MAP 4170 Name:_____
Test 4 Date: August 1, 2012

Show sufficient work and clearly mark your answers. Each problem is worth 10 points.

1. The following are the current price of $1,000 zero-coupon bonds:

Term to maturity	Price
1	$937.52
2	$859.10
3	$x

If the one year forward rate for year 3 is 8%, determine x.

(A) 805.12 (B) 795.46 (C) 792.38 (D) 801.42 (E) 868.07

2. You are given the information about two bonds that will mature in four years at par

	Bond A	Bond B
Par value	$1,000	$750
Annual coupon rate	4%	8%
Price	$700	$500

Determine the four-year spot rate.

(A) 9.33% (B) 8.06% (C) 10.67% (D) 10.32% (E) 9.52%

3. The modified duration of a perpetuity immediate with level annual payments of 1 is 16, when calculated using an annual effective interest rate of i. Determine i.

(A) 6.05% (B) 6.15% (C) 6.25% (D) 6.35% (E) 6.45%

4. Data for a two bond portfolio is:

Bond A is a 100 face value 20 year bond, redeemable at par, with annual coupons of 9. It has Macaulay duration of 8 years using an annual effective interest rate of 9%.

Bond B is a 10 year zero-coupon bond that can be purchased for 100 to yield 9%.

In what range is the Modified duration, D, of the two bond portfolio using an annual effective interest rate of 9%?

(A) $D < 8.2$ (B) $8.2 < D < 9.2$ (C) $9.2 < D < 10.2$
(D) $10.2 < D < 11.2$ (E) $11.2 < D < 12.2$

5. A 20 – year annuity immediate has annual payments that increase by 5% each year. Determine the duration of this annuity using an annual effective interest rate of 5%.

(A) 10 (B) 10.5 (C) 11 (D) 11.5 (E) 12

6. A $1,000 4 – year par value bond yields an effective annual interest rate of 6%. Coupons are paid on an annual basis at a rate of 4% per year. Determine the convexity of the bond.

(A) 15.96 (B) 15.90 (C) 16.30 (D) 16.42 (E) 16.60

7. An insurance company is making annual payments under the settlement provisions of a person injury lawsuit. A payment of 2,400 has just been made and twenty more payments are due. Future payments are indexed to the Customer Price Index, which is assumed to increase at 5% per year. Find the present value of the remaining obligation if the rate of interest assumed is 4%.

(A) 52,253 (B) 53,232 (C) 53,154 (D) 50,125 (E) 51,235

8. Sam has liabilities of $500 due in 2 year from now and another $1,000 3 years from now. He is to receive assets 2 years from now and 4 years from now in such a way as to have the assets have the same present value and duration as the liabilities when using an annual discount factor of 0.8. Determine the amount of assets Sam is to receive 4 years from now.

(A) 585 (B) 600 (C) 605 (D) 615 (E) 625

9. You are given the following table of interest rates:

Calendar year of original investment	Investment year rates		Portfolio rates
y	i_1^y	i_2^y	i^{y+2}
2000	5.25%	5.25%	5.40%
2001	5.35%	5.35%	5.65%
2002	5.45%	i	5.10%
2003	5.45%	5.45%	5.34%
2004	5.50%	5.35%	i
2005	5.50%	5.55%	5.65%

$1,000 is invested on 01/01/2002 and another $1,000 is invested on 01/01/2003. The total amount as of 12/31/2005 is $2397.2. Determine i.

(A) 5% (B) 5.1% (C) 5.2% (D) 5.3% (E) 5.6%

10. A deposit of $100 is made into an account on January 1, 2010. On April 1, 2010, a deposit of $X is made. The account value before the deposit is $120. There are no other transactions during 2010, and the account balance on December 31, 2010, is $130. The time weighted return during 2010 is 10%. Determine the dollar weighted return during 2010.

(A) 7.03% (B) 6.98% (C) 7.05% (D) 7.01% (E) 7.00%

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Test 4

Name: KEY
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Show sufficient work and clearly mark your answers. Each problem is worth 10 points.

1. The following are the current price of $1,000 zero-coupon bonds:

Term to maturity	Price
1	$937.52
2	$859.10
3	$x

If the one year forward rate for year 3 is 8%, determine x. $f_{[2,3]} = .08$

(A) 805.12 (B) 795.46 (C) 792.38 (D) 801.42 (E) 868.07

$$859.10(1+S_2)^2 = 1000$$

$$1000 = x(1+S_3)^3 = x(1+S_2)^2(1+f_{[2,3]})$$

$$\Rightarrow x\left(\frac{1000}{859.10}\right)(1.08) = 1000$$

$$\Rightarrow x \doteq 795.46$$

2. You are given the information about two bonds that will mature in four years at par

	Bond A	Bond B
Par value	$1,000	$750
Annual coupon rate	4%	8%
Price	$700	$500

Determine the four-year spot rate.

Fr = 40, C = 1000 Fr = 60, C = 750

(A) 9.33% (B) 8.06% (C) 10.67% (D) 10.32% (E) 9.52%

$$\begin{cases} (700 = 40a_{\overline{4}|} + 1000v^4) * 1.5 \\ 500 = 60a_{\overline{4}|} + 750v^4 \end{cases}$$

$$1050 = 60a_{\overline{4}|} + 1500v^4$$
$$-(500 = 60a_{\overline{4}|} + 750v^4)$$
$$550 = 750v^4 = \frac{750}{(1+S_4)^4} \Rightarrow S_4 \doteq 8.06\%$$

3. The modified duration of a perpetuity immediate with level annual payments of 1 is 16, when calculated using an annual effective interest rate of i. Determine i.

(A) 6.05% (B) 6.15% (C) 6.25% (D) 6.35% (E) 6.45%

$$\text{Mac}D = \frac{(Ia)_{\overline{\infty}|}}{a_{\overline{\infty}|}} = \ddot{a}_{\overline{\infty}|}$$

$$\text{Mod}D = \text{Mac}D \cdot v = \ddot{a}_{\overline{\infty}|} \cdot v = a_{\overline{\infty}|} = \frac{1}{i} = 16$$

$$\Rightarrow i = .0625$$

4. Data for a two bond portfolio is:

Bond A is a 100 face value 20 year bond, redeemable at par, with annual coupons of 9. It has Macaulay duration of 8 years using an annual effective interest rate of 9%.

Bond B is a 10 year zero-coupon bond that can be purchased for 100 to yield 9%.

In what range is the Modified duration, D, of the two bond portfolio using an annual effective interest rate of 9%?

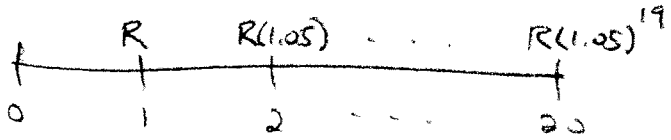
(A) D < 8.2 (B) 8.2 < D < 9.2 (C) 9.2 < D < 10.2
(D) 10.2 < D < 11.2 (E) 11.2 < D < 12.2

$$\text{Mac}D_A = 8 \qquad P_A = 9\,a_{\overline{20}|.09} + 100\,v_{.09}^{20} = 100$$

$$\text{Mac}D_B = 10 \qquad P_B = 100$$

$$\therefore \text{Mac}D_{\text{Portfolio}} = \frac{100}{200}(8) + \frac{100}{200}(10) = 9$$

$$\therefore \text{Mod}D_{\text{Portfolio}} = 9\,v_{.09} \doteq 8.26$$



5. A 20 – year annuity immediate has annual payments that increase by 5% each year. Determine the duration of this annuity using an annual effective interest rate of 5%.

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$$MacD = \frac{Rv + 2R(1.05)v^2 + \cdots + 20R(1.05)^{19}v^{20}}{Rv + R(1.05)v^2 + \cdots + R(1.05)^{19}v^{20}} \qquad v = \frac{1}{1.05}$$

$$= \frac{Rv(1 + 2 + \cdots + 20)}{Rv(1 + 1 + \cdots + 1)} = \frac{\frac{21}{2}(20)}{20} = 10.5$$

6. A \$1,000 4 – year par value bond yields an effective annual interest rate of 6%. Coupons are paid on an annual basis at a rate of 4% per year. Determine the convexity of the bond.

(A) 15.96 (B) 15.90 (C) 16.30 (D) 16.42 (E) 16.60

$$P(i) = 40(1+i)^{-1} + 40(1+i)^{-2} + 40(1+i)^{-3} + 1040(1+i)^{-4}$$

$$P'(i) = -40(1+i)^{-2} - 80(1+i)^{-3} - 120(1+i)^{-4} - 4160(1+i)^{-5}$$

$$P''(i) = 80(1+i)^{-3} + 240(1+i)^{-4} + 480(1+i)^{-5} + 20800(1+i)^{-6}$$

$$P(.06) \doteq 930.70$$

$$P''(.06) \doteq 15279.14$$

$$C = \frac{P''}{P} \doteq 16.42$$

7. An insurance company is making annual payments under the settlement provisions of a person injury lawsuit. A payment of 2,400 has just been made and twenty more payments are due. Future payments are indexed to the Customer Price Index, which is assumed to increase at 5% per year. Find the present value of the remaining obligation if the rate of interest assumed is 4%.

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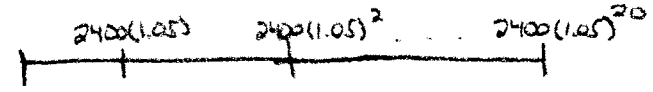

$$PV \overset{VEP}{=} \frac{2400(1.05)}{1.04}\left(1 + \frac{1.05}{1.04} + \cdots\right) \text{ (20 terms)}$$

$$= \frac{2400(1.05)}{1.04} s_{\overline{20}|\left(\frac{1.05}{1.04}-1\right)} \doteq 53154$$

8. Sam has liabilities of \$500 due in 2 year from now and another \$1,000 3 years from now. He is to receive assets 2 years from now and 4 years from now in such a way as to have the assets have the same present value and duration as the liabilities when using an annual discount factor of 0.8. Determine the amount of assets Sam is to receive 4 years from now.

$v = .08 =$ annual discount factor

(A) 585 (B) 600 (C) 605 (D) 615 (E) 625

$$PV: \quad 500v^2 + 1000v^3 = Xv^2 + Yv^4 \quad \times 2$$

$$PV * MacD: \quad 500(2)v^2 + 1000(3)v^3 = X(2)v^2 + Y(4)v^4$$

$$-(1000v^2 + 2000v^3 = 2Xv^2 + 2Yv^4)$$

$$1000v^2 + 3000v^3 = 2Xv^2 + 4Yv^4$$

$$+$$

$$1000v^3 = 2Yv^4$$

$$\Rightarrow Y = 500(1+v) = 500 \cdot \frac{1}{v} = \frac{500}{.8}$$

$$\therefore Y = 625$$

9. You are given the following table of interest rates:

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(A) 5% (B) 5.1% (C) 5.2% (D) 5.3% (E) 5.6%

$$2397.2 = 1000(1.0545)(1+i)(1.0510)(1.0534) + 1000(1.0545)^2(1.0534)$$

$$\Rightarrow i \doteq .05$$

10. A deposit of $100 is made into an account on January 1, 2010. On April 1, 2010, a deposit of $X is made. The account value before the deposit is $120. There are no other transactions during 2010, and the account balance on December 31, 2010, is $130. The time weighted return during 2010 is 10%. Determine the dollar weighted return during 2010.

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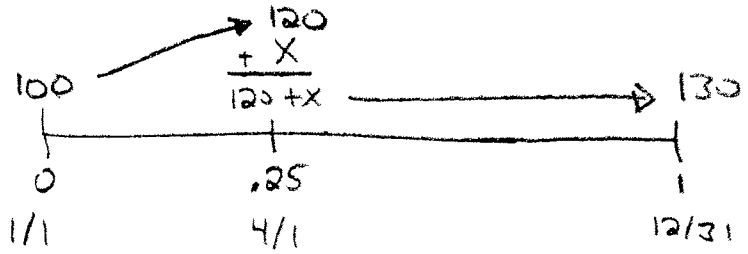

$$1.1 = \frac{120}{100} \cdot \frac{130}{120+X} \Rightarrow X \doteq 21.82$$

$i = i_{DW}$: $100(1+i) + X(1+.75i) = 130$

$$X = 21.82$$

$$\Rightarrow i \doteq .0703$$