.4

Payoff diagram for a long S&R forward contract, together with a zero-coupon bond that pays $1020 at maturity. Summing the value of the long forward plus the bond at each S&R Index price gives the line labeled "Forward + Bond."



Note: Payoff to long forward + long bond (i.e. long zero-coupon bond)
$= (S_T - 1020) + 1020 = S_T$
= payoff to long underlying (long position in underlying asset)

From the note we have
long forward + long bond = long underlying
⇒ long forward = long underlying – long bond
⇒ long forward = long underlying + short bond

Being in a short bond position is equivalent to taking out a loan. Therefore the verbal interpretation of the last line in the implications is that we can replicate a forward contract by borrowing the money to buy the underlying asset. Replicating a contract by using combinations of other contracts is called creating a synthetic contract. Therefore, we have that a loan and asset purchase forms a **synthetic forward contract**.

cash-and-carry and **reverse cash-and-carry** are the following hedged positions:

cash-and-carry = long underlying + short forward
reverse cash-and-carry = short underlying + long forward

cash-and-carry and reverse cash-and-carry payoff:
cash-and-carry payoff = $S_T + (F_{0,T} - S_T) = F_{0,T}$
reverse cash-and-carry payoff = $-S_T + (S_T - F_{0,T}) = -F_{0,T}$

If the asset pays continuous dividends at rate δ, which may be 0 in which case there are no dividends, then under a no arbitrage assumption $F_{0,T} = S_0 \cdot e^{(r-\delta)\cdot T}$. Therefore it is possible to create cash-and-carry arbitrage if a forward price $F_{0,T}$ is available such that $F_{0,T} > S_0 \cdot e^{(r-\delta)\cdot T}$, and it is possible to create reverse cash-and-carry arbitrage if a forward price $F_{0,T}$ is available such that $F_{0,T} < S_0 \cdot e^{(r-\delta)\cdot T}$.

Futures Contracts

A futures contract is similar to a forward contract. Futures, however, are generally restricted to certain financial instruments and commodities. Also, with a futures contract, the investor is required to maintain a **margin account**, that is used to give assurances that the contract will be completed. Interest is earned on the margin. The balance of the margin account is updated periodically, usually daily, by what is called a **mark to market**. There is a **maintenance margin**, which is an amount that the balance of the margin account is required to exceed. If the balance of the margin account drops below the maintenance margin, then there will be a **margin call** to the investor, which means the investor will be required to deposit additional funds into the margin account in order to keep the contract open. The contract can be closed by the investor at any time.

Example: Gold futures are traded through 100 ounce contracts. Per contract, the initial margin is \$3375 and the maintenance margin is \$2500. At the end of trading on December 5, 2006, an investor takes a long position on one gold futures contract that has a June 2007 delivery. Mark to market is performed monthly and interest on the margin is credited at $i^{(12)} = 12\%$, none of which is realistic. You are given the following gold futures prices on monthly anniversary dates. The calculations of the margin balance are described below the table.

	Gold Futures Price (per ounce)	Price Change	Margin Balance
December 5, 2006	660.30	–	3,375
January 5, 2007	648.20	–12.10	2,198.75 3,375
February 5, 2007	649.20	1.00	3,508.75

The contract is opened with a deposit of \$3375 into the margin account. The initial contract value is \$66,030. At the close of trading on January 5, 2007, the contract value is \$64,820 and after marking to market the margin balance is $3375(1.01) - 100(12.1) = 2198.75 < 2500$. A margin call is made, and if the investor wants to keep the contract open, a deposit of $3375 - 2198.75 = 1176.25$ is required. Similarly, at the close of trading on February 5, 2007, the contract value is \$64,920 and after marking to market the margin balance is $3375(1.01) + 100(1.00) = 3508.75$. No margin call is made.

Options

A **call option** is an agreement made today that gives the buyer of the option the right, but not the obligation, to buy the underlying asset at a specified price, K, called the strike or exercise price, at or by a specified date (see Exercise Style below). *We will focus only on European-Style options.* If the option is not exercised at the expiration date, then it becomes worthless. The buyer of the call option has a long position in the call and the seller of the call option has a short position in the call. If the buyer of the call option wants to exercise the option, then the seller of the call option must sell the underlying asset at the strike price. Clearly, the buyer of the call option will exercise the option if the spot price, S_T, at time T, the exercise date, exceeds the strike price; i.e. if $S_T > K$. If $S_T < K$, then the buyer of the call option will let the option expire without exercising it. The buyer of the call option must pay the seller of the call option an option premium, $C_0 = \text{Call}(K,T)$, when the option is purchased. (Compare to forward contract.)

Exercise Style:

European-Style: the option can be exercised only at the expiration date

American-Style: the option can be exercised at any time during the life of the option (at any time prior to the expiration date)

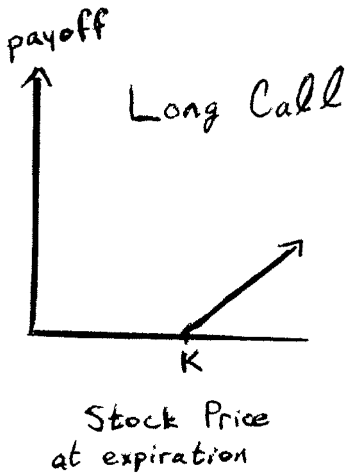
Bermudan-Style: the option can be exercised during specific periods during the life of the option, but not at any time

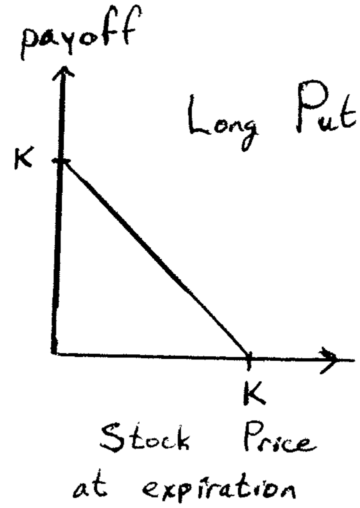
A **put option** is an agreement made today that gives the buyer of the option the right, but not the obligation, to sell the underlying asset at the strike price, K, at time T, the exercise date. If the option is not exercised at time T, then it becomes worthless. The buyer of the put option has a long position in the put and the seller of the put option has a short position in the put. If the buyer of the put option wants to exercise the option, then the seller of the put option must buy the underlying asset at the strike price. Clearly, the buyer of the put option will exercise the option if $S_T < K$. If $S_T > K$ then the buyer of the put option will let the option expire without exercising it. The buyer of the put option must pay the seller of the put option an option premium, $P_0 = \text{Put}(K,T)$, when the option is purchased. (Compare to short-selling.)

An option buyer has purchased the option. The option seller is said to be the *option writer* and is said to have *written* the option. An option writer has a short position in the option.

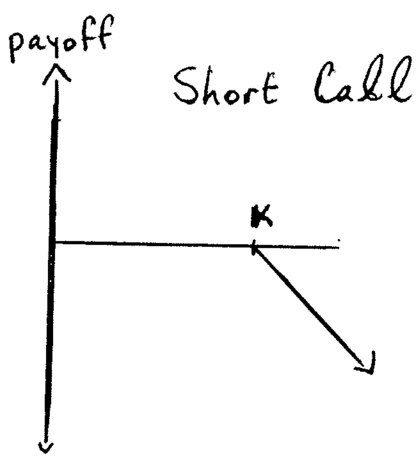
Payoff and Payoff Diagrams for Call and Put Options:

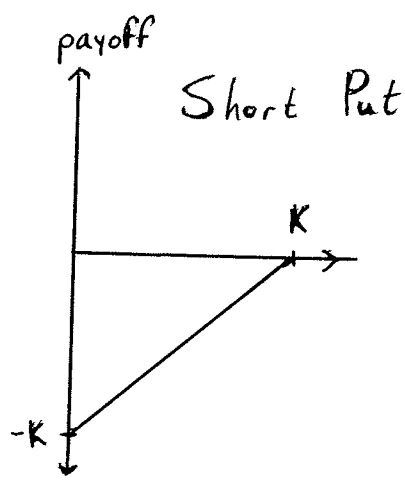
Long call payoff = max [0 , S_T – K]
Long put payoff = max [0, K – S_T]





Short call payoff = – max [0, S_T – K]
Short put payoff = – max [0, K – S_T]





Notes:

1. Long call payoff + Short call payoff = 0 and
   Long put payoff + Short put payoff = 0. (It's a zero-sum game.)
2. An option is **in-the-money** if it would have a positive payoff if exercised immediately. Similar definitions apply for **out-of-the-money** and **at-the-money** options

Profit and Profit Diagrams for Call and Put Options:

Denote by FV(option premium) the future value at the exercise date of the option premium, using the risk-free interest rate. Often the risk-free interest rate is given as a continuously compounded interest rate, r, and so

$$\text{FV(call option premium)} = C_0\, e^{rT}$$
$$\text{FV(put option premium)} = P_0\, e^{rT}$$

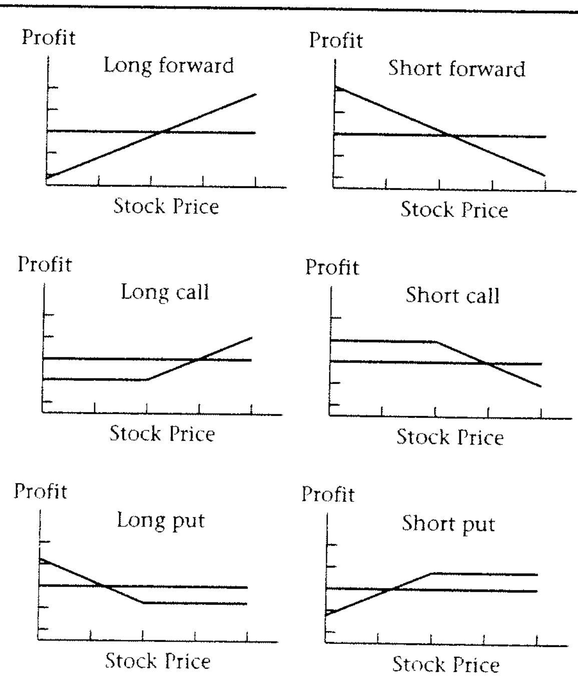
Long option profit = Long option payoff – FV(option premium) since the one with the long option position has purchased the option by *paying* the option premium when the option was purchased.

Likewise, Short option profit = Short option payoff + FV(option premium) since the one with the short option position has written the option and *collected* the option premium when the option was purchased.

Profit Diagrams:

FIGURE 2.15

The basic profit diagrams: long and short forward, long and short call, and long and short put.



Long/Short Positions with respect to the underlying asset

The purchaser of a forward contract is said to have a long forward position and has the obligation to *buy* the underlying asset at a future date. Therefore a long forward position implies a long position in the underlying asset. Likewise a short forward position implies a short position in the underlying asset.

The purchaser of a call option is said to have a long call position and has the right, but not the obligation, to *buy* the underlying asset at a future date. Therefore a long call position implies a long position in the underlying asset. Likewise a short call position implies a short position in the underlying asset.

The purchaser of a put option is said to have a long put position and has the right, but not the obligation, to *sell* the underlying asset at a future date. Therefore a long put position implies a *short* position in the underlying asset. Likewise a short put position implies a *long* position in the underlying asset.

Note: An investor who is long in the underlying asset will profit with an increase in asset value while an investor who is short in the underlying asset will profit with a decrease in asset value.
(See profit diagrams on previous page.)