Section 5 - Bonds

Bond Notation

P = the price
F = the par value, or face amount
C = the redemption value
r = the coupon rate (problems always give nominal rates, convert to eir's)
Fr = the coupon amount
g = the modified coupon rate, obtained by solving $Fr = Cg$ for g
i = the yield rate
n = the number of coupons

The following formula is used to find the price of a bond. Note that the only interest rate involved in this calculation is the yield rate i.

(Basic Formula) $P = Fr \cdot a_{\overline{n}|} + C\upsilon^n$

Remarks:
1. If $P = C$ then the coupons consisting entirely of interest.

2. If $P > C$ then the bond is said to be bought at a premium of $P - C$. Each coupon payment consists of partly interest earned on the bond and partly an adjustment of the price downward (a write-down of the premium) toward C. This write-down of premium is also called amortization of premium.

3. If $P < C$ then the bond is said to be bought at a discount of $C - P$. Each coupon payment consists of partly interest earned on the bond and partly an adjustment of the price upward (a write-up of the discount) toward C. This write-up of discount is also called amortization of discount.

Bond Amortization Schedule

We capture interest earned and amortization of premium (or discount) in a bond amortization schedule. Notice that this amortization schedule is completely analogous to the loan amortization schedule from last session.

Period	Coupon	Interest Earned	Principal Adjustment	Book Value of Bond
0				$BV_0 = P = Fr \cdot a_{\overline{n}\vert} + C\upsilon^n$
1	Fr	$I_1 = i \cdot BV_0$	$P_1 = Fr - I_1$	$BV_1 = BV_0 - P_1 = Fr \cdot a_{\overline{n-1}\vert} + C\upsilon^{n-1}$
2	Fr	$I_2 = i \cdot BV_1$	$P_2 = Fr - I_2$	$BV_2 = BV_1 - P_2 = Fr \cdot a_{\overline{n-2}\vert} + C\upsilon^{n-2}$
. . .	. . .	. . .	. . .	. . .
n	Fr	$I_n = i \cdot BV_{n-1}$	$P_n = Fr - I_n$	$BV_n = C$
Total	$n \cdot Fr$	$n \cdot Fr - (P - C$	$P - C$	

Remarks:
1. The book value is the value just after the coupon is paid.
2. The Principal Adjustment column will consist of negative values if the bond was bought at a discount.

Bond Values Between Coupon Payment Dates:

When a bond is sold between coupon payment dates, the new owner will receive the entire coupon on the following coupon payment date. The price the new owner pays reflects this fact.

B_{t+k}^{f} = the flat price (the amount actually paid) of a bond sold k periods after the t^{th} coupon has been paid. $(0 < k < 1)$

Fr_k = the accrued coupon, which is the amount of the next coupon payment that is allocated to the seller of the bond

$B_{t+k}^{m} = B_{t+k}^{f} - Fr_k$ = the market price, which is the price of a bond sold k periods after the t^{th} coupon has been paid, excluding the accrued coupon. The graph of the market price over time is continuous.

The following table shows three different methods for calculating bond values between coupon payment dates.

	Flat Price B_{t+k}^{f}	Accrued Coupon Fr_k	Market Price B_{t+k}^{m}
Theoretical Method	$B_t(1+i)^k$	$Fr \cdot s_{\overline{k}\vert}$	$B_t(1+i)^k - Fr \cdot s_{\overline{k}\vert}$
Practical Method	$B_t(1+k \cdot i)$	$Fr \cdot k$	$B_t(1+k \cdot i) - Fr \cdot k$
Semi-Theoretical Method	$B_t(1+i)^k$	$Fr \cdot k$	$B_t(1+i)^k - Fr \cdot k$

Remark: The value B_t is the book value just after the t^{th} coupon payment.

$$B_t = Fr \cdot a_{\overline{n-t}\vert} + C\upsilon^{n-t}$$

We will illustrate the concept of a callable bond in the examples.