Section 4 – Yield Rates and Loan Repayments

Yield Rate – The yield rate on an investment (aka the internal rate of return) is the interest rate at which the present value of returns from an investment is equal to the present value of contributions into the investment.

Reinvestment Rates Problems – These are problems in which interest on an investment is calculated based on a given rate, but then the interest amounts are reinvested in another account at a different reinvestment rate.

Interest Measurement Of A Fund:

1. Dollar Weighted Interest Rate
   (use simple interest from transaction date)

   $i_{DW} = \dfrac{I}{Exposure}$, where

   I = the amount of interest earned in the fund during the year, and

   Exposure = the weighted value of the amounts earning interest, where the weight on the amount at time t is the number of years that interest is being earned on that particular amount.

2. Time Weighted Interest Rate

   $i_{TW} = \dfrac{B_1^{bef}}{B_0} \cdot \dfrac{B_2^{bef}}{B_1^{aft}} \cdot \cdots \cdot \dfrac{B_n}{B_{n-1}^{aft}} - 1$, where

   B_k^{bef} = the balance in the account before the k^{th} transaction, and

   B_k^{aft} = the balance in the account after the k^{th} transaction

3. Portfolio Method and Investment Year Method

   We will illustrate these methods in the examples.

Loans:

Repayments Methods:

1. Amortization Method – The loan is repaid with equal periodic payments. The amount of interest paid each period and the amount of principal repaid each period change with each payment.

2. Sinking Fund Method – The amount of interest paid on the principal each period is constant, and the principal is repaid in full at the end of the term by accumulating the necessary amount in a separate (sinking) fund. The net amount of interest during each installment is the difference between the amount of interest paid on the principal and the amount of interest earned on the sinking fund for that installment.

3. No-Name Method – The amount of principal repaid each period is constant, and interest on the unpaid balance is also paid each period.

Note: If the interest rate on the sinking fund is equal to the loan interest rate, then the sinking fund method is equivalent to the amortization method.

More on the Amortization Method
Notation:

L = original loan amount
R = the periodic payment
n = number of payments
i = loan interest rate (as an eir with period matching payment period)
B_k = loan balance just after the payment at time k
P_k = amount of principal repaid by the payment at time k
I_k = amount of interest paid by the payment at time k

Calculating Outstanding Loan Balances

$B_k = L(1+i)^k - Rs_{\overline{k|}}$ (Retrospectively)

$B_k = Ra_{\overline{n-k|}}$ (Prospectively)

Loan Amortization Schedule

Time	Payment Amount	Interest Paid	Principal Repaid	Outstanding Balance
0				$L = B_0 = Ra_{\overline{n}\vert} = R\dfrac{1-v^n}{i}$
1	R	$I_1 = i \cdot B_0 = R(1-v^n)$	$P_1 = R - I_1 = Rv^n$	$B_1 = Ra_{\overline{n-1}\vert} = R\dfrac{1-v^{n-1}}{i}$
2	R	$I_2 = i \cdot B_1 = R(1-v^{n-1})$	$P_2 = R - I_2 = Rv^{n-1}$	$B_2 = Ra_{\overline{n-2}\vert} = R\dfrac{1-v^{n-2}}{i}$
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n	R	$I_n = i \cdot B_1 = R(1-v)$	$P_n = R - I_n = Rv$	$B_n = Ra_{\overline{0}\vert} = 0$
Total	$n \cdot R$	$n \cdot R - Ra_{\overline{n}\vert}$	$Ra_{\overline{n}\vert}$	