Extra Problems for Test 4

1. An insurance company is making annual payments under the settlement provisions of a personal injury lawsuit. A payment of 24,000 has just been made and ten more payments are due. Future payments are indexed to the Consumer Price Index which is assumed to increase at 5% per year. Find the present value of the remaining obligation if the rate of interest assumed is 8%.

   a. 202,325 b. 204,725 c. 206,225 d. 210,875 e. 212,125

2. Consider a yield curve defined by the following equation:

$$i_k = 0.09 + 0.002k - 0.001k^2$$

where i_k is the annual effective rate of return for zero coupon bonds with maturity of k years.

Let j be the one-year effective rate during year 5 that is implied by this yield curve.

Calculate j.

   a. 4.7% b. 5.8% c. 6.6% d. 7.5% e. 8.2%

3. A bond will pay a coupon of 100 at the end of each of the next three years and will pay the face value of 1000 at the end of the three-year period. The bond's duration (Macaulay Duration) when valued using an annual effective interest rate of 20% is X.

Calculate X.

   a. 2.61 b. 2.70 c. 2.77 d. 2.89 e. 3.00

Extra Problems for Test 4

4. John purchased three bonds to form a portfolio as follows:

Bond A has semiannual coupons at 4%, a duration of 21.46 years, and was purchased for 980.

Bond B is a 15 year bond with a duration of 12.35 years and was purchased for 1015.

Bond C has a duration of 16.67 years and was purchased for 1000.

Calculate the duration (in years) of the portfolio at the time of purchase

a. 16.62 b. 16.67 c. 16.72 d. 16.77 e. 16.82

5. The current price of an annual coupon bond is 100. The derivative of the price of the bond with respect to the yield to maturity is –700. The yield to maturity is an annual effective rate of 8%.

Calculate the duration of the bond.

a. 7.00 b. 7.49 c. 7.56 d. 7.69 e. 8.00

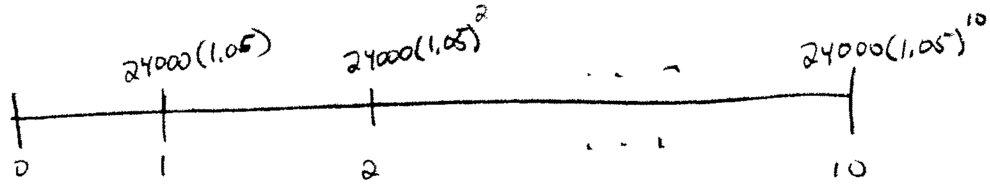
6. An insurance company accepts an obligation to pay 10,000 at the end of each year for 2 years. The insurance company purchases a combination of the following two bonds at a total cost of X in order to exactly match its obligation:

(i) 1-year 4% annual coupon bond with a yield rate of 5%
(ii) 2-year 6% annual coupon bond with a yield rate of 5%

Calculate X.

a. 18,564 b. 18,574 c. 18,584 d. 18,594 e. 18,604

1)



$v = \frac{1}{1.08}$

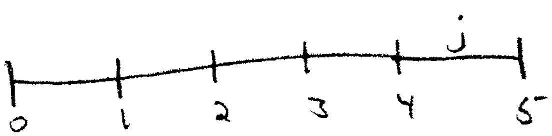
$$PV \overset{VEP}{=} 24000(1.05)v + 24000(1.05)^2 v^2 + \cdots + 24000(1.05)^{10} v^{10}$$

Since $1.05v = \frac{1.05}{1.08} < 1$, think of $v^{new} = \frac{1.05}{1.08}$

$$\Longrightarrow i^{new} = \frac{1.08}{1.05} - 1 = \frac{.03}{1.05}$$

$$= 24000 \cdot a_{\overline{10}|i^{new}} \doteq 206{,}225$$

2) The i_k are k-year spot rates



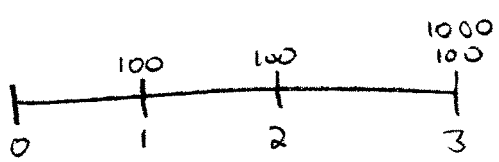
$$(1+i_5)^5 = (1+i_4)^4(1+j)$$

$i_5 = .075$

$i_4 = .082$

$$\therefore j = \frac{(1.075)^5}{(1.082)^4} - 1 \doteq 4.79\%$$

3)



$$X = MaD = \frac{100v + 2(100)v^2 + 3(1100)v^3}{100v + 100v^2 + 1100v^3} \overset{v=\frac{1}{1.2}}{=\!=} 2.70$$

4) $$MaD_P = \frac{P_A}{P_P} \cdot MaD_A + \frac{P_B}{P_P} \cdot MaD_B + \frac{P_C}{P_P} \cdot MaD_C$$

$$= \frac{980}{980+1015+1000}(21.46) + \frac{1015}{980+1015+1000}(12.35) + \frac{1000}{980+1015+1000}(16.67)$$

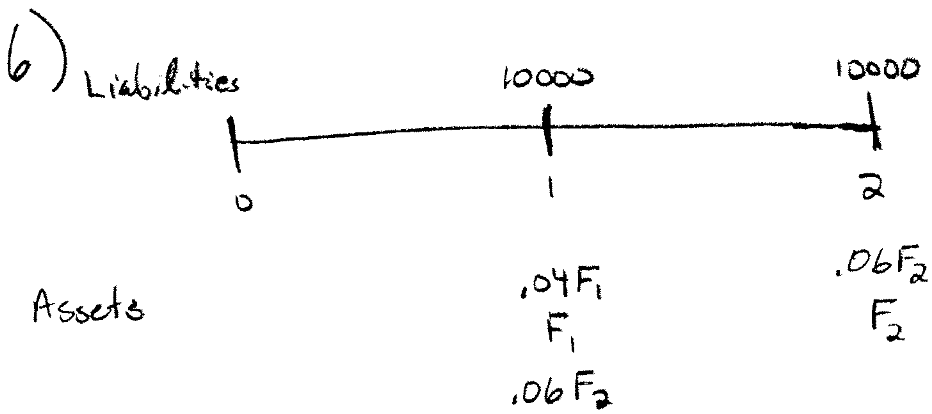
$$\doteq 16.77$$

5) $$MaD = MoD \cdot (1+i)$$

$$MoD = -\frac{P'(i)}{P(i)} = -\frac{-700}{100} = 7$$

$$\therefore MaD = 7(1.08) = 7.56$$

6)



F_1 = face value of the 1-year bond

F_2 = face value of the 2-year bond

$$10000 = .04F_1 + F_1 + .06F_2 = 1.04F_1 + .06F_2$$

$$\therefore 10000 = .06F_2 + F_2 = 1.06F_2$$

$$\Rightarrow F_1 = 9071.12, \quad F_2 = 9433.96$$

$$\therefore P_1 = 9071.12(.04)a_{\overline{1}|5\%} + 9071.12v_{.05} = 1.04(9071.12)v_{.05} = 8984.73$$

and $$P_2 = 9433.96(.06)a_{\overline{2}|5\%} + 9433.96v_{.05}^2 = 9609.94$$

$$X = P_1 + P_2 \doteq 18594$$

Note: Since both bonds are bought to yield 5%, $X = 10000v_{.05} + 10000v_{.05}^2$