Section 3 – Tweaks to Basic Level Annuities

Tweak 1: Annual payments vary from year to year (Two ways)

1) Payments form a geometric progression. In this case value each payment and use facts about valuing geometric sums and series.

2) Payments form an arithmetic progression. The general annuity immediate case is payments of P at time 1, $P + Q$ at time 2, $P + 2Q$ at time 3, …, and a final payment of $P + (n\text{-}1)Q$ at time n. The formulas for the present value, PV at time 0, and the accumulated value, AV at time n, of this annuity are:

$$PV = Pa_{\overline{n}|} + Q\frac{a_{\overline{n}|} - n\upsilon^n}{i}$$

$$AV = PV(1+i)^n = Ps_{\overline{n}|} + Q\frac{s_{\overline{n}|} - n}{i}$$

There are three special cases of the above formulas:

Case 1: $P = 1$ and $Q = 1$ (Increasing Annuity)

$$PV = (Ia)_{\overline{n}|} = \frac{\ddot{a}_{\overline{n}|} - n\upsilon^n}{i}$$

$$AV = (Is)_{\overline{n}|} = (Ia)_{\overline{n}|}(1+i)^n = \frac{\ddot{s}_{\overline{n}|} - n}{i}$$

Case 2: $P = n$ and $Q = -1$ (Decreasing Annuity)

$$PV = (Da)_{\overline{n}|} = \frac{n - a_{\overline{n}|}}{i}$$

$$AV = (Ds)_{\overline{n}|} = (Da)_{\overline{n}|}(1+i)^n = \frac{n(1+i)^n - s_{\overline{n}|}}{i}$$

Case 3: $n = \infty$ (Increasing Perpetuity)

$$PV = \frac{P}{i} + \frac{Q}{i^2}$$